3 Vols 1076

THE MATHEMATICAL REPOSITORY.

CONTAINING

Analytical Solutions

OF NEAR

Five Hundred Questions,

MOSTLY SELECTED FROM

SCARCE and VALUABLE AUTHORS.

DESIGNED

As Examples to MAC-LAURIN's and other Elementary Books of Algebra;

AND

To conduct BEGINNERS to the more difficult

Properties of Numbers.

By JAMES DODSON,
Late Master of the Royal Mathematical School in Christ's Hospital, and F.R.S.

The SECOND EDITION.

LONDON,
Printed for J. NOURSE, in the Strand,
Bookseller to His MAJESTY.
MDCCLXXV.

TO

Mr. Abraham De Moivre,

FELLOW of the ROYAL SOCIETIES

OF LONDON and BERLIN.

Sir,

IT is not a new custom for authors to dedicate their mathematical works, to gentlemen who are the most illustrious ornaments of mathematical sciences; and as the learned world have long since thought it justice to rank You among that number, it will, I hope, sufficiently apologize for this address, which I flatter myself you will favourably receive, on account of the likelihood there is, that these sheets will prove beneficial to students.

BESIDES the help that your celebrated works in general have afforded me, the appendix to your *Miscellanea Analytica* has furnished me with the method of approximating the sums of such series as are the reciprocals of the powers of arithmetical progressions.

M. *James Bernoulli* could not sum the series of the reciprocals of square numbers *; this was discovered by M. *John Bernoulli*, who, on that occasion, expresses a singular satisfaction, but confesses his inability to sum the series of the reciprocals of cube numbers †: The sum of both these series, and many others, naturally flow from an approximation you have given to sum the series $\frac{1}{1^m}+\frac{1}{2^m}+\frac{1}{3^m}$, *&c. ad infinitum*; and although you well knew that the *desiderata* of Messrs *Bernoulli* were included therein, yet you waved, not only the application, but the assumption of the honour thereof.

This, Sir, as it sufficiently shews the modest opinion you have of your own excellent performances, so it gives me this opportunity of declaring to the public, that I think myself singularly happy in being permitted to subscribe myself,

Sir,

Your most obliged,

And most humble Servant,

JAMES DODSON.

* *See his* Works, Vol. I. Page 398.

† *See his* Works, Vol. IV. Page 20, *&c.*

PRE-

PREFACE.

THE following ſheets contain a large collection of queſtions in common Algebra; *which (ſome few excepted) are diſpoſed in the following order.*

1. *Such queſtions, in the ſolutions of which only* Addition *and* Subtraction *of quantities are uſed.*

2. *Queſtions, which beſide the former operations, require the uſe of* Multiplication *and* Diviſion.

3. *Queſtions, wherein the doctrine of* Proporportion *is requiſite.*

4. *Such queſtions, producing* ſimple Equations, *wherein a more complex proceſs is neceſſary:* Where any of theſe ſeemed likely to occur in practice frequently, a *general Solution* is given.

5. *Queſ-*

5. *Questions, producing* Equations *of* simple Powers.

6. *Questions, that produce* adfected quadratic Equations, *solved by* completing the Square: *With a few Examples of Dr.* Halley's *Method of finding their roots by a*-logarithmical Process.

7. *Questions, that produce* adfected Equations of higher Powers.

THE method of finding divisors, delivered in Sir *Isaac Newton's* UNIVERSAL ARITHMETIC is here principally used; because it seems best adapted to find the roots of such equations, when those roots are whole numbers, or rational fractions: There are however, some examples of finding the roots of such cubic equations as want the second term by a method similar to *Cardan's*, or by Dr. *Halley's* logarithmical process depending thereon; but where neither of these methods appeared practicable, without a previous reduction, one of Dr. *Halley's* approximations to the root (called his *rational* and *irrational* theorems) is used, according as *this*, or *that*, seemed most applicable to the given numbers.

8. Indetermined questions; *as well those that are capable of innumerable answers, as those where*

where the number of answers in integers is limited.

The solutions of these questions are attempted, for the most part, in a manner different from what has been commonly used; and some solutions are given at large, which other writers have thought too operose to be inserted: A few of the questions usually called *Diophantine* are introduced toward the end of these, and are solved by the same principles.

9. *Questions, relating to* Arithmetical Progressions, *and other series derived from them: such as, their* squares, cubes, *&c. the different Series of* Figurate numbers; *and of those* Numbers *whose* second, third, fourth, *&c.* Differences *are equal*; *the* Combinations, Elections, Permutations *of Quantities*, &c.

10. *Questions, relating to* Simple Interest, Discompt, *&c.*

11. *Questions, relating to* Geometrical Progressions.

12. *Questions, relating to* Compound Interest; *and to the values of* Annuities *for* Time certain; *both in* Possession *and* Reversion.

13. *Ques-*

13. *Questions, relating to* Geometrical Progressions infinitely decreasing; *and to series of Fractions, the* Numerators *of which are* Numbers, *whose* 1st, 2d, 3d, &c. Differences *are equal, and their* Denominators *a* Geometrical Progression.

14. *The* Summation *of the several series of the* Reciprocals *of* Figurate Numbers; *and of other* Series *which can be obtained by a similar Process.*

15. *The several* Series *that are commonly used for making* Logarithms, *investigated by* common Algebra.

These Series were first exhibited by *Mercator*, from the Quadrature of the *Hyperbola*; afterward by Dr. *Halley*, from the Doctrine of *Ratiunculæ*, and by extracting the *Root* of an *Infinite Power*; by others, from the Doctrine of *Fluxions*, &c. Now, as Beginners in Mathematical Learning cannot soon be acquainted with any of the above Principles; and since a Table of *Logarithms* is useful, even at the first Entranee into these Studies, this Solution obtained by the Assumption of a Series, although it may not give sufficient Satisfaction as to the Invention, will prove the

Truth

Truth of those Methods which are used in constructing such Tables; and by Consequence enable the Operator to correct them if erroneous.

16. Approximations *to the* Sums *of the several* Series *of the* Reciprocals *of the* 1st, 2d, 3d, &c. Powers *of an* arithmetical Progression.

This manner of ranging the questions has been sometimes dispensed with, when there was an Opportunity thereby of bringing more Matter into less Space.

Variety of Authors have been carefully examined to collect Materials for this work; viz. Oughtred, Leybourn, Moore, Kersey, Wallis, Harriott, Parsons, Newton, Halley, Jones, De Moivre, Ward, Ronayne, Simpson, *&c. of our own Nation; and* Descartes, Alexander, Ozanam, John *and* James Bernoulli, Wolfius, Euler, *&c. of other Nations.*

From these Authors, and many others, has been selected much the greater Part of this Work. It is true, neither the Words made use of by an Author in expressing a Question, nor his Manner of Solution, have been strictly retained; it having

ing been endeavoured, in the Diction, to avoid both Ambiguity, and Superfluity; and in the Operation, to use that Kind of Process, which appeared to be most conducive to render the Solution intelligible, elegant, and concise; but how well this has been performed, must be left to superior Judgment.

In the Division and Arrangement of the several Solutions, the Author has endeavoured to imitate the Manner *he has observed in the Papers of the most excellent Mathematician* William Jones, *Esq.*

Thus are Students *furnished with a great Number of Examples, applicable to any Book of* Theory, *so disposed, that they may be* gradually *applied, and by that Means the Learner may be relieved, at* proper Intervals, *from the Fatigue of going through a whole* System *of* Operations, *before he can have so much as a Glimpse of their* Application *or* Use; *a Disadvantage very common to Books of Theory.*

Those Persons, *also, whose* rapid Genius, *carries them, almost as soon as they can solve an Equation, to the sublimer Speculations of* Fluxions, Quadratures, *&c. will possibly find* here, *something, even in* Common Algebra *worthy Notice*;

Notice; which they may have hitherto either neglected, or overlooked.

And if what is here done, should meet with a favourable Reception, from those, who (having duly examined the several Parts of Mathematical Learning) are the proper Judges *of Performances of this Kind, all the Purposes of this Publication will be answered; and Encouragement thereby given, for the farther Prosecution of a* Scheme *which the Author, and some of his* Mathematical Friends *have long since projected; and which, when accomplished, will be truly (what this is but in Part) The* MATHEMATICAL REPOSITORY.

January, 1747.

ERRATUM. Quest. II. line 2. *for* 16 *read* 18.

THE MATHEMATICAL REPOSITORY.

QUESTION I.

TWO travellers set out at the same time from *London* and *York*, whose distance is 150 miles; one of them goes 8, the other 7 miles a day: In what time will they meet?

Let - - -	$x =$	the time of their meeting;
Then - - -	$8x =$	the miles which the first travels;
And - - -	$7x =$	the miles which the second travels;
But (their sum)	$15x = 150$	(the distance of the two cities) by question:
Therefore	$x = 10$.	

QUEST. II. One bought 12 yards of cloth at a certain price; and another time 18 yards thereof at the same price; at which last time he paid 48 shillings more than before: What did he pay a yard?

Let - - - - - - -	$x =$	the price of 1 yd of cloth,
Then - - - -	$12x =$	the sum paid for 12 yds,
And - - - - -	$18x =$	the sum paid for 18 yds;
But (the difference of those sums)	$6x =$	48 by question.
Therefore	$x =$	8.

QUEST. III. Four men A, B, C, and D, built a ſhip, which coſt 2607 *l.* whereof B paid twice as much as A; C paid as much as A and B; and D paid as much as C and B: What did each pay?

Suppoſe A paid	- - - - - -	x, pounds;
Then B paid	- - - - -	$2x$,
- - C,	- - - - - -	$3x$,
And D,	- - - - - -	$5x$;

Whence (the whole ſum paid) $11x = 2607$ *l.* by queſt.
Therefore - - - - - - $x = 237$.

QUEST. IV. A charitable lady relieving four poor perſons, gave among them 6 *s.* 8 *d.* to the ſecond ſhe gave, twice; to the third, thrice; and to the fourth, four times as much as to the firſt: What did ſhe give to each?

Suppoſe ſhe gave	x	pence to the firſt;
Then ſhe gave	$2x$	pence to the ſecond;
- - - - -	$3x$	- - to the third;
And - - -	$4x$	- - to the fourth:

And (ſhe gave in all) $10x$ (= 6*s.* 8*d.* =) 80 by queſtion;
Therefore - - $x = 8$.

QUEST. V. Four days after a courier, who travels 30 miles a day, had been diſpatched, a ſecond was ſent with orders to overtake him; in order to which, the latter goes 42 miles a day: In what time will he overtake the former?

Let - - -	$x =$	the days in overtaking;
Then - -	$42x =$	miles the ſecond travelled,
And - - -	$30x =$	{ miles travelled by the firſt in the ſame time;
But $(30 \times 4 =)$	$120 =$	{ miles the firſt had gone before the ſecond ſet out.
Th. -	$30x + 120 =$	miles the firſt travelled in all:
But -	$30x + 120 =$	$42x$ by queſtion,
Or - - -	$120 =$	$12x$ by ſubtraction.
Th. - -	$10 =$	x, the days ſought.

QUEST.

QUEST. VI. A cask which held 126 gallons, was filled with a mixture of brandy, wine, and cyder; in it there were 13 gallons of wine, more than there were of brandy; and as much cyder as of both wine and brandy: What quantity was there of each?

Suppose there were - - x gallons of brandy;
Then there were - - $x+13$ of wine;
And - - - - - - - $2x+13$ of cyder;

Whence (their sum) - $4x+26=126$ by question;
Therefore (by subtraction) - - $4x=100$,
And - - - - - - - - $x=25$.

QUEST. VII. In a lump of mixed metal, weighing 29 lb. there were 2 lb. of silver more than of gold; 4 lb. of copper more than of silver; and 3 lb. of brass more than of copper: How many pounds are there of each?

Suppose there were - - x lb. of gold;
Then there were - - $x+2$ of silver;
And - - - - - - - $x+6$ of copper;
Also - - - - - - - $x+9$ of brass;

But (their sum) - - $4x+17=29$ by question.
Therefore (by subtraction) - $4x=12$,
And - - - - - - - - $x=3$.

QUEST. VIII. A detachment of four regiments consisted of 5219 men; colonel *A*'s regiment exceeded colonel *B*'s by 22 men; colonel *C*'s by 73 men; and colonel *D*'s by 130 men: How many were in each regiment?

Let - - - - x the men in col. *A*'s regiment;
Then - - - - $x-22=$ those in col. *B*'s;
And - - - - $x-73=$ those in col. *C*'s;
Also - - - - $x-130=$ those in col. *D*'s;
But (their sum) $4x-225=5219$ by question;
Th. (by addition) - $4x=5444$,
And - - - - - $x=1361$.

QUEST. IX. Six men were employed at the same kind of work; of whom, the second, earned 13 pence; the third, 14 pence; the fourth, 17 pence; the fifth, 23 pence; and the sixth 29 pence respectively, less than the first; also the five last earned, in all, three times as much as the first: What did each earn?

Suppose the first earned - - x pence;

Then the second earned - - $x-13$ pence,
the third - - - - - $x-14$,
the fourth - - - - - $x-17$,
the fifth - - - - - $x-23$,
the sixth - - - - - $x-29$;

But (the sum of the five last) $5x-96=3x$ by quest.
Therefore (by transposition) - $2x=96$,
And - - - - - - - - - $x=48$.

QUEST. X. Being to buy a suit of cloaths for each of my six children, I propose to lay out four times as much on the eldest as I do on the youngest; and to bestow 12 shillings a suit less, on each, than on the next elder: What will each suit cost?

Suppose the youngest's suit cost x shillings,
Then the second's will cost $x+12$,
the third's - - - - $x+24$,
the fourth's - - - $x+36$,
the fifth's - - - - $x+48$,
And the eldest's - - - - $x+60$;
But - - - - - - - - - $x+60=4x$ by qu.
Therefore (by subtraction) - - $60=3x$,
And - - - - - - - - - - $20=x$.

QUEST.

QUEST. XI. *A* and *B* began trade with equal stocks; *A* in the first year tripled his stock, all but 30 *l*. *B* doubled his stock, and had 50 *l*. to spare; now the amount of both their gains was 4 times the stock of each: What stock is required?

Let $x=$ required stock.

$$\left.\begin{array}{ll} \text{Now} & 3x-30= A\text{'s stock,} \\ \text{And} & 2x-30= A\text{'s gain,} \\ \text{Also} & 2x+50= B\text{'s stock,} \\ \text{And} & x+50= B\text{'s gain,} \end{array}\right\} \text{at the year's end.}$$

Then $3x+20=4x$ (the sum of the gains) by quest.

Therefore $20=x$.

QUEST. XII. At a certain election 375 persons voted, and the candidate chosen had a majority of 91: How many voted for each?

Suppose the person chosen had x votes,

And the other - - - - y votes;

$$\left.\begin{array}{ll} \text{Then (all the persons who voted)} & x+y=375, \\ \text{And (the majority) - - -} & x-y=\ 91, \end{array}\right\} \text{by qu.}$$

The sum of both equations is - $2x=466$,

And their difference - - - $2y=284$;

Therefore - - - - - - - - $x=233$,

And - - - - - - - - - - $y=142$.

QUEST. XIII. Two men, who had between them 35 guineas, played together till one of them had won 4 guineas of the other; and then the winner had twice as many guineas as the loser had at first; How many had each?

Let $x=$ the guineas which the winner had;

And $y=$ those which the loser had:

$$\left.\begin{array}{ll} \text{Then} & x+y=35 \\ \text{And} & x+4=2y \end{array}\right\} \text{by question.}$$

$$\left.\begin{array}{ll} \text{Th.} & x=35-y \\ \text{And} & x=2y-4 \end{array}\right\} \text{by transposition.}$$

Th. $2y-4=35-y$.

Th. $3y=39$ by transp.

Th. $y=13$ by division.

Wh. $x=22$.

QUEST. XIV A gentleman being asked the age of his two sons, replied, that, if to the sum of their ages 25 be added, the number arising will be double the age of the eldest; but if 8 be taken from the difference of their ages, the remainder will be the youngest's age: How old was each?

Let $x=$ eldest's, and $y=$ youngest's age;

$$\left.\begin{array}{ll}\text{Then} & x+y+25=2x \\ \text{And} & x-y-8=y\end{array}\right\}\text{by question.}$$

$$\left.\begin{array}{lll}\text{Th.} & y+25=x, & \text{in 1st.} \\ \text{And} & x=2y+8, & \text{in 2d}\end{array}\right\}\text{step.}$$

$$\begin{array}{ll}\text{Hence} & 2y+8=y+25. \\ \text{Th.} & y=17. \\ \text{And} & x=42.\end{array}$$

QUEST. XV. A merchant received a bill of exchange in pistoles (at 16*s.* 6*d.*), guineas (at 21*s.*), and moidores (at 27*s.*); the sum of the pistoles and guineas was 40; the sum of the pistoles and moidores was 36; and the sum of the guineas and moidores was 30: What was the value of the bill?

Suppose he received x pistoles; y guineas; z moidores:

$$\left.\begin{array}{ll}\text{Then first, - - - - - -} & x+y=40; \\ \quad\text{secondly, - - - - -} & x+z=36; \\ \quad\text{thirdly, - - - - -} & y+z=30;\end{array}\right\}\text{by quest.}$$

$$\begin{array}{ll}\left.\begin{array}{l}\text{Whence by subtr. the 2d.} \\ \text{from the 1st. - - -}\end{array}\right\} & y-z=4; \\ \text{And (by adding the two last -} & 2y=34; \\ \text{Therefore - - - - - - - -} & y=17; \\ \text{Now by first - - - - - - -} & x=(40-17=)23; \\ \text{And by second - - - - - -} & z=(36-23=)13;\end{array}$$

Now 23 pistoles - - - - - -	= *L.* 18	19	6,
17 guineas - - - - - -	= 17	17	0,
13 moidores - - - - - -	= 17	11	0;
Therefore the bill was for - - -	£. 54	07	6.

QUEST. XVI. The ſtock of three traders amounted to 780*l* the ſhares of the firſt and ſecond exceeded the third's by 220*l*. and the ſhares of the ſecond and third was 380*l*. more than the firſt's: Each ſhare is required?

Let x, y, and z, repreſent the required ſhares;

$$\left.\begin{array}{ll}\text{Then firſt,} & x+y+z=780,\\ \text{ſecondly,} & x+y=z+220,\\ \text{thirdly,} & y+z=x+380,\end{array}\right\}\text{by queſt.}$$

Whence (by taking the difference of the firſt and ſecond equations) $z=560-z$,

Or $2z=560$; therefore $z=280$;

And (by taking the difference of the firſt and third equations) $x=400-x$,

Or $2x=400$; whence $x=200$:

And $y=(780-280-200=)300$.

QUEST. XVII. Two perſons began play with equal ſums of money; the firſt won 11 ſhillings, the other loſt 7 ſhillings; and then the firſt had twice as many ſhillings as the ſecond: What ſum had each at firſt?

Suppoſe they had x ſhillings each;

$$\left.\begin{array}{l}\text{Then } x+11,\\ \text{And } x-7,\end{array}\right\}\text{were the ſums they had after playing;}$$

Therefore $x+11=(\overline{x-7}\times 2=)\ 2x-14$ by queſt.

Whence $25=x$ (by tranſpoſition.)

QUEST. XVIII. A mercer having cut 12 yards off each of three equal pieces of silk, found that the remnants taken together were 126 yards: What was the length of each piece?

Let $x=$ length of each piece;
Then $\underline{x-12}=$ length of each remnant;
Now $3\times\overline{x-12}=126$ by quest.
Or $3x-36=126$ by multip.
Th. $3x\quad=162$ by transp.
And $x\quad=54$.

QUEST. XIX. After A had won a shilling of B, he had as many shillings as B had left; but had B won a shilling of A, then he would have had twice as many as A would have had left: How many had each?

Suppose A, had x, and B, y shillings;

$$\left.\begin{array}{l}\text{Then } x+1=y-1 \\ \text{And } y+1=(\overline{x-1}\times 2=)2x-2\end{array}\right\}\text{by quest.}$$

$$\left.\begin{array}{l}\text{Th. } y = x+2, \text{ in 1st.} \\ \text{And } y =2x-3, \text{ in 2d}\end{array}\right\}\text{by transp.}$$

Th. $2x-3=x+2$;
Th. $x\quad=5$;
And $y\quad=7$.

QUEST. XX. The paving of a square at 2*s.* a yard, cost as much as the inclosing it at 5*s.* a yard: The side of that square is required?

Let $x=$ side of the square;
Then $4x=$ yards of inclosure;
And $xx=$ yards of pavement;
Whence $20x=(4x\times 5=)$ price of inclosing;
And $\underline{2xx=(xx\times 2=)}$ price of paving:
But $2xx=20x$ by quest.

$$\left.\begin{array}{l}\text{Th. } xx=10x \\ \text{And } x=10.\end{array}\right\}\text{by division.}$$

QUEST. XXI. A person, whose ability was known to be greater than his Industry, was hired to work for a year at 7*s*. a day; but with this condition, that for every day he played, he should forfeit 3*s*. now at the year's end he had neither money to receive nor to pay: How many days did he work?

If he worked x days, he played $365-x$;
Then $7x=$ sum earned;
And $1095-3x=(\overline{365-x}\times 3=)$ sum forfeited;
But $7x=1095-3x$, by quest.
Th. $10x=1095$,
And $x=109\frac{1}{2}$ days.

QUEST. XXII. A general, disposing his army into a square battle, finds he has 284 men more than a perfect square; but increasing the side by 1 man, he will want 25 men: How many had he?

Let $x=$ side of first square;
Then $xx+284=$ army;
And $\overline{x+1}\times\overline{x+1}-25=$ army;
Hence $xx+2x-24=xx+284$;
Then $2x = 308$;
Th. $x = 154$;
And he had $(154\times 154+284=)$ 24000 men.

QUEST. XXIII. 'Tis required to divide the number 14 into two such parts, that the difference of the squares of those parts may be 56?

Let x represent the greater, and y the lesser of those parts;

$$\left.\begin{array}{ll}\text{Then} & xx-yy=56\\ \text{And} & x+y=14\end{array}\right\}\text{ by quest.}$$

Whence $x-y=4$ (for $\frac{xx-yy}{x+y}=x-y$)
Th. $2x = 18$ by adding 2d. and 3d.
Th. $x = 9$.

QUEST. XXIV. *A* borrowed of *B* as much money as *A* had, and spent 6*d*. to treat him; after which meeting with *C*, *A* borrowed of him twice as much money as he had left, and treated him with 12*d*. lastly, *A* borrowed of *D* three times as much money as he had left, and spent on him 18*d*. after which he had 30*d*. left: What had he at first?

Suppose he had x pence at first;
Then he borrowed x pence of *B*,
And (spending 6*d*.) had $2x - 6$ left;
Then he borrowed $4x - 12$ of *C*,
And (spending 12*d*.) had $6x - 30$ left;
Then he borrowed $18x - 90$ of *D*,
And (spending 18*d*.) had $24x - 138$ left:
But $24x - 138 = 30$ by quest.
Theref. $24x = 168$,
And $x = 7$ pence.

QUEST. XXV. Upon measuring the corn produced by a field, being 8 quarters, it appeared that it had yielded but $\frac{1}{3}$ part more than was sown: How much was that?

If x represent the quantity of corn sown;

Then by question $x + \frac{x}{3} = 8$;
But (by multiplication) $3x + x = (8 \times 3 =) 24$,
That is - - - - - - $4x = 24$:
Th. - - - - - - - $x = 6$.

QUEST. XXVI. A gentleman left 210*l*. between his two children; to his daughter he left half as much as he left to his son: What did he leave to each?

Let the son's legacy $= x$,

Then the daughter's $= \frac{x}{2}$;

Now - - - $210 = x + \frac{x}{2}$ by quest.

And (by multip.) $420 = (2x + x =) 3x$;
Th. - - - - $140 = x$.

QUEST.

QUEST. XXVII. Being sent to market, to buy a certain quantity of meat, I found that if I bought beef, which was then 4*d.* a lb. I should lay out all the money I was entrusted with; but if I bought mutton, then $3\frac{1}{2}$ a lb. I should have 2 shillings left: How much meat was sent for?

If x be the lbs. of meat required;

Then x lb. at 4*d.* will cost $4x$ pence,

And x lb. at $3\frac{1}{2}d.$ - - - $\frac{7}{2}x$ pence,

Now $4x=\frac{7x}{2}+24$ by quest.

Whence $8x=7x+48$ (by multiplication)

And $x=48$ (by subtraction.)

QUEST. XXVIII. A fish was caught whose tail weighed 9 lb, his head weighed as much as his tail and $\frac{1}{2}$ his body; and his body weighed as much as his head and tail: What did the fish weigh?

Suppose the body weighed x lb.

Then - - - - - - $9+\frac{x}{2}=$ weight of the head;

And (head + tail) $9+9+\frac{x}{2}=x$ by quest.

That is - - - - - $18+\frac{x}{2}=x$;

But - - - - - - $36+x=2x$ by multip.

Th. - - - - - - - $36=x$ by subtraction.

Hence the fish weighed 72*lb.*

QUEST. XXIX. One being asked how old he was, answered; that the product of $\frac{1}{20}$ of the years he had lived, being multiplied by $\frac{5}{8}$ of the same, would be his age; What was it?

Suppose his age was x years;

Then - - - $\frac{x}{20}\times\frac{5x}{8}=x$ by quest.

That is $\left(\frac{5xx}{20\times 8}=\right)\frac{xx}{4\times 8}=x$;

But (by multiplication) $xx=32x$,

Th. (by division) - - $x=32$.

QUEST. XXX. Some persons agreed to give six-pence each to a waterman for carrying them from London to Gravesend; but with this condition, that for every other person taken in by the way, three-pence should be abated in their joint fare; now the waterman took in 3 more than a fourth part of the number of the first passengers, in consideration of which he took of them but five pence each: How many persons were there at first?

Suppose there were x passengers at first,

Then $\frac{x}{4}+3$ were taken in afterwards;

And $\frac{x}{4}+3$ persons at $3d.$ each comes to $\frac{3x}{4}+9$:

But $6x-\frac{3x}{4}-9 = 5x$ by question;

And $24x-3x-36 = 20x$ by multiplication,

That is, $24x-3x-20x=36$ by transposition,

Th. - - - - $x=36$.

QUEST. XXXI. From each of 16 pieces of gold, an artist filed the worth of half a crown, and then offered them in payment for their original value; but being detected, and the pieces weighed, they were found to be worth no more than 8 guineas: Their original worth is required?

Suppose they were each worth x shillings.

Then the value of each after filing $\Big\} =(x-2\frac{1}{2}=x-\frac{5}{2}=) \frac{2x-5}{2}$;

But (8 guin.=) $168=(\frac{2x-5}{2}\times 16=)\overline{2x-5}\times 8$.

That is - $168=16x-40$,

Or (by addit.) $208=16x$;

Th. - - - $13=x$.

QUEST. XXXII. What sum of money is that, from which 5 *l.* being subtracted, two thirds of the remainder will be 40 *l*?

Suppose - $x=$ the sum required.

Then - $x-5=$ remainder, when 5 is subtracted;

And $\overline{x-5}\times\frac{2}{3}=$ two thirds of that remainder;

But $\overline{x-5}\times\frac{2}{3}=$ 40 by question,

Th. $x-5=(40\times\frac{3}{2}=)\ 60$;

Th. - - $x=(60+5=)65$.

QUEST. XXXIII. One being asked the hour of the day, replied, that the time then passed from noon, was equal to $\frac{29}{43}$ of the time remaining until midnight: What time was that?

Suppose - - - - $x=$ the required time;

Then - - $12-x=$ the remaining time to mid.

And - - - - - $x=\overline{12-x}\times\frac{29}{43}$ by quest.

Or - - - - - $43x=(\overline{12-x}\times 29=)\ 348-29x$,

Or $(43x+29x=)\ 72x=348$;

Th. - - - - - $x=(\frac{348}{72}=\frac{29}{6}=)\ 4^{h.} : 50^{min.}$

QUEST. XXXIV. A man dying, his wife being with child, ordered by will, that if the child proved a daughter, then his wife should have $\frac{2}{3}$ and the child $\frac{1}{3}$ of his estate; but if it was a son, then he should have $\frac{2}{3}$ and the mother $\frac{1}{3}$ thereof; now it happened that the mother was delivered of a son and a daughter: How must the estate (which was 6300*l.*) be divided between them?

Suppose the daughter's share was x *l.*

Then the mother's would be $2x$,

And then the son's - - - $4x$;

For then $\left\{\begin{array}{l}\text{the son's share is to the mother's}\\ \text{the mother's to the daughter's}\end{array}\right\}$ as $\frac{2}{3}$ to $\frac{1}{3}$:

But (the whole estate) $6300=7x$, by question;

Th. - - $(\frac{6300}{7}=)\ 900=x$.

QUEST. XXXV. It is required to divide 55, into two ſuch parts, that the greater of them divided by their difference, may quote 6?

If x be the greater, and y the leſſer part,

$$\left.\begin{array}{lr} \text{Then} \; - \; - & x+y=55 \\ \text{And} \; - \; - & \dfrac{x}{x-y}=6 \end{array}\right\} \text{by queſt.}$$

But - (by 2d.) $x=(\overline{x-y}\times 6=)6x-6y$,
Or - - - - $6y=(6x-x=)5x$,
Th. - - - - $y=\dfrac{5x}{6}$:

And (by 1ſt.) $x+\dfrac{5x}{6}=55$,

Or $(6x+5x=)11x=55\times 6$ by multiplication.
Th. - - - $x=(5\times 6=)30$.

QUEST. XXXVI. *A* having about him 240, and *B* having 96 *l.* were met by ſome thieves; who took from *A* twice as much as from *B*, and left *A* three times as much as they left *B*: What ſum was each robbed of?

Suppoſe *A* was robbed of x, and *B*, of y, pounds:

Then (by queſtion) $\left\{\begin{array}{l} x=2y, \\ 240-x=3\times\overline{96-y}; \end{array}\right.$

And (writing $2y$ for x, in 2d. equat.) $\}$ $240-2y=288-3y$,

Th. (by tranſp. $3y-2y=)y=(288-240=)48$.

QUEST. XXXVII. A company of 18 perſons, men and women, clubbing for a reckoning of 9 *l.* 18 *s.* paid each as many ſhillings as there were men in company: How many were there?

Let the number of men - $=x$,
And the number of women - $=y$;
Then the ſum paid by the men $=xx$ ſhillings,
And - - - by the women $=xy$ ſhillings.

$$\left.\begin{array}{lr} \text{Now firſt} \; - \; - \; - \; - & x+y=18, \\ \text{And ſecondly} \; - \; - \; - & xx+xy=198(=9l.\ 18s.) \end{array}\right\} \text{by q.}$$

Th. (dividing the 2d. equat. by 1ſt.) $\Big\}$ $\dfrac{xx+xy}{x+y}=\dfrac{198}{18}$;

That is - - - - - - - - $x=11$,
And - - - - - - - - - $y=7$.

QUEST.

QUEST. XXXVIII. At an election the number of voters was three times the majority by which the choice was carried; and the product of the numbers which voted for each, was 122 times the said majority: How many votes had each?

Let x be the greater, and y the lesser, N° of voters.

Then the whole number of voters $=x+y$.

And the majority - - - - $=x-y$,

$$\left.\begin{array}{l}\text{Now - - } (\overline{x-y}\times 3=)3x-3y=x+y,\\ \text{And } (\overline{x-y}\times 122=)\ 122x-122y=xy,\end{array}\right\}\text{ by qu.}$$

But (by transposing the first) $2x=4y$,

Th: - - - - - - - - - - $x=2y$,

And (writg. $2y$ for x) $244y-122y=2yy$ by second;

Or (divid. by y) $224-122=)\ 122=2y$,

Th. - - - - - - - - - $61=y$.

QUEST. XXXIX. Bought 8 yards of cloth for 62 shillings; for part of it, I gave 9*s.* a yard; and for the rest 7*s.* How much was bought of each?

Suppose x yards at 9*s.* and y yards at 7*s.* were bought;

Then x yards cost $9x$, And y yards cost $7y$;

$$\text{But by question - - - - }\left\{\begin{array}{l}x+y=8,\\ 9x+7y=62;\end{array}\right.$$

And the first mult. by 7 gives $7x+7y=56$,

Th. (the diff. of the two last) - $2x=6$;

Whence - - - - - - - - - - $x=3$,

And - - - - - - - - - - - $y=5$.

QUEST. XL. A son asking his father how old he was, the father replied, "My age 7 years ago, was just "four times as great as your age, at that time; but 7 "years hence, if you and I live, my age will be only "double to yours:" The age of each person is required?

Suppose the father was x, and the son y years old:

$$\left.\begin{array}{l}\text{Then first - } x-7=(\overline{y-7}\times 4=)4y-28\\ \text{And secondly } x+7=(\overline{y+7}\times 2=)2y+14\end{array}\right\}\text{ by quest.}$$

$$\left.\begin{array}{l}\text{Th. - - - - } x=4y-21\\ \text{And - - - - } x=2y+7\end{array}\right\}\text{ by transposition;}$$

But - - $4y-21=2y+7$,

And - - - - $2y=28$ by transposition,

Th. - - - - $y=14$.

QUEST.

QUEST. XLI. A person paid a bill of 50*l.* with half guineas and crowns, using 101 pieces in all: How many of each sort did he pay?

Let x represent the number of $\frac{1}{2}$ guineas,
And y - - the number of crowns;
Then - - - - - - $x+y=101$,
Then - - - $21x+10y=2000(50l.\times40)$;
But 1st. eq. $\times10$, gives $10x+10y=1010$,
Whence (by subtraction) $11x=(2000-1010=)990$;
Th. - - - - - - - - - $x=90$,
And - - - - - - - - $y=11$.

QUEST. XLII. Two remnants of cloth, which together measure 40 yards, were of equal value; and the one sold at 3*s.* the other at 7*s.* a yard; How many yards were there of each?

Suppose x yards at 3*s.* a yard, and y yards at 7*s.*
Then the value of the first was $3x$, and of the 2d. $7y$;

$$\left.\begin{array}{lr}\text{Whence 1.} & x+y=40 \\ \text{And 2.} & 3x=7y\end{array}\right\}\text{ by question;}$$

Th. (from first) - - $x=40-y$,
But - - $(3x=)120-3y=7y$ by 2d.
And (by transposition) $120=10y$,
Th. - - - - - - $12=y$.

QUEST. XLIII. A person exchanges 6 French crowns and 2 dollars for 45 shillings; and at another time 9 French crowns and 5 dollars, for 76 shillings: What were the values of the crown and dollar?

$$\left.\begin{array}{lr}\text{Suppose a French crown is worth} & x, \\ \text{And a dollar} & y,\end{array}\right\}\text{ pence;}$$

$$\text{Then (by question)}\begin{cases}6x+2y=45\times12=540, \\ 9x+5y=76\times12=912,\end{cases}$$

1st. $\times$ by 3 gives - $18x+6y=1620$,
2d. $\times$ by 2 gives - $18x+10y=1824$,
Their difference is - - $4y=204$,
And - - - - - - - $y=51$,
Hence $(6x+2y=)$ $6x+102=540$,
Then - - - - - $x=\left(\frac{438}{6}=\right)73$.

QUEST.

QUEST. XLIV. Having laid out 37 shillings in brandy at 2*s.* and rum at 3*s.* a quart; I find that I could have bought as many quarts of rum as I now have of brandy, and as many of brandy as now of rum, for 4 shillings less: How much was bought?

If x, quarts of brandy; and y, of rum were bought,

Then - - - $2x+3y=37$, } by qu.
And - - - $2y+3x=(37-4=)33$ }

But (by subtraction) $y-x=4$,
Th. - - - - - - $y=x+4$,
And (by multiplication) $3y=3x+12$;
But - - $2x+3x+12=37$ by first,
And (by transposition) $5x=25$,
Th. - - - - - - $x=5$,
And - - - - - $y=9$.

QUEST. XLV. A gentleman gave to 3 persons 56*l.* the second received $\frac{4}{9}$ of the sum given to the first, and the third $\frac{1}{4}$ of what the second had; How much had each?

Suppose the first received x *l.*

Then the second received $\frac{4x}{9}$, and the third $\frac{x}{9}$:

Th. $(x+\frac{4x}{9}+\frac{x}{9}=)\ x+\frac{5x}{9}=56$ by quest.

Whence - $(9x+5x=)\ 14x=56\times 9$;

Therefore (dividing by 14) $x=(4\times 9=)\ 36$.

QUEST. XLVI. Out of a cask of liquor, 63 gallons were sold to two persons (between them); to the first $\frac{1}{3}$, and to the second $\frac{1}{6}$, of what the cask contained: What did the cask hold?

If the cask hold x gallons;

Then the first bought $\frac{x}{3}$ galls. and the second $\frac{x}{6}$ galls.

Therefore - - - $\frac{x}{3}+\frac{x}{6}=63$ by quest.

Which multiplied by 6 produces $\frac{6x}{3}+\frac{6x}{6}=(63\times 6=)378$.

That is - - $(2x+x=)3x=378$;

Th. - - - - - - - $x=126$.

QUEST. XLVII. Out of a cask of wine, which had leaked away $\frac{1}{3}$, 21 gallons were drawn; and then being gauged, it appeared to be $\frac{1}{2}$ full: How much did it hold?

Suppose it held x gallons, and had leaked $\frac{x}{3}$ galls.

And when gauged $21+\frac{x}{3}$ had been taken out,

But - - - - $21+\frac{x}{3}=\frac{x}{2}$ by question;

Which $\times$d. by 3×2 gives $126+\frac{6x}{3}=\frac{6x}{2}$.

That is - - $126+2x=3x$;

Th. (by subtraction) - $126=(3x-2x=)\ x$.

QUEST. XLVIII. After paying away $\frac{1}{4}$ and $\frac{1}{5}$ of my money, I found 66 guineas left in my bag: What was in it at first?

Suppose x guineas,

Then (by question) $x-\frac{x}{4}-\frac{x}{5}=66$;

Which $\times$d. by 4×5, gives $20x-\frac{20x}{4}-\frac{20x}{5}=(66\times 20=)\ 1320$;

That is $(20x-5x-4x=)\ 11x=1320$;

Th. - - - - - - - - - $x=120$.

QUEST. XLIX. A gentleman gave in charity 46 pounds; a part thereof, in equal portions, to 5 poor men; and the rest, in equal portions, to 7 poor women: now a man and a woman had between them 8 pounds; What was given to the men, and what to the women?

Suppose the men had $x\,l.$ then the women had $46-x$;

Whence one man had $\frac{x}{5}$, and one woman $\frac{46-x}{7}$;

Now by question - - $\frac{x}{5}+\frac{46-x}{7}=8$;

Which multiplied by $(5\times 7=)$ 35 produces $7x+\overline{46-x}\times 5=(8\times 35=)280$,

That is $(7x+230-5x=)\ 2x+230=280$,

Or (by transposition) - - - $2x=50$;

Th. - - - - - - - - - - - $x=25$.

QUEST. L. What fraction is that, to the numerator of which if 1 be added, the value will be $\frac{1}{3}$; but if 1 be added to the denominator, its value will be $\frac{1}{4}$?

Let $\frac{x}{y}$ represent the fraction required;

$$\left.\begin{array}{l}\text{Then } \frac{x+1}{y}=\frac{1}{3};\ \text{Or } 3x+3=y, \\ \text{And } \frac{x}{y+1}=\frac{1}{4};\ \text{Or } 4x=y+1,\end{array}\right\}\text{by question:}$$

But writing $3x+3$ for y, $4x=3x+3+1=3x+4$;
Th. (by subtraction) - $x=4$,
And - - - - - - $y=(3\times4+3=)\ 15$.

QUEST. LI. Two remnants of cloth were bought, which measured one 7, the other 5 yards; the first and 1 yard of the second cost 3*l*. 8*s*. also the second and 1 yard of the first cost the same sum: What were each valued at a yard?

Suppose the first remnant cost y s. and the 2d. z s. a yard;
Then the value of the first was $7y$, and of the 2d $5z$;

But by question - - - - - - $\left\{\begin{array}{l}7y+z=68, \\ 5z+y=68:\end{array}\right.$

Th. (by transposing the first) - - - - - $z=68-7y$,
And - - - - - $(68-7y\times5=)\ 340-35y=5z$;
Th. (by the second $5z+y=$) $340-35y+y=68$,
That is - - - - - - - - - $340-34y=68$,
Or (by transposition) - - $(340-68=)\ 272=34y$;
Th. - - - - - - - - - - $\left(\frac{272}{34}=\right)\ 8=y$.

QUEST. LII. The expence of building a ship (which cost 3900*l*) was defrayed by 2 merchants, whose shares in her were as 4 to 9: What did each pay?

Suppose one of them paid x *l*.

Then $(4:9::x:)\ \frac{9x}{4}=$ the other's payment;

And - - - $x+\frac{9x}{4}=3900$ by question;

Or $(4x+9x=)$ $13x=3900\times4$;
Th. - - - - - $x=(300\times4=)\ 1200$,
And $4:9::1200:y=$ [redacted]. 2700

QUEST. LIII. A cask containing 120 gallons, was filled with brandy, white wine, cyder, and water; the brandy and wine (taken together) make $\frac{1}{2}$ the content of the cask; the brandy and cyder make $\frac{2}{3}$ of the content: and the brandy and water $\frac{3}{4}$ thereof: The quantities of each are required?

Let x, y, u, and z, be gallons of brandy, wine, cyder, and water:

$$\left.\begin{aligned} \text{Then } x+y+u+z &= 120, \\ x+y &= (120\times\tfrac{1}{2}=)\ 60, \\ x+u &= (120\times\tfrac{2}{3}=)\ 80, \\ x+z &= (120\times\tfrac{3}{4}=)\ 90, \end{aligned}\right\}\text{by question;}$$

(By addition) $4x+2y+2u+2z=350$;
(By first) $2x+2y+2u+2z=240$;
Th. - - $2x=110$ (by subtraction)
And - - - $x=55$, brandy;
$y=(60-55=)\ 5$, wine;
$u=(80-55=)\ 25$, cyder;
$z=(90-55=)\ 35$, water.

QUEST. LIV. A draper sold a $\frac{1}{4}$ of a piece of cloth at 5*s.* $\frac{1}{5}$ of it at 4*s.* and $\frac{1}{6}$ of it at 4*s.* 6*d.* a yard; by doing which he took 8 guineas: How many yards did the piece contain?

If it contained x yards;

Then $\frac{x}{4}\times 5+\frac{x}{5}\times 4+\frac{x}{6}\times\frac{9}{2}=(8\times 21=)\ 168$ by q.

Or - - - - - $\frac{5x}{4}+\frac{4x}{5}+\frac{3x}{4}=168$;

And (multiplying by 20) $\left\{\frac{100x}{4}+\frac{80x}{5}+\frac{60x}{4}=168\times 20,\right.$

Or - - - - $25x+16x+15x=3360$,
Or - - - - - - - - $56x=3360$;
Th. - - - - - - - - - $x=(\frac{3360}{56}=)\ 60$.

QUEST.

QUEST. LV. The governors of Christ's-Hospital in London, bestowed $\frac{1}{2}$ of a legacy committed to their trust, among 3 of their boys who were sent to the university; $\frac{1}{4}$ of it, among 7 boys sent to sea; and $\frac{1}{6}$ of it, among 11 boys bound to trades; now 1 scholar, 1 sea-boy, and 1 apprentice, received among them 10*l.* 1*s.* What was the legacy?

If the legacy was x shillings;

Then $\frac{1}{3}$ of $\frac{x}{2}+\frac{1}{7}$ of $\frac{x}{4}+\frac{1}{11}$ of $\frac{x}{6}=$ (10 *l.* : 1 *s.* =) 201 *s.*

That is - - - - $\frac{x}{6}+\frac{x}{28}+\frac{x}{66}=201$ by quest.

Which being multiplied by $2\times 14\times 33$ produces

$$\frac{2\times 14\times 33x}{6}+\frac{2\times 14\times 33x}{28}+\frac{2\times 14\times 33x}{66}=201\times 2\times 14\times 33;$$

Or $14\times 11x+33x+14x=201\times 2\times 14\times 33.$

Or $(154x+33x+14x=)\ 201x=201\times 2\times 14\times 33;$

Th. - - - - - - $x=(2\times 14\times 33=924=)$ 46*l.* 4*s.*

QUEST. LVI. Some boys A, B, C, D, E, F, G, H, and I, robbed an orchard; A had for his share $\frac{1}{5}$, B $\frac{1}{12}$, C $\frac{1}{8}$, D $\frac{1}{20}$, E $\frac{1}{7}$, and F $\frac{1}{4}$ part of the whole; G had 310, H 425, and I 140 apples: How many apples had they in all?

Suppose the whole number of apples was x,

$$\text{Then } \left.\begin{array}{l}\frac{x}{5}+\frac{x}{12}+\frac{x}{8}+\frac{x}{20}+\frac{x}{7}+\frac{x}{4}+310\\ +425+140\ \text{- - - - - - - -}\end{array}\right\}=x;$$

Or $\frac{x}{5}+\frac{x}{12}+\frac{x}{8}+\frac{x}{20}+\frac{x}{7}+\frac{x}{4}+875=x:$

Which multiplied by $(5\times 3\times 8\times 7=)\ 840;$

$$\left.\begin{array}{l}\text{produces, } 168x+70x+105x+42x+\\ 120x+210x+875\times 840\end{array}\right\}=840x;$$

That is - - - - - - $715x+735000=840x;$

And (by transposit.) $735000=(840x-715x=)\ 125x;$

Th. $\left(\frac{735000}{125}=\frac{147000}{25}=\frac{29400}{5}=\right)\ 5880=x.$

QUEST.

QUEST. LVII. A person at play won twice as much as he began with, and then lost 16 shillings; after which he lost $\frac{4}{5}$ of what remained; lastly, he won as much as he began with; then counting his money found he had 80 shillings: I demand what sum he began with?

Suppose he began with x shillings;
Then he had $3x$, after he had won $2x$;
And $3x-16$, when $16s.$ was lost;
Also $(3x-16-\frac{\overline{3x-16}\times 4}{5}=)\frac{3x-16}{5}$ remained when $\frac{4}{5}$ of the former was lost:
Lastly by question $\frac{3x-16}{5}+x=80$;
And $(3x-16+5x=)8x-16=80\times 5$;
Now (dividing by 8) $x-2=(10\times 5=)50$,
Th. (by addition) $x=52$.

QUEST. LVIII. A man and his wife did usually drink out a vessel of beer in 12 days; but when the man was out, the vessel lasted the woman 30 days: In how many days would the man alone be drinking it out?

Suppose the man could drink it out in x days:

Then $(\overset{\text{D.}}{x} : \overset{\text{V.}}{1} :: \overset{\text{D.}}{12} :)\ \frac{12}{x}=$ the part drank by the man in 12 days;
And $(30 : 1 :: 12 :)\ \frac{12}{30}=$ by the woman in 12 days;
But - - - $\frac{12}{x}+\frac{12}{30}=1$ by question;
Or - - $360+12x=30x$;
Or - - - - $360=(30x-12x=)\ 18x$;
Th. - - - - $20=x$.

QUEST.

QUEST. LIX. A ciftern into which water may be let by two cocks A and B, will be filled by them both running together in 12 hours; and by the cock A alone in 20 hours: In what time will it be filled by the cock B alone?

Suppofe in x time;

$$\left.\begin{array}{l} \text{H.Cift.H.} \\ \text{Then } (x : 1 :: 12 :) \frac{12}{x} = \\ (20 : 1 :: 12 :) \frac{12}{20} = \end{array}\right\} \text{the quantity of water fupplied in 12 hours by} \left\{\begin{array}{l} B, \\ A; \end{array}\right.$$

But $\frac{12}{20} + \frac{12}{x} = 1$ (ciftern full) by queftion;

That is - $12x + 240 = 20x$;

Or - - - $240 = (20x - 12x =)\ 8x$;

Th. - - - $30 = x$.

QUEST. LX. A fum of money is to be fhared between two perfons, A and B; fo that as often as A takes 9*l.* B is to take 4*l.* now it happened, that A received 15*l.* more than B: Their refpective fhares are required?

Suppofe A received x *l.* then B received $x - 15$;

Thence $x : x - 15 :: 9 : 4$ by queftion:

But the product of the extremes of 4 proportionals, is equal to the product of the means;

Th. - - - - - $4x = (\overline{x - 15} \times 9 =)\ 9x - 135$;

And (by tranfpofition) $135 = (9x - 4x =)\ 5x$;

Th. - - - - - $27 = x$.

QUEST LXI. A footman, who contracted for 8*l.* a year, and a livery fuit, was turned away at the end of 7 months, and received only 2*l.* 3*s.* 4*d.* and his livery: What was its value?

Suppofe it was x pounds;

Then (becaufe 2*l.* 3*s.* 4*d.* $= \frac{13}{6}$ *l.*)

$$\begin{array}{llll} \text{Mo.} & \text{£.} & \text{Mo.} & \text{£.} \\ 12 : & 8 + x :: & 7 : & \frac{13}{6} + x \end{array} \text{ by queftion;}$$

That is $(12 \times \frac{13}{6} + x =)\ 26 + 12x = (\overline{8 + x} \times 7 =)\ 56 + 7x$;

Or $(12x - 7x =)\ 5x = (56 - 26 =)\ 30$;

Th. - - - - - $x = (\frac{30}{5} =)\ 6$.

QUEST.

QUEST. LXII. A market-woman bought a certain number of eggs, at 2 a penny, and as many at 3 a penny, and sold them all out again at the rate of 5 for two-pence; after which she found that (instead of making just her money again, as she expected) she had lost 4 pence by them: What number of eggs had she?

Suppose she bought x eggs of each sort;

Then x eggs at 2 a penny cost $\frac{x}{2}$ pence;

And x ditto at 3 a penny cost $\frac{x}{3}$ pence;

Also $2x$ eggs at 5 for two-pence sold for $\frac{4x}{5}$ pence;

(For $5 : 2 :: 2x : \frac{4x}{5}$);

Now $\frac{x}{2}+\frac{x}{3}+\frac{4x}{5}=4$ by question;

That is $15x+10x-24x=(4\times 30=)\ 120$;

Th. - - - - - $x=120$.

QUEST. LXIII. What number is that, to which if 3, 5, and 8, be severally added; the first sum shall be to the second, as the second to the third?

If x = the number sought?

Then $\overline{x+3} : \overline{x+5} :: \overline{x+5} : \overline{x+8}$ by question;

Th. $\overline{x+3}\times\overline{x+8}=\overline{x+5}\times\overline{x+5}$.

Or $xx+11x+24=xx+10x+25$;

Or - - $11x+24=10x+25$;

Th. $(11x-10x=)\ x=(25-24=)\ 1$.

QUEST. LXIV. A hare 50 of her leaps before a greyhound, takes 4 leaps to the greyhound's 3; but 2 of the greyhound's leaps are as much as 3 of the hare's: How many leaps must the greyhound take to catch the hare?

Suppose the greyhound must take x leaps,
and in that time the hare takes y leaps;
Then $3 : 4 :: x : y$ by question;

Th. $3y=4x$, and $y=\frac{4x}{3}$;

Now the hare takes $50+\frac{4x}{3}$ leaps in the whole;

Also $2 : 3 :: x : \overline{50 + \frac{4x}{3}}$ by question;

That is $(50+\frac{4x}{3}\times 2=)\ 100+\frac{8x}{3}=3x$;
Or - - - - - - - $300+8x=9x$;
Th. (by subtraction) - - - $300=x$.

QUEST. LXV. The joint stock of two partners, (whose particular shares differed by 40*l.*) was to the share of the lesser; as 14 to 5: Their particular shares are desired?

Suppose $x=$ the lesser share;
Then $x+40=$ the greater;
And $2x+40=$ their joint stock;
Whence $\overline{2x+40} : x :: 14 : 5$ by question;
Th. $(2x+40\times 5=)\ 10x+200=14x$;
And - - $200=(14x-10x=)\ 4x$;
Th. - - - $(\frac{200}{4}=)50=x$.

QUEST. LXVI. A bankrupt owed to two creditors 140*l* the difference of which debts, was to the greater of them, as 4 to 9: What were those debts?

Suppose $x=$ the greater, and y the lesser debt;
Then $x-y : x :: 4 : 9$ by question;
That is $(\overline{x-y}\times 9=)\ 9x-9y=4x$;
Or (by transposition $9x-4x=$) $5x=9y$;
Th. - - - - - - - - - - - $x=\frac{9y}{5}$:
But $(x+y=140$; that is) $\frac{9y}{5}+y=140$ by quest.
And (by multip.) $(9y+5y=)\ 14y=140\times 5$;
Th. - - - - - - - - - - $y=(10\times 5=)\ 50$.

QUEST. LXVII. *A*, *B*, and *C*, make a joint stock; *A* put in 17*l*. less than *B*, and 34*l*. less than *C*; and the sum of the shares of *A* and *B*, is to the sum of the shares of *B* and *C*, as 6 to 7: What did each put in?

Suppose *A* put in - - - - - - - - - - x*l*.
Then *B* put in - - - - - - - - - - $x+17$;
And *C* - - - - - - - - - - - - - $x+34$;
Whence the sum of the shares $\begin{cases} A \text{ and } B = 2x+17, \\ C \text{ and } B = 2x+51; \end{cases}$
But $2x+17 : 2x+51 :: 6 : 7$ by question;
That is $\overline{2x+17}\times 7=\overline{2x+51}\times 6$,
Or $14x+119=12x+306$,
Or $(14x-12x=)\ 2x=(306-119=)\ 187$;
Th. - - - - $x=\left(\frac{187}{2}=\right)\ 93$*l*. 10*s*.

QUEST.

QUEST. LXVIII. It is required to find two numbers, the greater whereof shall be to the lesser, as their sum is to 45; and as their difference is to 9?

Let x represent the greater, and y the lesser number;

Then $x : y :: x+y : 45$ } by question;
And $x : y :: x-y : 9$ }

Th. - - - $45x = (\overline{x+y} \times y =)\ xy + yy$,

And - - - $9x = (\overline{x-y} \times y =)\ xy - yy$;

The sum of which $54x = 2xy$,

Whence - - $27x = xy$,

And - - - $27 = y$:

Now - - - $9x = 27x - 27 \times 27$,

And - - - $x = 3x - 3 \times 27$;

Th. - - - $81 = 2x$,

And - - - $40\frac{1}{2} = x$.

QUEST. LXIX. It is required to divide 60*l.* so, between two men *A* and *B*, that the difference between *A*'s share and 31, shall be to the difference between 31 and *B*'s share, as 6 to 7?

Suppose *A*'s share to be x *l.* and *B*'s y *l.*

Then $x - 31 : 31 - y :: 6 : 7$ by question;

That is $(\overline{x-31} \times 7 =)\ 7x - 217 = (\overline{31-y} \times 6 =) 186 - 6y$,

Or (by transposition) $7x = 403 - 6y$;

Th. - - - - - - - - $x = \frac{403-6y}{7}$;

But ($x+y=60$; that is) $\frac{403-6y}{7} + y = 60$ by question;

And (by multip.) $403 - 6y + 7y = (60 \times 7 =)\ 420$;

Th. (by transposition) $y = (420 - 403 =)\ 17$,

And $x = \left(\frac{403 - 6 \times 17}{7} = \frac{301}{7}\right) = 43$.

QUEST. LXX. Sold a quantity of tobacco for 19 shillings, part at 1*s.* a lb. and the rest at 15*d.* now the first part was to the latter, as $\frac{3}{4}$ to $\frac{2}{3}$: How much was sold of each?

Let $x=$ the quantity sold at 1*s.* a lb.

Then $\left(\frac{3}{4}:\frac{2}{3}::x:\right)\frac{2\times4x}{3\times3}=$ the other quantity at 15*d.*

Also $12x$, is the value of x lb. at 12*d.*

And $\left(\frac{2\times4\times15x}{3\times3}=\right)\frac{2\times4\times5x}{3}$, the val. of $\frac{2\times4x}{3\times3}$ lb. at 15*d.*

Now $12x+\frac{2\times4\times5x}{3}=(19\times12=)228$ by question;

Or $(36x+40x=)\ 76x=228\times3$;

Th. - - - - $x=(3\times3=)\ 9.$

QUEST. LXXI. What two numbers are as 3 to 1; and the sum of their squares is to their sum as 15 to 1?

Let $x=$ the greater number;

Then $(3:1::x:)\ \frac{x}{3}=$ the lesser number;

Whence $\left(x+\frac{x}{3}=\right)\frac{4x}{3}=$ their sum;

And $\left(xx+\frac{xx}{9}=\right)\frac{10xx}{9}=$ sum of their squares;

Now $\frac{10xx}{9}:\frac{4x}{3}::15:1$ by question;

That is $\frac{10xx}{9}=\frac{4\times15x}{3}$;

Or $10xx=3\times4\times15x$;

Or $10x=3\times4\times15$;

Th. $x=(3\times2\times3=)\ 18.$

QUEST.

QUEST. LXXII. What 2 numbers are as 3 to 2; whose sum is equal to the square of their difference?

Let x represent one of those numbers;

Then $(3:2::x:)\frac{2x}{3}=$ the other;

Now $(x+\frac{2x}{3}=\frac{3x+2x}{3}=)\frac{5x}{3}=$ their sum;

And $(x-\frac{2x}{3}=\frac{3x-2x}{3}=)\frac{x}{3}=$ their difference;

But (by question) - - $\frac{5x}{3}=\frac{x}{3}\times\frac{x}{3}$;

And (by division) - - $5=\frac{x}{3}$;

Th. (by multiplication) - $15=x$.

QUEST. LXXIII. What 2 numbers are as 2 to 3, to each of which if 4 be added, the sums will be as 5 to 7?

Let x equal one of the required numbers;

Then $(2:3::x:)\frac{3x}{2}=$ the other number;

Also $x+4:\frac{3x}{2}+4::5:7$ by question;

Th. $(\overline{x+4}\times 7=)\ 7x+28=\overline{\frac{3x}{2}+4}\times 5$,

That is - - - $7x+28=\frac{15x}{2}+20$

Or - - - - - - $7x+8=\frac{15x}{2}$;

And - - - - - $14x+16=15x$;

Th. - - - - - - - $16=x$.

QUEST. LXXIV. There are three numbers whose differences are equal (that is, the second exceeds the first, as much as the third exceeds the second); and the first is to the third as 5 to 7; also the sum of the three numbers is 324: What are those numbers?

Let the second number be represented by y,
And the difference of those numbers by x;
Then - - - $y-x=$ the first number,
$y =$ the second,
And - - - $y+x=$ the third;
But (their sum) $3y=324$ by question;
Therefore - - $y=108$ (by division):
But - - $y-x:y+x::5:7$ by question;
That is $108-x:108+x::5:7$;
Th. - $\overline{108-x}\times 7=\overline{108+x}\times 5$,
Or - - $756-7x=540+5x$,
Or $(756-540=)$ $216=(7x+5x=)$ $12x$:
Th. - $\left(\frac{216}{12}=\right)$ $18=x$;
And the numbers required are 90, 108, and 126.

QUEST. LXXV. What two numbers are those, whose difference, sum, and product, are to each other, as the numbers 2, 3, and 5, respectively?

If $x=$ the greater, and $y=$the lesser of the numbers required,
Then $x-y:x+y::2:3$ } by question;
And $x+y:xy::3:5$ }
That is $3x-3y=2x+2y$; Or $x=5y$:
And $5x+5y=3xy$; Or $5\times 5y+5y=3y\times 5y$,
That is - - - - - $(25y+5y=)$ $30y=15yy$;
And (by division) - - - - - - - $30=15y$,
Th. - - - - - - - - - - - $\left(\frac{30}{15}=\right)$ $2=y$;
And - - - - - - - - - - - - $x=(5\times 2=)10$.

QUEST.

QUEST. LXXVI. A butcher being asked, what number of calves and sheep he had bought? replied, if I had bought 4 more of each, I should have had 4 sheep for every 3 calves; and if I had bought 4 less of each, I should have had 3 sheep for every 2 calves: How many of each did he buy?

If he bought x sheep, and y calves;

$$\left.\begin{array}{l}\text{Then } x+4 : y+4 :: 4 : 3 \\ \text{And } \ x-4 : y-4 :: 3 : 2\end{array}\right\} \text{by question;}$$

$$\text{That is} \left\{\begin{array}{l}(\overline{x+4}\times 3=)\ 3x+12=(\overline{y+4}\times 4=)\ 4y+16, \\ (\overline{x-4}\times 2=)\ 2x-\ 8=(\overline{y-4}\times 3=)\ 3y-12;\end{array}\right.$$

$$\text{Whence} \ \cdots\cdots \left\{\begin{array}{l}x=\frac{4y+4}{3}, \\ x=\frac{3y-4}{2},\end{array}\right.$$

$$\text{But} \ \cdots\cdots \ \frac{3y-4}{2}=\frac{4y+4}{3},$$

$$\text{Or} \ \cdots\cdots \ 9y-12=8y+8;$$

$$\text{Th.} \ \cdots \ (9y-8y=)\ y=(8+12=)\ 20:$$

$$x=\left(\frac{3\times 20-4}{2}=\frac{56}{2}=\right) 28.$$

QUEST. LXXVII. One has 2 sorts of wine A and B, the wine A is worth 6*d.* a quart; and B 10*d.* a quart; he would mix 100 quarts of those, so that each quart of the mixture may be afforded for 7*d.* How many quarts of each may be taken?

Suppose x quarts of A, and y quarts of B;

Also put $6=a$, $10=b$, $100=t$, and $7=m$:

$$\left.\begin{array}{l}\text{Then } x+y=t \\ \text{And } ax+by=mt\end{array}\right\} \text{by question;}$$

But $ax+ay=at$ by multiplication;

And $by-ay=mt-at$ by subtraction;

$$\text{Th.} \ - \ y=\left(\frac{mt-at}{b-a}=\right)\frac{\overline{m-a}\times t}{b-a}.$$

$$\text{In this exam. } y=\left(\frac{7-6}{10-6}\times 100=\tfrac{1}{4}\times 100=\right) 25.$$

QUEST. LXXVIII. Four persons, *A*, *B*, *C*, *D*, spent 20 shillings in company, together; whereof *A* proposed to pay $\frac{1}{3}$, *B* $\frac{1}{4}$, *C* $\frac{1}{5}$, and *D* $\frac{1}{6}$ part; but, when the money came to be collected, they found it was not sufficient to answer the intended purpose: How much must each person contribute to make up the whole reckoning, supposing their several shares to be, still, to each other, in the proportion above specified?

Suppose *A* was to pay x shillings;

Then - - $(\frac{1}{3} : \frac{1}{4} :: x :) \frac{3x}{4} =$ the sum *B* must pay;

And - - - $(\frac{1}{3} : \frac{1}{5} :: x :) \frac{3x}{5} =$ the sum *C* must pay;

And - - - $(\frac{1}{3} : \frac{1}{6} :: x :) \frac{x}{2} =$ the sum *D* must pay;

But - - - - $x + \frac{3x}{4} + \frac{3x}{5} + \frac{x}{2} = 20$ by question;

Or $20x + 15x + 12x + 10x = (20 \times 20 =)\ 400$,

That is - - - - - - $57x = 400$;

Th. - - - - - - $x = \left(\frac{400}{57} =\right) 7s.\ \frac{1}{57}$;

B must pay - - $\left(\frac{400}{57} \times \frac{3}{4} = \frac{300}{57} =\right) 5\ \frac{15}{57}$;

C ditto $\left(\frac{400}{57} \times \frac{3}{5} = \frac{80 \times 3}{57} = \frac{240}{57} =\right) 4\ \frac{12}{57}$;

D ditto - - $\left(\frac{400}{57} \times \frac{1}{2} = \frac{200}{57} =\right) 3\ \frac{29}{57}$.

QUEST.

QUEST. LXXIX. What two numbers are as 7 to 5, whose product is to their sum as 35 to 3?

If x be the greater required number;

Then - - $(7 : 5 :: x :)\frac{5x}{7}=$ the lesser;

Now - - $\left(\frac{5x}{7}\times x=\right)\frac{5xx}{7}=$ their product;

And $\left(\frac{5x}{7}+x=\frac{5x+7x}{7}=\right)\frac{12x}{7}=$ their sum:

But - - - - - $\frac{5xx}{7} : \frac{12x}{7} :: 35 : 3$ by question;

That is - $\left(\frac{5xx}{7}\times 3=\right)\frac{15xx}{7}=\frac{12x}{7}\times 35$;

Or (by division) - - $15x=12\times 35$;

Th. - - - - - - - - $x=(4\times 7=)\ 28$.

QUEST. LXXX. *A* and *B* severally cut packs of cards, so as to cut off less than they left; now what *A* left, added to what *B* cut off, make 50; also the cards left by both exceed those cut off by 64: How many did each cut off?

Suppose *A* cut off x, and *B* y cards;

Then *A* left $52-x$, and *B* $52-y$;

And the cards left by both $(=52-x+52-y)=104-x-y$;

From which if the cards cut off by both be taken } $104-2x-2y$ remains;

But - - - - $52-x+y=50$ } by question:

And - - - $104-2x-2y=64$ }

But (from first) $104-2x+2y=100$, by multiplication,

And (by subtraction) - $4y=(100-64=)\ 36$;

Th. - - - - - - - $y=9$:

And - $(52-50+9=)\ x=11$.

QUEST. LXXXI. Two pieces of cloth, of equal goodness, but of different lengths, were bought, the one for 5*l.* and the other for 6*l.* 10*s.* now if the lengths of both pieces were increased by 10, the numbers resulting will be in proportion as 5 to 6: How long was each piece, and how much did they cost a yard?

Suppose the price of 1 yard of each was x shillings;

Then $\frac{100}{x}$ will be the length of the least piece,

And $\frac{130}{x}$ ditto - - - - - - - - greatest;

Now $\frac{100}{x}+10 : \frac{130}{x}+10 :: 5 : 6$ by question;

Th. - $\frac{600}{x}+60=\frac{650}{x}+50$;

Or $600+60x=650+50x$;

Or $(60x-50x=)\ 10x=(650-600=)\ 50$;

Th. - - - - $x=\left(\frac{50}{10}=\right) 5$:

And - - $\left\{\begin{array}{l}\frac{100}{5}=20\\ \frac{130}{5}=26\end{array}\right\}$ were the lengths of the pieces.

QUEST. LXXXII. Suppose that for every 10 sheep a farmer kept, he should plow an acre of land; and be allowed one acre of pasture for every 4 sheep: How many sheep may that person keep, who farms 700 acres of land?

If x be the number of sheep required,

Then $(10 : 1 :: x :)\ \frac{x}{10}=$ acres plowed;

And $(4 : 1 :: x :)\ \frac{x}{4}=$ acres of pasture;

Now - - $\frac{x}{10}+\frac{x}{4}=700$ by question;

Or $(4x+10x=)\ 14x=700\times 40$;

Th. - - - - $x=(50\times 40=)\ 2000$.

QUEST. LXXXIII. What 3 numbers are those whose sum is 78; and $\frac{1}{3}$ of the first is to $\frac{1}{4}$ of the second as 1 to 2; also $\frac{1}{4}$ of the second is to $\frac{1}{5}$ of the third as 2 to 3?

If x, y, and z be the numbers required,

Then $\frac{x}{3} : \frac{y}{4} :: 1 : 2$; Th. $\frac{2x}{3}=\frac{y}{4}$:

And $\frac{y}{4} : \frac{z}{5} :: 2 : 3$; Th. $\frac{3y}{4}=\frac{2z}{5}$.

Whence $\frac{8x}{3}=y$; And $y=\frac{8z}{15}$:

Th. $\frac{8x}{3}=\frac{8z}{15}$; Th. $5x=z$:

But - - - - - - $x+y+z=78$ by question;

That is - - - - $x+\frac{8x}{3}+5x=78$,

Or - - - - - - $3x+8x+15x=(78\times3=)\ 234$;

Or - - - - - - - - - $26x=234$;

Th. - - - - - - - - - $x=\left(\frac{234}{26}=\right)9$.

QUEST. LXXXIV. It is required to find the value of y in the following equations, *viz.*

$\left.\begin{array}{l} ax+by=m \\ cx+dy=n \end{array}\right\}$ $\left\{\begin{array}{l}\text{where } a, b, c, d, m, \text{ and } n, \text{ are known} \\ \text{numbers?}\end{array}\right.$

Now $\left\{\begin{array}{l} cax+cby=cm \\ acx+ady=an \end{array}\right\}$ by multiplication;

And $cby-ady=cm-an$ by subtraction;

Th. - $y=\frac{cm-an}{cb-ad}$.

QUEST. LXXXV. There are four numbers, the first of which is to the second, as the third to the fourth; the sum of the first and second is 20, and of the third and fourth 12; but the difference of the second and third is 4: What are those numbers?

If x, y, u, and z be the numbers required;
Then $x : y :: u : z$; Or $xz = yu$;
$x + y = 20$; Or $y = 20 - x$:
$u + z = 12$; Or $z = 12 - u$:
$y - u = 4$; Or $y = u + 4$:
Now - - - - - - - - $u + 4 = 20 - x$;
Th. - - - - - - - - $x = 16 - u$:
But ($yu = xz$; that is) $\overline{u+4} \times u = \overline{16-u} \times \overline{12-u}$,
Or - - - - - - - - $uu + 4u = 192 - 28u + uu$,
Or - - - - - - - - - $4u = 192 - 28u$,
Or - - - ($28u + 4u =$) $32u = 192$;
Th. - - - - - - - - - $u = 6$.

QUEST. LXXXVI. The value of y in the following equations is required, *viz.*

$$\left.\begin{array}{l} \frac{x}{a} + \frac{y}{b} = m \\ \frac{x}{c} + \frac{y}{d} = n \end{array}\right\} ?$$

$$\text{Now}\left\{\begin{array}{l} x + \frac{ay}{b} = am \\ x + \frac{cy}{d} = cn \end{array}\right\} \text{by multiplication;}$$

And $\frac{ay}{b} - \frac{cy}{d} = am - cn$ by subtraction;

Or $day - bcy = \overline{am - cn} \times bd$;

Th. - - $y = \frac{am - cn}{da - bc} \times bd$.

QUEST.

QUEST. LXXXVII. Fifteen guineas are to be divided among four persons, A, B, C, and D, in such a manner, that A's share may be to B's, as 2 to 3, B's share to C's as 4 to 5, and C's share to D's as 6 to 7: What had each? If $x=A$'s share;

Then - - - - - $(2:3::x:)\frac{3x}{2}=B$'s share;

And - - - $(4:5::\frac{3x}{2}:)\frac{3\times5x}{2\times4}=C$'s share;

Also - $(6:7::\frac{3\times5x}{2\times4}:)\frac{3\times5\times7x}{2\times4\times6}=\frac{5\times7x}{4\times4}=D$'s;

Now - - - - $x+\frac{3x}{2}+\frac{15x}{8}+\frac{35x}{16}=(15\times21=)315$;

And $(16x+24x+30x+35x=)105x=315\times16$;

Therefore - - - - - - - $x=(3\times16=)48$.

QUEST. LXXXVIII. It is required to divide the number 128 into 4 such parts, that the first added to 7, the second less 7, the third multiplied by 7, and the fourth divided by 7; shall be equal among themselves?

Let u, x, y, and z, be the parts required;

Then $u+7=x-7=7y=\frac{z}{7}$ by question;

Th. $u+14=x$; $\frac{u+7}{7}=y$; and $\overline{u+7}\times7=z$;

But - - $u+x+y+z=128$ by question;

That is $u+\overline{u+14}+\frac{u+7}{7}+\overline{7u+49}=128$;

Or - $u+u+\frac{u+7}{7}+7u=(128-14-49=)65$;

Or - - - $9u+\frac{u+7}{7}=65$,

Or - - $63u+u+7=(65\times7=)455$,

Or - - - - - - $64u=(455-7=)448$;

Th. - - - - - - - $u=\left(\frac{448}{64}=\right)7$;

Whence - - - - - $x=7+14=21$;

And - - - - - - $y=\left(\frac{7+7}{7}\right)=2$;

Also - - - - - - $z=\overline{7+7}\times7=98$.

QUEST. LXXXIX. A gentleman had three horses, *A*, *B*, and *C*, and a saddle with its furniture worth 55*l.* now if the saddle be put on *A*'s back, he will be worth as much as *B* and *C*; if the saddle be put on *B*'s back, he will be worth twice as much as *A* and *C*; and if the saddle be put on *C*'s back, he will be worth thrice as much as *A* and *B*: What is each horse worth?

Suppose x, y, and z, to be the values of the three horses;

$$\left.\begin{array}{ll} \text{Then} & x+55=y+z \\ & y+55=(\overline{x+z}\times 2=)\ 2x+2z, \\ \text{And} & z+55=(\overline{x+y}\times 3=)\ 3x+3y, \end{array}\right\}\text{by quest.}$$

$$\text{By equation,}\left\{\begin{array}{ll} \text{first} \text{ - - } & x=y+z-55, \\ \text{second} \text{ - } & x=\dfrac{y+55-2z}{2}, \\ \text{third} \text{ - - } & x=\dfrac{z+55-3y}{3}; \end{array}\right.$$

$$\begin{array}{ll} \text{Th.} \text{ - } & y+z-55=\dfrac{y+55-2z}{2}, \\ \text{Or} & 2y+2z-110=y+55-2z; \\ \text{Whence} \text{ - - } & y=165-4z: \\ \text{Also} \text{ - } & y+z-55=\dfrac{z+55-3y}{3}, \\ \text{Or} & 3y+3z-165=z+55-3y; \\ \text{Whence} \text{ - - } & y=\dfrac{220-2z}{6}: \end{array}$$

$$\begin{array}{ll} \text{Now} \text{ - } & 165-4z=\dfrac{220-2z}{6}, \\ \text{Or} \text{ - } & 990-24z=220-2z, \\ \text{Or} \text{ - - - } & 770=22z; \\ \text{Th.} \text{ - - - } & 35=z; \\ & y=(165-140=)\ 25; \\ \text{And} \text{ - - - } & x=(25+35-55=)\ 5. \end{array}$$

QUEST. XC. *A* and *B* being at play, severally cut packs of cards, so as to take off more than they left; now it happened, that *A* cut off twice as many as *B* left; and *B* cut off 7 times as many as *A* left: How were the cards cut by each?

Suppose *A* cut off x cards, and *B* y cards;
Then *A* left $52-x$;
And *B* $52-y$.

But by question - $\begin{cases} x=(\overline{52-y}\times 2=)\ 104-2y, \\ y=(\overline{52-x}\times 7=)\ 364-7x; \end{cases}$

Now from the second $7x=364-y$;

Th. - - - - - - - $x=\dfrac{364-y}{7}$;

But - - - - $\dfrac{364-y}{7}=104-2y$;

And - - - $364-y=(\overline{104-2y}\times 7=)\ 728-14y$,

Or - $(14y-y=)\ 13y=(728-364=)\ 364$;

Th. - - - - - - $y=\left(\dfrac{364}{13}=\right)\ 28$;

And - - - - - - $x=\left(\dfrac{364-28}{7}=\dfrac{336}{7}=\right)\ 48$.

QUEST. XCI. Having first lost $\frac{1}{3}$ of my money at play, I won 3 times as much as I had left, $\frac{1}{2}$ as much money as I began with, and 50*l.* and then found, that I had as much above a 100*l.* as the sum I began with was below 100*l.* What sum did I begin with?

Suppose x the sum began with;

Then - - - $\left(x-\frac{x}{3}=\right)\frac{2x}{3}$ was left after $\frac{1}{3}x$ was lost;

And $\left(\frac{2x}{3}\times 3+\frac{x}{2}+50=\right)\frac{5x}{2}+50$ was afterwards won;

Th. - $\left(\frac{2x}{3}+\frac{5x}{2}+50=\right)\frac{19x}{6}+50$ $\left\{\begin{array}{l}\text{was the sum in}\\ \text{hand after wins.}\end{array}\right.$

But - $\frac{19x}{6}+50-100=100-x$ by question;

Or - - - $\frac{19x}{6}-50=100-x$;

Or - - - - - - $\frac{19x}{6}=150-x$;

And (by multip.) $19x=(\overline{150-x}\times 6=)\ 900-6x$;
Or $(19x+6x=)$ $25x=900$;
Th. - - - - - - $x=36$.

QUEST. XCII. It is required to divide 21 into two such parts, that the quotient of the greater part, divided by the lesser, may be to the quotient of the lesser part, divided by the greater, as 25 to 4?

If x be the greater part,
Then $21-x$ is the lesser;

And $\frac{x}{21-x}:\frac{21-x}{x}::25:4$, by question;

That is $\frac{4x}{21-x}=\frac{25\times\overline{21-x}}{x}$,

Or - - $4xx=25\times\overline{21-x}^2$;
Whence - $2x=(5\times\overline{21-x}=)\ 105-5x$,
And - - $7x=105$;
Th. - - $x=15$.

QUEST. XCIII. What 3 numbers are thoſe, the difference of whoſe differences is 2, the ſum of the two firſt numbers is equal to the third, and the ſum of the firſt and third is to the ſecond, as 17 to 8?

If x, y, and z, be the numbers required;
Then $x+z : y :: 17 : 8$ by queſtion;
That is - $8x+8z=17y$.
But - - - $x+y=z$ by queſtion;
And - - $8x+8y=8z$:
Th. - $8x+8x+8y=17y$;
Or. - - - $16x=9y$:
Th. - - - - $x=\frac{9y}{16}$;

And $\left(\frac{9y}{16}+y=\right)\frac{25y}{16}=z$: Becauſe $x+y=z$:

Now - - - $z-y=\left(\frac{25y}{16}-y=\right)\frac{9y}{16}$;

And - - - $y-x=\left(y-\frac{9y}{16}=\right)\frac{7y}{16}$:

But (their diff.) $\frac{2y}{16}=2$ by queſtion;

Th. - - - - - $y=16$;

$x=\frac{9\times 16}{16}=9$;

And - - - - $z=\left(\frac{25\times 16}{16}=\right) 25$.

QUEST.

QUEST. XCIV. A lady being asked how many yards of silk she had in her gown, replied, that if she had bought 2 yards more, at a price greater by three shillings a yard, the gown would have cost 64 shillings more than it did; and if she had bought 3 yards more at an advance of 2 shillings a yard, it would have cost 68 shillings more than it did: How many yards were in it?

Let x be the yards in the gown, at y shillings a yard.
Then the gown cost xy shillings;

$$\left.\begin{array}{l} \text{But } (\overline{x+2}\times\overline{y+3}=)\ xy+3x+2y+6=xy+64, \\ \text{And } (\overline{x+3}\times\overline{y+2}=)\ xy+2x+3y+6=xy+68, \end{array}\right\}\text{by q.}$$

$$\begin{array}{ll} \text{That is} & 3x+2y=(64-6=)\ 58, \\ \text{And} & 2x+3y=(68-6=)\ 62; \\ \text{Wh. (by sub.)} & y-x=(62-58=)\ 4; \\ \text{Th.} & y=x+4: \\ \text{But} & 3x+\overline{x+4}\times 2=58; \\ \text{That is} & 5x+8=58; \\ \text{Or} & 5x=50; \\ \text{Th.} & x=10. \end{array}$$

QUEST. XCV. What two numbers are those, whose sum, multiplied by the greater, produces 120; and by the lesser, 105?

If x be the greater, and y the lesser number;

$$\left.\begin{array}{ll} \text{Then} & (\overline{x+y}\times x=)\ xx+xy=120 \\ \text{And} & (\overline{x+y}\times y=)\ xy+yy=105 \end{array}\right\}\text{by question:}$$

$$\begin{array}{ll} \text{The sum of which equations is} & xx+2xy+yy=225; \\ \text{Th. its square root} & x+y=15: \\ \text{But (by first)} & (\overline{x+y}\times x=)\ 15x=120; \\ \text{Th.} & x=8; \\ \text{And} & y=7. \end{array}$$

QUEST.

QUEST. XCVI. Three men, *A*, *B*, and *C*, enter partnership; *A* paid in as much as *B* and $\frac{1}{3}$ of *C*; *B* paid in as much as *C* and $\frac{1}{3}$ of *A*; and *C* paid in 10*l*. and $\frac{1}{3}$ of *A*: What did each man contribute to the stock?

Suppose *A*, paid x; *B*, y; and *C*, z pounds;

$$\text{Then (by quest.)}\begin{cases} x = \left(y + \frac{z}{3} =\right) \frac{3y+z}{3}: \\ y = z + \frac{x}{3}; \text{ Or } 3y - 3z = x: \\ z = 10 + \frac{x}{3}; \text{ Or } 3z - 30 = x: \end{cases}$$

Whence $\frac{3y+z}{3} = 3y - 3z$,

Or - $3y + z = (\overline{3y - 3z} \times 3) = 9y - 9z$;

Th. - - $z = \frac{6y}{10}$:

Also - $\frac{3y+z}{3} = 3z - 30$,

Or - $3y + z = 9z - 90$,

Th. - $\frac{3y+90}{8} = z$:

But - $\frac{3y+90}{8} = \frac{6y}{10}$,

Or $30y + 900 = 48y$;

Th. $\left(\frac{900}{18} =\right) 50 = y$;

$$z = \left(\frac{6 \times 50}{10} =\right) 30:$$

And - - $x = (3 \times 30 - 30 =)\ 60$.

QUEST.

QUEST. XCVII. Three persons, *A*, *B*, and *C*, being at play, *A* won ½ the money that *B* and *C* had, and carried off 153*l*. now if *B* had won ⅓ of what *A* and *C* had, or if *C* had won ¼ of what *A* and *B* had, they would have carried off the same sum? How much had each?

If x be *A*'s money; y, *B*'s; and z, *C*'s:

Then (by quest.)
$$\begin{cases} x+\frac{y+z}{2}=153;\ \text{Or } 2x+y+z=306; \\ y+\frac{x+z}{3}=153;\ \text{Or } 3y+x+z=459; \\ z+\frac{x+y}{4}=153;\ \text{Or } 4z+x+y=612: \end{cases}$$

Th. $\frac{306-y-z}{2}=(x=)\ 459-3y-z,$

Or $306-y-z=918-6y-2z,$

Th. - - $y=\frac{612-z}{5}:$

Also $\frac{306-y-z}{2}=(x=)612-4z-y,$

Or $306-y-z=1224-8z-2y,$

Th. - - $y=918-7z,$

But $\frac{612-z}{5}=918-7z,$

Or $612-z=4590-35z,$

Th. - - $z=\left(\frac{3978}{34}=\frac{1989}{17}=\right)\ 117;$

$y=(918-819=)\ 99;$

$x=(459-297-117=)\ 45.$

QUEST.

QUEST. XCVIII. To find three numbers, so that $\frac{1}{2}$ the first, $\frac{1}{3}$ of the second, and $\frac{1}{4}$ of the third, shall be equal to 62; $\frac{1}{3}$ of the first, $\frac{1}{4}$ of the second, and $\frac{1}{5}$ of the third, equal to 47; and $\frac{1}{4}$ of the first, $\frac{1}{5}$ of the second, and $\frac{1}{6}$ of the third, equal to 38?

If x, y, and z, represent the numbers required;

$$\text{Th.}\ \frac{x}{2}+\frac{y}{3}+\frac{z}{4}=62;\ \text{or}\ 6x+4y+3z=(62\times12=)744;$$

$$\frac{x}{3}+\frac{y}{4}+\frac{z}{5}=47;\ \text{or}\ 20x+15y+12z=(47\times60=)2820;$$

$$\frac{x}{4}+\frac{y}{5}+\frac{z}{6}=38;\ \text{or}\ 15x+12y+10z=(38\times60=)2280;$$

$$\begin{array}{ll}
\text{But} & \left.\begin{array}{r}120x+80y+60z=(744\times20=)\ 14880\\ 100x+75y+60z=(2820\times5=)\ 14100\\ 90x+72y+60z=(2280\times6=)\ 13680\end{array}\right\}\text{by mul.}\\
\text{And} & \\
\text{Th.} & \left.\begin{array}{r}20x+5y=(14880-14100=)\ 780\\ 10x+3y=(14100-13680=)\ 420\end{array}\right\}\text{by sub.}\\
\text{And} & \\
\text{Now}\ \text{- - - -} & 20x+6y=(420\times2=)\ 840\ \text{by mul.}\\
\text{Th.}\ \text{- - - - - -} & y=(840-780=)\ 60\ \text{by subtr.}\\
\text{But}\ \text{- - -} & 10x+(3\times60=)\ 180=420,\\
\text{Or}\ \text{- - - - -} & 10x=(420-180=)\ 240;\\
\text{Th.}\ \text{- - - - - - - - - - - -} & x=\left(\frac{240}{10}=\right)24;\\
\text{Also}\ \text{- - - -} & 6\times24+4\times60+3z=744,\\
\text{Or}\ \text{- - - - - -} & 144+240+3z=744,\\
\text{Or}\ \text{- - - -} & 3z=(744-144-240=)\ 360;\\
\text{Th.}\ \text{- - - - - - - - - - - -} & z=\left(\frac{360}{3}=\right)120.
\end{array}$$

QUEST. XCIX. A gentleman left some money to be divided among four servants, so that the share of the first should be equal to $\frac{1}{2}$ the sum of the shares of the other three; that the share of the second should be $\frac{1}{3}$ of the sum of the shares of the other three; and that the share of the third should be $\frac{1}{4}$ of the sum of the shares of the other three; now, upon dividing the money in this manner, it was found, that the share of the first exceeded that of the last by 14 pounds: What was the share of each person?

If x, y, u, and z, represent the shares required?

$$\left.\begin{array}{lll} \text{Then} - & x=\dfrac{y+u+z}{2}; & \text{or } 2x=y+u+z, \\ & y=\dfrac{x+u+z}{3}; & \text{or } 3y=x+u+z, \\ & u=\dfrac{x+y+z}{4}; & \text{or } 4u=x+y+z, \\ & x-z=14; & \text{or } x=14+z, \end{array}\right\} \text{by question:}$$

$$\text{Now by adding} \left\{\begin{array}{c} x \\ y \\ u \end{array}\right\} \text{to the} \left\{\begin{array}{l} \text{first} \\ \text{second} \\ \text{third} \end{array}\right\} \text{equat.} \left\{\begin{array}{l} 3x=x+y+u+z, \\ 4y=x+y+u+z, \\ 5u=x+y+u+z; \end{array}\right.$$

Th. - - $3x=4y$, and $3x=5u$;

Whence - $\frac{3x}{4}=y$, and $\frac{3x}{5}=u$;

But (by the fourth) - $x-14=z$;

And (by the first) - - - $2x=\frac{3x}{4}+\frac{3x}{5}+x-14$,

Which (mult. by 20 give) $40x=15x+12x+20x-280$;

Or (by transposition) - $280=(47x-40x=)\ 7x$;

Th. - - - - - - - - $40=x$

$30=y$

$24=u$

$26=z$.

QUEST. C. A shepherd, in time of war, was plundered by a party of soldiers, who took $\frac{1}{4}$ of his flock, and $\frac{1}{4}$ of a sheep; another party took from him $\frac{1}{3}$ of what he had left, and $\frac{1}{3}$ of a sheep; also a third party took $\frac{1}{2}$ of what were then left, and $\frac{1}{2}$ a sheep; which being done, he had but 25 sheep left: How many had he at first?

Suppose he had x sheep at first;

Then the 1st party took $\left(\frac{x}{4}+\frac{1}{4}=\right)\frac{x+1}{4}$,

And there remained - $\left(x-\frac{x+1}{4}=\right)\frac{3x-1}{4}$,

The 2d party took - $\left(\frac{3x-1}{3\times4}+\frac{1}{3}=\frac{3x+3}{3\times4}=\right)\frac{x+1}{4}$,

And then remained $\left(\frac{3x-1}{4}-\frac{x+1}{4}=\frac{2x-2}{4}=\right)\frac{x-1}{2}$;

The 3d party took - $\left(\frac{x-1}{2\times2}+\frac{1}{2}=\right)\frac{x+1}{4}$,

And then remained $\left(\frac{x-1}{2}-\frac{x+1}{4}=\right)\frac{x-3}{4}$:

But (by question) - - - - $\frac{x-3}{4}=25$.

Or - - - - - - - - - - $x-3=(25\times4=)\ 100$;

Th. - - - - - - - - - - $x=103$.

QUEST. CI. A trader maintained himself for 3 years, at the expence of 50*l.* a year; and in each of those years augmented that part of his stock, which was not so expended, by $\frac{1}{3}$ thereof; at the end of the third year his original stock was doubled: What had he at first?

Suppose his original stock was x *l.*

Then - - - - - - - - - - $x-50=$ the sum traded with the 1st year,

And - - - - - - - - - - $\frac{x-50}{3}=$ the sum gained therein;

Th. - - $\left(x-50+\frac{x-50}{3}=\right)\frac{4x-200}{3}=$ sum possessed at the end of it;

Also - $\left(\frac{4x-200}{3}-50=\right)\frac{4x-350}{3}=$ sum traded with the 2d year,

And - - - - - - - - $\frac{4x-350}{9}=$ sum gained therein:

Th. $\left(\frac{4x-350}{3}+\frac{4x-350}{9}=\right)\frac{16x-1400}{9}=$ sum possessed at the end of it;

Again $\left(\frac{16x-1400}{9}-50=\right)\frac{16x-1850}{9}=$ sum traded with the 3d year,

And - - - - - - - - - $\frac{16x-1850}{27}=$ sum gained therein;

Th. - - - $\frac{16x-1850}{9}+\frac{16x-1850}{27}=$ sum possessed at the end of it:

That is - - - - - - - $\frac{64x-7400}{27}=2x$ by quest.

Or - - - - - - - - - $64x-7400=54x$,

Or - - - - - - - - - - - - $10x=7400$;

Th. - - - - - - - - - - - - - $x=740$.

QUEST.

QUEST. CII. A man being at play, loſt $\frac{1}{4}$ of his money, and then won 3 ſhillings; after which, he loſt $\frac{1}{3}$ of what he then had, and won 2 ſhillings; laſtly, he loſt $\frac{1}{7}$ of what he then had; this done, he had but 12 ſhillings left: What had he at firſt?

Suppoſe he had x ſhillings at firſt;

$$\text{Then after}\left\{\begin{array}{ll} \text{loſing} \;\text{- - - - -} & \frac{x}{4} \\ \text{winning} \;\text{- - - -} & 3 \\ \text{loſing}\left(\frac{3x+12}{3\times4}=\right) & \frac{x+4}{4} \\ \text{winning} \;\text{- - - -} & 2 \\ \text{loſing} \;\text{- - - -} & \frac{x+8}{7\times2} \end{array}\right\}\text{he had left}\left\{\begin{array}{l} x-\frac{x}{4}=\frac{3x}{4}, \\ \frac{3x}{4}+3=\frac{3x+12}{4}; \\ \frac{3x+12}{4}-\frac{x+4}{4}=\left(\frac{2x+8}{4}=\right)\frac{x+4}{2}, \\ \frac{x+4}{2}+2=\frac{x+8}{2}; \\ \frac{x+8}{2}-\frac{x+8}{14}=\left(\frac{6x+48}{14}=\right)\frac{3x+24}{7}: \end{array}\right.$$

But (by queſtion) . . . . . . $\frac{3x+24}{7}=12,$

Or . . . . $\frac{x+8}{7}=4,$

Or . . . . $x+8—28;$

Th. . . . . $x=20.$

QUEST. CIII. *A*, *B*, and *C*, (who had among them 480 shillings) went to play; the first game *A* lost ½ his money, equally to *B* and *C*; the second game *B* lost $\frac{1}{3}$ of the money he then had, equally to *A* and *C*; the third game *C* lost 40 shillings to *A*, and 40 shillings to *B*; now upon counting their money, it appeared, that they had each an equal sum; What had each at first?

Suppose *A* had x; *B*, y; and *C*, z shillings;

$$\text{Then after the 1st game}\left\{\begin{array}{l} A \text{ had } \frac{x}{2}, \\ B \text{ had } y+\frac{x}{4}, \\ C \text{ had } z+\frac{x}{4}; \end{array}\right.$$

$$\text{After the 2d game}\left\{\begin{array}{l} A \text{ had } \left(\frac{x}{2}+\frac{y}{6}+\frac{x}{24}=\right)\frac{13x}{24}+\frac{y}{6}, \\ B \text{ had } \left(y+\frac{x}{4}-\frac{y}{3}-\frac{x}{12}=\right)\frac{2y}{3}+\frac{x}{6}, \\ C \text{ had } \left(z+\frac{x}{4}+\frac{y}{6}+\frac{x}{24}=\right)z+\frac{7x}{24}+\frac{y}{6}; \end{array}\right.$$

$$\text{After the 3d game}\left\{\begin{array}{l} A \text{ had } \left(\frac{13x}{24}+\frac{y}{6}+40=\right)\frac{13x+4y+960}{24}, \\ B \text{ had } \left(\frac{2y}{3}+\frac{x}{6}+40=\right)\frac{16y+4x+960}{24}, \\ C \text{ had } \left(z+\frac{7x}{24}+\frac{y}{6}-80=\right)\frac{24z+7x+4y-1920}{24}; \end{array}\right.$$

$$\text{But by quest.}\left\{\begin{array}{l} \frac{13x+4y+960}{24}=\frac{16y+4x+960}{24}, \\ \frac{13x+4\ \ +960}{24}=\frac{24z+7x+4y-1920}{24}; \end{array}\right.$$

$$\text{Or} \quad - \quad - \quad \left\{\begin{array}{l} 13x+4y=16y+4x, \\ 13x+4y=24z+7x+4y-2880; \end{array}\right.$$

That

That is - $\begin{cases} 9x=12y; & \text{Or} - \frac{3x}{4}=y: \\ 6x=24z-2880; & \text{Or } \frac{480+x}{4}=z: \end{cases}$

But by queſt. $480=x+y+z$;

That is - $480=x+\frac{3x}{4}+\frac{480+x}{4}$,

Or - - $1920=(4x+3x+480+x=)\ 8x+480$,
Or - - $1440=8x$;
Th. - - $180=x$,

$$y=\left(\frac{3\times 180}{4}=\right)135;$$

And - - - $z=\left(\frac{480+180}{4}=\right)165$.

QUEST. CIV. There are two numbers, in proportion as 3 to 5; the ſum of whoſe ſquares is 306: What are thoſe numbers?

Suppoſe x the leſſer of thoſe numbers;

Then - - $(3:5::x:)\frac{5x}{3}=$ the greater;

But - - $x^2+\frac{5\times 5\times x^2}{3\times 3}=306$ by queſtion;

That is $\left(\frac{9x^2+25x^2}{9}=\right)\frac{34x^2}{9}=306$,

Or - - - - - - - $\frac{xx}{9}=\left(\frac{306}{34}=\right)9$;

Th. - - - - - - - $x^2=(9\times 9=)\ 9^2$,
And (the root thereof) - $x=9$.

QUEST. CV. If A and B together, can perform a piece of work in 8 days; A and C together in 9 days; and B and C in 10 days. How many days will it take each person alone to perform the same work?

Suppose A could perform it in x days, B in y, and C in z days; and because the work to be performed by all is the same, let it be represented by unity:

$$\left.\begin{array}{ll} \text{Then } (x : 1 :: 8 :) \frac{8}{x} = \\ \text{And } (y : 1 :: 8 :) \frac{8}{y} = \\ \text{Also } (x : 1 :: 9 :) \frac{9}{x} = \\ \text{And } (z : 1 :: 9 :) \frac{9}{z} = \\ \text{Again } (y : 1 :: 10 :) \frac{10}{y} = \\ \text{And } (z : 1 :: 10 :) \frac{10}{z} = \end{array}\right\} \text{the work performed by} \left\{\begin{array}{l} A \text{ in 8 days;} \\ B \text{ in 8 days;} \\ A \text{ in 9 days;} \\ C \text{ in 9 days;} \\ B \text{ in 10 days;} \\ C \text{ in 10 days;} \end{array}\right.$$

$$\left.\begin{array}{l} \text{But } \frac{8}{x} + \frac{8}{y} = 1; \text{ Or } \frac{1}{x} + \frac{1}{y} = \frac{1}{8} \\ \text{And } \frac{9}{x} + \frac{9}{z} = 1; \text{ Or } \frac{1}{x} + \frac{1}{z} = \frac{1}{9} \\ \text{Also } \frac{10}{y} + \frac{10}{z} = 1; \text{ Or } \frac{1}{y} + \frac{1}{z} = \frac{1}{10} \end{array}\right\} \text{by question.}$$

$$\text{Th. (their sum) } \frac{1}{x} + \frac{1}{y} + \frac{1}{z} \times 2 = \frac{1}{8} + \frac{1}{9} + \frac{1}{10};$$

$$\text{Or (by division) } \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{16} + \frac{1}{18} + \frac{1}{20}:$$

But

$$\text{But because}\left\{\begin{aligned}&\frac{1}{x}+\frac{1}{y}=\frac{1}{8},\\&\frac{1}{x}+\frac{1}{z}=\frac{1}{9},\\&\frac{1}{y}+\frac{1}{z}=\frac{1}{10};\end{aligned}\right\}\text{Therefore}\left\{\begin{aligned}&\frac{1}{z}=\frac{1}{16}+\frac{1}{18}+\frac{1}{20}-\frac{1}{8},\\&\frac{1}{y}=\frac{1}{16}+\frac{1}{18}+\frac{1}{20}-\frac{1}{9},\\&\frac{1}{x}=\frac{1}{16}+\frac{1}{18}+\frac{1}{20}-\frac{1}{10};\end{aligned}\right.$$

$$\text{Or}\ -\left\{\begin{aligned}&\frac{1}{z}=\left(\frac{1}{18}+\frac{1}{20}-\frac{1}{16}=\right)\frac{10\times8+9\times8-9\times10}{9\times10\times8\times2},\\&\frac{1}{y}=\left(\frac{1}{16}+\frac{1}{20}-\frac{1}{18}=\right)\frac{10\times9+8\times9-8\times10}{9\times10\times8\times2},\\&\frac{1}{x}=\left(\frac{1}{16}+\frac{1}{18}-\frac{1}{20}=\right)\frac{9\times10+8\times10-8\times9}{9\times10\times8\times2};\end{aligned}\right.$$

$$\text{Th.}\left\{\begin{aligned}&z=\left(\frac{9\times10\times8\times2}{10\times8+9\times8-9\times10}=\right)23\tfrac{7}{31},\\&y=\left(\frac{9\times10\times8\times2}{10\times9+8\times9-8\times10}=\right)17\tfrac{23}{41},\\&x=\left(\frac{9\times10\times8\times2}{9\times10+8\times10-8\times9}=\right)14\tfrac{34}{49}.\end{aligned}\right.$$

QUEST. CVI. How many yards were there in a piece of cloth, that cost 15 guineas, the price of a yard being to the number of yards as 5 to 7?

Suppose there were x yards;

Then. - $(7:5::x:)\ \frac{5x}{7}=$ price of a yard;

And - $(x\times\frac{5x}{7}=)\ \frac{5xx}{7}=15\times21$ by question;

Th. (by division) - $\frac{xx}{7}=(3\times21=)\ 3^2\times7$,

And (by multiplication) $xx=3^2\times7^2$;

Th. (the root thereof) $x=(3\times7=)\ 21$.

QUEST. CVII. *A*, *B*, and *C*, playing together; in the first game, *A* lost to *B*, as much as *B* began with, and to *C*, as much as *C* began with; in the second game, *B* lost to *C*, as much as *C* then had, and to *A*, as much as *A* had left; in the third game, *C* lost to *A*, as much as *A* then had, and to *B*, as much as *B* had left; and then each had 8 shillings: What had each at first?

Suppose *A* had x; *B*, y; and *C*, z shillings;

$$\left.\begin{array}{l}\text{Then}\\ \text{after}\\ \text{the 1st}\\ \text{game}\end{array}\right\}\begin{array}{l}A\text{ had } x-y-z,\\ B\text{ had } 2y,\\ C\text{ had } 2z;\end{array}$$

$$\left.\begin{array}{l}\text{after}\\ \text{the 2d}\\ \text{game}\end{array}\right\}\begin{array}{l}A\text{ had } 2x-2y-2z,\\ B\text{ had } (2y-2z-x+y+z=)\ 3y-z-x,\\ C\text{ had } 4z;\end{array}$$

$$\left.\begin{array}{l}\text{after}\\ \text{the 3d}\\ \text{game}\end{array}\right\}\begin{array}{l}A\text{ had } 4x-4y-4z,\\ B\text{ had } 6y-2z-2x,\\ C\text{ had } (4z-2x+2y+2z-3y+z+x=)\,7z-x-y:\end{array}$$

$$\left.\begin{array}{l}\text{but by}\\ \text{quest.}\end{array}\right\}\begin{array}{lll}4x-4y-4z=8; & \text{Or} \quad - \quad - & x=2+y+z:\\ 6y-2x-2x=8; & \text{Or} \quad - \quad - & x=3y-z-4:\\ 7z-x-y=8; & \text{Or} \quad - \quad - & x=7x-y-8:\end{array}$$

$$\begin{array}{ll}\text{Whence} \quad - & 2+y+z=3y-z-4,\\ \text{Th.} \quad - - - - & z=y-3;\\ \text{Also} \quad - - & 2+y+z=7z-y-8,\\ \text{Or} \quad - - & 10+2y=6z,\\ \text{Th.} \quad - - & \frac{5+y}{3}=y-3,\end{array}$$

$$\begin{array}{ll}\text{But} \quad - - - & \frac{5+y}{3}=y-3,\\ \text{Or} \quad - - - - & 5+y=3y-9;\\ \text{Th.} \quad - - - - & 7=y;\\ \text{And} \quad - - - - & z=(7-3=)\ 4;\\ & x=(2+7+4=)\ 13.\end{array}$$

QUEST.

QUEST. CVIII. If a vintner mixes ſherry and ſack in quantities proportional as 3 to 1, the mixture will be worth but 56 pence a gallon; but if the proportion were as 5 to 3, it would be worth 60 pence a gallon: Required the price of each wine?

Suppoſe there were x gallons of ſherry,
And that the ſherry was y, and the ſack z pence a gallon;

Then $(3:1::x:)\ \frac{x}{3}=$ gallons of ſack in the firſt mixture.

Th. $(x+\frac{x}{3}=)\frac{4x}{3}=$ gallons of both in ditto;

Alſo $(5:3::x:)\frac{3x}{5}=$ gall. of ſack in the ſecond mixture.

Th. $(x+\frac{3x}{5}=)\frac{8x}{5}=$ gallons of both in ditto.

Whence - $xy+\frac{xz}{3}=\left(\frac{4x}{3}\times 56=\right)\frac{224x}{3}$,

And - $xy+\frac{3xz}{5}=\left(\frac{8x}{5}\times 60=\frac{480x}{5}=\right)96x$;

$$\left.\begin{array}{ll}\text{That is} - & y+\frac{z}{3}=\frac{224}{3} \\ \text{And} - - & y+\frac{3z}{5}=96\end{array}\right\}\text{by diviſion;}$$

Alſo - - $\frac{3z}{5}-\frac{z}{3}=96-\frac{224}{3}$ by ſubtraction;

Or - - - $\frac{4z}{15}=\frac{64}{3}$;

Th. - - - $z=\frac{64\times 15}{4\times 3}=16\times 5=80$:

But - $y+\frac{3\times 80}{5}=96$; Or $y+3\times 16=96$;

Th. - - - $y=(96-48=)\ 48$.

QUEST. CIX. Three workmen can severally do a piece of work in the following times, *viz.* A can perform it once in 3 weeks, B thrice in 8 weeks, and C five times in 12 weeks: In what time will it be done if they all work together?

Or generally.

The forces of several agents being given, to determine (x) the time wherein they will jointly produce a given effect E?

If $\left\{\begin{matrix} A \\ B \\ C \\ \&c. \end{matrix}\right\}$ can perform $E \left\{\begin{matrix} a \\ b \\ c \end{matrix}\right\}$ times in the time $\left\{\begin{matrix} \mathfrak{a}, \\ \mathfrak{b}, \\ \mathfrak{c}: \end{matrix}\right.$

Then $\left\{\begin{matrix} (\mathfrak{a} : a :: x :)\dfrac{ax}{\mathfrak{a}} = \\ (\mathfrak{b} : b :: x :)\dfrac{bx}{\mathfrak{b}} = \\ (\mathfrak{c} : c :: x :)\dfrac{cx}{\mathfrak{c}} = \end{matrix}\right\}$ the effect which will be produced in the time x by $\left\{\begin{matrix} A, \\ B, \\ C. \end{matrix}\right.$

That is - $\dfrac{ax}{\mathfrak{a}} + \dfrac{bx}{\mathfrak{b}} + \dfrac{cx}{\mathfrak{c}} = E$:

Th. $x = \dfrac{E}{\dfrac{a}{\mathfrak{a}} + \dfrac{b}{\mathfrak{b}} + \dfrac{c}{\mathfrak{c}}}$.

In this example $x = \left(\dfrac{1}{\frac{1}{3} + \frac{3}{8} + \frac{5}{12}} = \dfrac{24}{8+9+10} = \right) \frac{8}{9}$.

QUEST.

QUEST. CX. If two poſt-boys *A* and *B*, being 59 miles aſunder, ſet out to meet each other; *A* going 7 miles in 2 hours, and *B* (who began his journey 1 hour later than *A*) going 8 miles in 3 hours: How many miles will each travel before they meet?

Or generally,

The velocities or celerities of two moveable bodies *A* and *B*, tending, in oppoſite directions, to the ſame place; together with the diſtance or interval of the places, and times, from, and in which, they begin to move, being given; To find the place of their meeting?

Let the diſtance of *A* from the place of meeting be x;
———— *A* from *B* ———— d;
And the interval of time between their ſetting out be t:

$$\left.\begin{matrix}\text{Now if } A \\ \text{And - } B\end{matrix}\right\} \text{ move through the ſpace } \left\{\begin{matrix} a \\ b \end{matrix}\right\} \text{ in the time } \left\{\begin{matrix} \mathfrak{a}; \\ \mathfrak{b}: \end{matrix}\right.$$

Then - - $(a : \mathfrak{a} :: x :)\ \frac{\mathfrak{a}x}{a} =$ the time *A* moves,

And $(b : \mathfrak{b} :: d-x :)\ \frac{\overline{d-x} \times \mathfrak{b}}{b} =$ the time *B* moves:

But - - $\frac{\mathfrak{a}x}{a} - \frac{\overline{d-x} \times \mathfrak{b}}{b} = t$ by queſtion;

Or - - - $b\mathfrak{a}x - a\mathfrak{b}d + a\mathfrak{b}x = abt$ by multiplication;

Or (by addition) - $b\mathfrak{a}x + a\mathfrak{b}x = abt + a\mathfrak{b}d$;

Th. - - - - - - - $x = \frac{bt + \mathfrak{b}d}{b\mathfrak{a} + a\mathfrak{b}} \times a$.

In the above example - $x = \left(\frac{8 \times 1 + 3 \times 59}{8 \times 2 + 7 \times 3} \times 7 =\right) 35$.

QUEST. CXI. The velocities or celerities of 2 movable bodies A and B, tending in the same direction, to the same place; together with the distance or interval, of the places, and times, from, and in which, they begin to move, being given: To find the place where they will be together?

Let $\left\{\begin{array}{l}\text{the distance of } A \text{ (the hindermost body) from} \\ \quad \text{the required place be - - - - - - - -} \quad x, \\ \text{the distance of } A \text{ from } B \text{ be - - - - - -} \quad d, \\ \text{the interval of time between their setting out be} \quad t;\end{array}\right.$

$\begin{array}{l}\text{Now if } A \\ \text{And} \quad B\end{array}\Big\}$ moves through the space $\left\{\begin{array}{c}a \\ b\end{array}\right\}$ in the time $\left\{\begin{array}{c}\mathfrak{a}: \\ \mathfrak{b}:\end{array}\right.$

(But $\dfrac{\mathfrak{a}}{a} \sqsupset \dfrac{\mathfrak{b}}{b}$, or the question will be impossible.)

Then - - $(a : \mathfrak{a} :: x :)\ \dfrac{\mathfrak{a}x}{a} =$ the time A moves,

And $(b : \mathfrak{b} :: x-d :)\ \dfrac{\overline{x-d} \times \mathfrak{b}}{b} =$ the time B moves,

Now CASE I. If A sets out before B;

$$\text{Then} \quad - \quad - \quad \frac{\mathfrak{a}x}{a} - \frac{\overline{x-d} \times \mathfrak{b}}{b} = t,$$

$$\text{Or} \quad - \quad - \quad - \quad b\mathfrak{a}x - a\mathfrak{b}x + a\mathfrak{b}d = abt;$$

$$\text{Or} \quad - \quad - \quad - \quad - \quad b\mathfrak{a}x - a\mathfrak{b}x = abt - a\mathfrak{b}d;$$

$$\text{Th.} \quad - \quad - \quad - \quad - \quad - \quad - \quad x = \frac{bt - \mathfrak{b}d}{b\mathfrak{a} - a\mathfrak{b}} \times a:$$

But CASE II. If B sets out before A;

$$\text{Then} \quad - \quad - \quad \frac{\overline{x-d} \times \mathfrak{b}}{b} - \frac{\mathfrak{a}x}{a} = t,$$

$$\text{Or} \quad - \quad - \quad - \quad - \quad - \quad a\mathfrak{b}x - b\mathfrak{a}x = abt + a\mathfrak{b}d;$$

$$\text{Th.} \quad - \quad - \quad - \quad - \quad - \quad - \quad - \quad x = \frac{bt + \mathfrak{b}d}{a\mathfrak{b} - b\mathfrak{a}} \times a.$$

EXAM-

EXAMPLE to CASE I.

B, travelling 5 miles in 2 hours, sets out 4 hours after *A*, who was 59 miles behind him, and goes 10 miles in 3 hours: How many miles must *A* go to overtake *B*?

Here $x = (\frac{5\times4-2\times59}{5\times3-10\times2}\times10=\frac{-98}{-5}\times10=98\times2=)196.$

EXAMPLE to CASE II.

B, travelling 5 miles in 2 hours, sets out 4 hours before *A*, who was 59 miles behind him, and goes 10 miles in 3 hours: How many miles must *A* travel to overtake *B*?

Here $x = (\frac{5\times4+2\times59}{10\times2-5\times3}\times10=\frac{138}{5}\times10=138\times2=)276.$

QUEST. CXII The weight (w) of any mixture, together with the specific gravity thereof (s), and of each of the two things mixed $(\mathfrak{a}, \mathfrak{b})$, being given: To find the quantity of each of the things mixed?

Let x be the weight of that simple whose specific gravity $\mathfrak{a}$ is greatest;

Then $w-x=$ the weight of the other simple:

$$\left.\begin{aligned}\text{But}\quad \frac{x}{\mathfrak{a}}&=\\ \frac{w-x}{\mathfrak{b}}&=\\ \text{And}\quad \frac{w}{s}&=\end{aligned}\right\}\text{the magnitude of the body whose weight is}\left\{\begin{aligned}&x,\\ &w-x,\\ &w;\end{aligned}\right.$$

Whence $\frac{x}{\mathfrak{a}}+\frac{w-x}{\mathfrak{b}}=\frac{w}{s}$;

Or $s\mathfrak{b}x+s\mathfrak{a}w-s\mathfrak{a}x=\mathfrak{a}\mathfrak{b}w$;

Or - $s\mathfrak{a}w-\mathfrak{a}\mathfrak{b}w=s\mathfrak{a}x-s\mathfrak{b}x$;

Th. - $\frac{\overline{s-\mathfrak{b}}\times\mathfrak{a}w}{\overline{\mathfrak{a}-\mathfrak{b}}\times s}=x$.

EXAMPLE.

What quantity of gold is there, in a mixture of gold and silver whose weight is 85 ounces, and specific gravity 15; the specific gravity of gold being 19; and of silver $10\frac{1}{2}$?

Here $x=\left(\frac{\overline{15-\frac{21}{2}}\times19\times85}{\overline{19-\frac{21}{2}}\times15}=\frac{9\times19\times85}{17\times15}=19\times3=\right)57$.

QUEST.

QUEST. CXIII. Two partners, *A* and *B*, dividing their gain (60*l.*) *B* took 20; *A*'s money continued in trade 4 months; and if the uumber 50 be divided by *A*'s money, the quotient will be the time that *B*'s money (*viz.* 100*l.*) continued in trade: What was *A*'s money?

Suppose *A*'s money was x pounds;

Then $\frac{50}{x}=$ the time *B*'s money was in trade;

And $4x+\frac{50}{x}\times 100 : 60 :: \frac{50}{x}\times 100 : 20$;

That is $80x+\frac{100000}{x}=\frac{300000}{x}$,

Or - $80xx+100000=300000$,

Or - - - $80xx=(300000-100000=)\ 200000$,

Or - - - - - $xx=\frac{200000}{80}=2500$;

Th. (the root) - $x=50$.

QUEST. CXIV. What 2 numbers are those, whose sum is to the greater as 11 to 7; and the difference of their squares is 132?

If $x=$ the greater, and y the lesser number;

$$\left.\begin{array}{ll}\text{Then} & \overline{x+y} : x :: 11 : 7 \\ \text{That is} & (\overline{x+y}\times 7=)\ 7x+7y=11x \\ \text{And} & xx-yy=132\end{array}\right\}\text{by question.}$$

But (by the first) - - $7y=(11x-7x=)\ 4x$;

Th. - - - - - - - - $y=\frac{4x}{7}$;

And - - - - - - - - $yy=\frac{16xx}{49}$;

Now (by the sec.) $xx-\frac{16xx}{49}=132$,

Or - $(49xx-16xx=)\ 33xx=132\times 49$.

Or (by division) - - $xx=(4\times 49=)\ 2^2\times 7^2$;

Th. (the root thereof) - $x=(2\times 7=)\ 14$.

QUEST. CXV. It is required to find two numbers, whose difference may be to the lesser, as 48 is to the greater; and to the greater, as 3 to the lesser?

Suppose x the greater, and y the lesser;

$$\left.\begin{array}{l}\text{Then } x-y : y :: 48 : x \\ \text{And } x-y : x :: 3 : y\end{array}\right\} \text{by question:}$$

That is - $\overline{x-y}\times x=48y$;

And - - $\overline{x-y}\times y=3x$:

The product of which two equations will be

$$\overline{x-y}^{\,2}\times xy=48\times 3xy;$$

Whence - $\overline{x-y}^{\,2}=(48\times 3=)\ 144$,

And - - - - $x-y=12$ by extraction;

Th. - - - - - $x=12+y$,

(From first) $144+12y=48y$,

Whence - - $144=36y$;

And - - - - - $4=y$:

Th. - - - - - $x=(12+4=)\ 16$.

QUEST. CXVI. What two numbers are those, whose sum is to the greater as 7 to 5, and their sum multiplied by the lesser produces 126?

If x be the greater, and y the lesser number;

$$\left.\begin{array}{ll}\text{Then - - - -} & x+y : x :: 7 : 5 \\ \text{That is - - - -} & \dfrac{x}{x+y}=\dfrac{5}{7} \\ \text{And - - - -} & y\times\overline{x+y}=126\end{array}\right\} \text{by question:}$$

The product of which two equations is $\Big\}$ $xy=(\frac{5}{7}\times 126=5\times 18=)\ 90$:

But - $(y\times\overline{x+y}=)\ xy+yy=126$;

Whence - - - $90+yy=126$,

Or - - - - - - $yy=(126-90=)\ 36$;

Th. (its root) - - - $y=6$.

QUEST.

QUEST. CXVII. What two numbers are those, whose product is 63; and the square of their sum is to the square of their difference, as 64 to 1?

Suppose $x=$ the greater, and $y=$ the lesser of those numbers;

Then $xy=63$, - - - - } by question.
And $\overline{x+y}^2 : \overline{x-y}^2 :: 64 : 1$, }

Or $x+y : x-y :: \sqrt{64} : \sqrt{1}$;

By last - $x+y=(\overline{x-y}\times 8=)\ 8x-8y$,

Or - - $9y=7x$;

Th. - - - $y=\dfrac{7x}{9}$:

But $(xy=)\ \dfrac{7xx}{9}=63$ by first,

Or - - $xx=9\times 9$,

Th. - - $x=9$.

QUEST. CXVIII. What two numbers are those, whose sum, multiplied by the greater produces 209, and by their difference 57?

If x be the greater, and y the lesser number;

Then - - - $(\overline{x+y}\times x=)\ xx+xy=209$ } by quest.
And - - $\overline{x+y}\times\overline{x-y}=)\ xx-yy=57$ }

The diff. of which equations is $xy+yy=152$;

And the sum of the first and last } $xx+2xy+yy=361$;

Th. the root thereof - - - $x+y=19$:

But by first - - $(\overline{x+y}\times x=)\ 19x=209$;

Th. - - - - - - - - $x=(\dfrac{209}{19}=)\ 11$;

And - - - - - - - - $y=8$.

QUEST.

QUEST. CXIX. What 2 numbers are thoſe, whoſe ſum being multiplied by the greater, and the product divided by the leſſer, quotes 24; but if their ſum be multiplied by the leſſer, and the product divided by the greater, the quotient will be but 6?

Suppoſe $x=$ the greater, and $y=$ the leſſer number;

Then by queſtion - - $\begin{cases} \overline{x+y}\times\dfrac{x}{y}=24, \\ \overline{x+y}\times\dfrac{y}{x}=6; \end{cases}$

The product of theſe equations is $\overline{x+y}^2=(24\times6=)144$:
Th. - - - - - - - - - - $x+y=12$:
But - - - (by firſt $\overline{x+y}\times\dfrac{x}{y}=$) $\dfrac{12x}{y}=24$;
Th. - - - - - - - - - $x=2y$:
But - - - ($x+y=2y+y=$) $3y=12$;
Th. - - - - - - - - - $y=4$.

QUEST. CXX. It is required to find three numbers, x, y, and z, ſo that x multiplied by y, may be 12; x, multiplied by z, 18; and y, multiplied by z, 24?

$$\left.\begin{array}{ll} \text{That is } xy=12, & \text{Th. } x=\dfrac{12}{y} \\ xz=18, & \text{Th. } x=\dfrac{18}{z} \\ yz=24, & \text{Th. } y=\dfrac{24}{z} \end{array}\right\}\text{by queſtion:}$$

But $\dfrac{12}{y}=\dfrac{18}{z}$; Or $\dfrac{2}{y}=\dfrac{3}{z}$:

And $2z=3y$; Or $\dfrac{2z}{3}=y$:

Alſo $\dfrac{24}{z}=\dfrac{2z}{3}$; Or $\dfrac{12}{z}=\dfrac{z}{3}$:

Th. $36=zz$; And $6=z$.

QUEST.

QUEST. CXXI. What three numbers are thoſe, whoſe differences are equal, whoſe ſum is 15, and the ſum of their cubes 495?

Let y repreſent the ſecond of thoſe numbers;
And x their difference;

Then $\left\{\begin{array}{l} y-x \\ y \\ y+x \end{array}\right\}$ will be the numbers; and $\left\{\begin{array}{l} y^3-3y^2x+3yx^2-x^3 \\ y^3 \\ y^3+3y^2x+3yx^2+x^3 \end{array}\right\}$ their cubes:

And $3y=$ ſum; Alſo $3y^3+6yx^2=$ ſum of cubes.
But $3y=15$; And $3y^3+6yx^2=495$ by queſt.
Th. $y=5$; And $3\times125+30x^2=495$;
Now (by tranſp.) $30x^2=(495-375=)\ 120$,
Or (by diviſion) - - - - - $x^2=4$;
Th. - - - - - - - - $x=2$.

QUEST. CXXII. The ſum of three numbers, which have equal differences, is 21; and the ſum of their ſquares is equal to their product? What are thoſe numbers?

If $y=$ the ſecond number, and x the difference of them;

Then $\left\{\begin{array}{l} y-x \\ y \\ y+x \end{array}\right\}$ will be the numbers required; and $\left\{\begin{array}{l} yy-2xy+xx \\ yy \\ yy+2xy+xx \end{array}\right\}$ their ſquares:

And $3y=$ their ſum; Alſo $3yy+2xx=$ ſum of ſquares;
Again $(\overline{y-x}\times y\times\overline{y+x}=)\ yyy-yxx=$ their product:
But $3y=21$ by queſt. And $3yy+2xx=y^3-yx^2$:
Th. $y=7$; And $3\times49+2xx=343-7x^2$;
Or (by tranſpoſition $7xx+2xx=$) $9xx=(343-147=)$ [196;
Th. - - - - - - - - - $3x=14$;
And - - - - - - - - - $x=\frac{14}{3}$.

QUEST.

QUEST. CXXIII. A prize being taken at sea, was equally divided among the ship's company; and each man received 1*l.* and the $\frac{1}{100}$ part of the remaining sum; now if the number of men, be added to the pounds that each man received, the square of that sum will be equal to four times the value of the prize; That value is required?

If the number of men. - - - $=x$,
And the share of each - - - $=y$;
Then the prize was - - - - $=xy$:
But $(\overline{x+y}^2=)\ xx+2xy+yy=4xy$ by question;
And (subtract. $4xy$) $xx-2xy+yy=0$,
Th. - - - - - - - - $x-y=0$; or $x=y$:
Now the prize was - - $(xy=)\ xx$;
And each man's share - - - $=1+\frac{xx-1}{100}$ by quest.
That is - - - - - $(y=)\ x=1+\frac{xx-1}{100}$,
Or - - - - - - - - $x-1=\frac{xx-1}{100}$:
Now (dividing by $x-1$) - $1=\frac{x+1}{100}$,
Or - - - - - - - - - $100=x+1$;
Th. - - - - - - - - - $99=x$.

QUEST. CXXIV. The sum of three numbers (whose differences are equal) is 21; and the sum of the squares of those numbers is 155: What are those numbers?

If y equal the second number, and $x=$ the difference of those numbers;

Then $\left\{\begin{array}{l} y-x \\ y \\ y+x \end{array}\right\}$ will be the numbers required; and - - $\left\{\begin{array}{l} yy-2xy+xx \\ yy \\ yy+2xy+xx \end{array}\right\}$ their squares:

And $3y=$ their sum; Also $3yy+2xx=$ sum of squares;
But $3y=21$ by qu. And $3yy+2xx=155$:
Th. $y=7$; And $3\times49+2xx=155$,
Or - - - - - - - - $2xx=(155-147=)8$;
Th. - - - - - - - - $x=2$.

QUEST.

QUEST. CXXV. *A*, sets out from *London* for *York*; and at the same time, *B*, sets out from *York* for *London*; they meet in the road, and find *A*, had travelled 30 miles more than *B*, had; *A*, expected to reach *York* in 4 days, and *B* to reach *London* in 9 days, each travelling at the same rate he had hitherto done: Required the distance of *London* and *York*.

Suppose - - - - - - - $x=$ miles *A* } had travelled;
Then - - - - - - $x-30=$ miles *B* }

And - - - - - - $2x-30=$ } distance of the two cities:

Now - - - - - - { $x-30=$ miles *A* ; $x=$ miles *B* } had yet to travel;

Therefore - - - { $\frac{x-30}{4}=$ miles *A* ; $\frac{x}{9}=$ miles *B* } travels a day;

And { $\frac{x-30}{4}:x::1:\frac{4x}{x-30}=$ time *A* ; $\frac{x}{9}:x-30::1:\frac{\overline{x-30}\times 9}{x}=$ time *B* } spent before they met;

Th. the times being equal $\frac{4x}{x-30}=\frac{\overline{x-30}\times 9}{x}$ by quest.

Or - - - - - - - - - $4xx=\overline{x-30}^2\times 9$;

Th. - - - - - - - - - $2x=(\overline{x-30}\times 3=)3x-90$,

And - - - - - - - - $90=x$.

QUEST. CXXVI. What two numbers are those, whose difference is to the greater as 5 to 6; and their product multiplied by the lesser produces 384?

Suppose $x=$ the greater, and y the lesser,

Then $x-y:x::5:6$,

That is $(\overline{x-y}\times 6=)\ 6x-6y=5x$, } by question.
And $(xy\times y=)\ xyy=384$: }

But (by first $6x-5x=$) $x=6y$;

And (by second $xyy=$) $6yyy=384$,

Or (by division) - - $y^3=(64=)\ 4^3$;

Th. - - - - - - - - $y=4$.

QUEST.

QUEST. CXXVII. *A* and *B*, set out the one from *London*, the other from *Lincoln*, at the same time; they met in the road, and at that time *A* had travelled 20 miles more than *B*, and had gone in $6\frac{2}{3}$ days as far as *B* had gone in all; lastly, *B* continuing the same pace would get to *London* in 15 days: Required the distance of the two cities?

Let $x=$ the time of their meeting,
And $y=$ the miles travelled by *B* in 1 day;

Then the miles travelled by - $\left\{\begin{array}{l} B=xy, \\ A=15y, \end{array}\right.$ {for *A* had already gone those miles which *B* could go in 15 days;

Also - - - - $\frac{15y}{x}=$ miles *A* went *per* day;

But - - - - - $6\frac{2}{3}=\frac{20}{3}$;

Th. $\left(\frac{20}{3}\times\frac{15y}{x}=\right)\frac{100y}{x}=xy$ {for *A* went in $6\frac{2}{3}$ days as far as *B* went in all;

Or - - - - - $\frac{100}{x}=x$ by division,

Or - - - - - $100=xx$;

Th. - - - - - $10=x$:

But - - - - - $15y=xy+20$ {for *A* travelled 20 miles more than *B*;

That is - - - $15y=10y+20$,

Or $(15y-10y=)\ 5y=20$;

Th. - - - - - $y=4$.

Now - - $(xy+15y=10\times4+15\times4=25\times4=)\ 100$ is the distance required.

QUEST. CXXVIII. It is required to divide 16 into 2 such parts, that the squares of those parts may be in proportion as 25 to 9?

Let x be the greater part;
Then $16-x$ will be the lesser;
Whence $xx:\overline{16-x}^2::25:9$;
That is $9xx=25\times\overline{16-x}^2$;
Th. - $3x=(5\times\overline{16-x}=)\ 80-5x$;
And $(3x+5x=)\ 8x=80$,
Th. - $x=10$,

QUEST.

QUEST. CXXIX. What three numbers are those whose sum, multiplied by the lesser, produce $\frac{133}{90}$; and by the mean $\frac{133}{80}$; and by the greater $\frac{133}{75}$?

If $x=$ the greatest; $y=$ the mean; and $z=$ the least N° required.

Then (by question) $\begin{cases} z\times\overline{x+y+z}=\dfrac{133}{90}, \\ y\times\overline{x+y+z}=\dfrac{133}{80}, \\ x\times\overline{x+y+z}=\dfrac{133}{75}, \end{cases}$

The sum of these 3 equations is $\left.\right\}\ \overline{x+y+z}\times\overline{x+y+z}=\dfrac{133}{90}+\dfrac{133}{80}+\dfrac{133}{75},$

That is - - - - - - $\overline{x+y+z}^2=\dfrac{17689}{3600};$

Th. - - - - - - - - $x+y+z=\dfrac{133}{60}:$

Now (by Equation $\begin{cases} \text{first)}\ z\times\dfrac{133}{60}=\dfrac{133}{90}, \\ \text{2d)}\ y\times\dfrac{133}{60}=\dfrac{133}{80}, \\ \text{3d)}\ x\times\dfrac{133}{60}=\dfrac{133}{75}; \end{cases}$

Th. - - - - - - - - - - $\begin{cases} z=\left(\dfrac{60}{90}\right)=\dfrac{2}{3}, \\ y=\left(\dfrac{60}{80}\right)=\dfrac{3}{4}, \\ z=\left(\dfrac{60}{75}\right)=\dfrac{4}{5}. \end{cases}$

QUEST. CXXX. There are 3 numbers, x, y, z; the sum of x and y multiplied by z will produce $\frac{57}{40}$; the sum of x and z multiplied by y, $\frac{13}{10}$; and the sum of y and z multiplied by x, $\frac{11}{8}$: What are those numbers?

$$\left.\begin{array}{ll}\text{Now} & (\overline{x+y}\times z=)\ xz+yz=\frac{57}{40}, \\ \text{And} & (\overline{x+z}\times y=)\ xy+zy=\frac{13}{10}, \\ \text{Also} & (\overline{y+z}\times x=)\ xy+xz=\frac{11}{8},\end{array}\right\}\text{by question:}$$

$$\left.\begin{array}{l}\text{The sum of} \\ \text{the 3 equa-} \\ \text{tions is}\end{array}\right\}\ 2xy+2xz+2yz=\frac{57}{40}+\frac{13}{10}+\frac{11}{8},$$

$$\text{That is}\ \text{-}\quad 2xy+2xz+2yz=\frac{164}{40}$$

$$\text{Th.}\ \text{- - -}\quad xy+xz+yz=\frac{82}{40}:$$

$$\text{If the three given equa-tions be severally ta-ken from the last}\left\{\begin{array}{l} xy=\left(\frac{82}{40}-\frac{57}{40}=\right)\frac{5}{8} \\ xz=\left(\frac{82}{40}-\frac{13}{10}=\right)\frac{3}{4} \\ yz=\left(\frac{82}{40}-\frac{11}{8}=\right)\frac{27}{40}\end{array}\right.$$

$$\left.\begin{array}{l}\text{And the product of} \\ \text{the 3 last equat. is}\end{array}\right\}\ x^2y^2z^2=\left(\frac{5}{8}\times\frac{3}{4}\times\frac{37}{40}=\right)\frac{27\times 3}{8\times 4\times 8};$$

$$\text{Th.}\ \text{- - - - - -}\quad xyz=\left(\sqrt{\frac{27\times 3}{8^2\times 4}}=\right)\frac{9}{16}:$$

$$\text{But (be-cause}\left\{\begin{array}{l} xy=\frac{5}{8};\ \text{Th.})\ \frac{5z}{8}=\frac{9}{16}, \\ xz=\frac{3}{4};\ \text{Th.})\ \frac{3y}{4}=\frac{9}{16}, \\ yz=\frac{27}{40};\ \text{Th.})\ \frac{27x}{40}=\frac{9}{16};\end{array}\right.$$

$$\text{Th.}\ x=\left(\frac{8\times 9}{16\times 5}=\right)\frac{9}{10};\ y=\left(\frac{4\times 9}{3\times 16}=\right)\frac{3}{4};\ z=\left(\frac{40\times 9}{27\times 16}=\right)\frac{5}{6}.$$

QUEST.

QUEST. CXXXI. There are three numbers whose continual product being divided by the sum of the two greater will quote 480; if by the sum of the two lesser 672, and if by the greatest and least 560: What are those numbers?

If x represent the greater, y the mean, and z the lesser;

$$\text{by question;}\left\{\begin{array}{l}\frac{xyz}{x+y}=480;\\ \frac{xyz}{x+z}=560;\\ \frac{xyz}{y+z}=672;\end{array}\right\}\text{ That is }\left\{\begin{array}{l}xyz=480x+480y,\\ xyz=560x+560z,\\ xyz=672y+672z,\end{array}\right.$$

Th. - - - - $480x+480y=560x+560z$,

Or - - - - - - $6x+6y=7x+7z$,

Th. - - - - - - $6y-7z=(7x-6x=)\ x$:

Also - - - - $560x+560z=672y+672z$,

Or - - - - - - $5x+5z=6y+6z$;

Th. - - - - - - - - - $x=\frac{6y+z}{5}$:

But - - - - - - $6y-7z=\frac{6y+z}{5}$,

Or - - - - - - $30y-35z=6y+z$;

Th. - - - - - - - $\frac{2y}{3}=z$:

Again (bec. $6y-7z=x$) $6y-\frac{14y}{3}=x$,

Or - - - - - $18y-14y=3x$;

Th. - - - - - - - - $\frac{4y}{3}=x$:

But (by first $\frac{4y}{3}\times y\times\frac{2y}{3}=$) $\frac{8y^3}{9}=\overline{\frac{4y}{3}+y}\times 480$,

That is (by division) - $\frac{yy}{9}=(\overline{\tfrac{4}{3}+1}\times 60=)\ \frac{7}{3}\times 60$,

Or - - - - - - - - - $yy=7\times 3\times 60=3^2\times 2^2\times 35$;

Th. - - - - - - - - - $y=(3\times 2\sqrt{35}=)\ 6\sqrt{35}$,

And - - - - - - - - - $x=\left(\frac{4\times 6\sqrt{35}}{3}=\right)\ 8\sqrt{35}$,

Also - - - - - - - - - $z=\left(\frac{2\times 6\sqrt{35}}{3}=\right)\ 4\sqrt{35}$.

QUEST. CXXXII. What two numbers are as 3 to 2; the ſum of whoſe cubes is 280?

If $x=$ the greater of thoſe numbers,

Then - $(3 : 2 :: x :) \frac{2x}{3} =$ the leſſer;

Whence $(x^3 + \frac{8x^3}{27} =) \frac{35x^3}{27} = 280$, by queſtion;

Or (by diviſion) - - $\frac{x^3}{27} = (8=) \, 2^3$,

Or (by multiplicat.) - $x^3 = (2^3 \times 27 =) \, 2^3 \times 3^3$;

Th. - - - - - - - $x = (2 \times 3 =) \, 6$.

QUEST. CXXXIII. What two numbers are as 5 to 4; and their product multiplied by the leſſer produces 2160?

If x be the greater number,

Then - $(5 : 4 :: x :) \frac{4x}{5} =$ the leſſer number,

Alſo - $(x \times \frac{4x}{5} =) \frac{4xx}{5} =$ their product.

But $\left(\frac{4xx}{5} \times \frac{4x}{5}\right) = \frac{16x^3}{25} = 2160$, by queſtion,

Or (by diviſion) - $\frac{x^3}{25} = (135=) \, 27 \times 5$,

Or (by multipl.) - $x^3 = (27 \times 5 \times 25 =) \, 3^3 \times 5^3$;

Th. - - - - - - $x = (3 \times 5 =) \, 15$.

QUEST. CXXXIV. Some gentlemen travelling; each had twice as many guineas as there were ſervants in all, and each had as many ſervants as there were gentlemen, and all their money was 3456 guineas: How many gentlemen and ſervants were there?

If the number of gentlemen $=x$,
Then the numb. of ſervants $=x^2$;
And - $(1 : 2xx :: x :) \, 2x^3 =$ number of guineas:
But - - - - - - $2x^3 = 3456$ by queſtion,
Or - - - - - - - $x^3 = (1728=) \, 12^3$;
Th. - - - - - - - $x = 12$.

QUEST. CXXXV. What two numbers are those, the product of the greater and square root of the lesser of which is 18; and the product of the lesser and square root of the greater is 12?

If $x=$ the greater, and y the lesser of those numbers;

$$\left.\begin{array}{ll}\text{Then } x\sqrt{y}=18; & \text{Or } x=\frac{18}{\sqrt{y}} \\ \text{And } y\sqrt{x}=12; & \text{Or } \sqrt{x}=\frac{12}{y}\end{array}\right\}\text{by question.}$$

Now - $x=\frac{12^2}{y^2}$; Th. $\frac{18}{\sqrt{y}}=\frac{12^2}{yy}$,

Or - $\frac{1}{\sqrt{y}}=\frac{8}{y^2}$; Th. $y^2=8\sqrt{y}$,

Or - $y^4=64y$; Th. $y^3=(64=)\ 4^3$;

And - - - - - - - $y=4$.

QUEST. CXXXVI. What two numbers are those, the square of the greater of which multiplied by the lesser, produces 147; and the square of the lesser multiplied by the greater 63?

If x, be the greater; and y, the lesser;

Then by question $\left\{\begin{array}{l}xxy=147, \\ xyy=\ 63:\end{array}\right.$

And the first equat. divided by the 2d, is $\Big\}\ \frac{x}{y}=\left(\frac{147}{63}=\frac{49}{21}=\right)\frac{7}{3}$;

Th. - - - - - - $x=\frac{7y}{3}$:

But (by 2d $xyy=$) $\frac{7yyy}{3}=63$,

That is - - - - - $y^3=(63\times\frac{3}{7}=9\times3=)\ 3^3$;

Th. - - - - - - $y=3$.

QUEST. CXXXVII. Several merchants making a joint stock, paid each 65 times as many pounds as there were partners; and having traded therewith, gained as many pounds *per Cent.* as there were partners; now if 10 guineas be added to and taken from their gain in pounds, the product of that sum and difference will be $6491\frac{5}{16}$: How many partners were there?

Suppose there were - - - - - x partners;
Then each paid - - - - - $65x$ pounds,
And their whole stock was - $65xx$ pounds:

But $(100 : x :: 65xx :)\frac{13x^3}{20} =$ their gain;

And $\overline{\frac{13x^3}{20}+\frac{21}{2}} \times \overline{\frac{13x^3}{20}-\frac{21}{2}} = 6491\frac{5}{16}$ by question.

That is - $\frac{169x^6}{400} - \frac{441}{4} = \frac{103861}{16}$,

Or - - - - - $\frac{169x^6}{400} = \frac{103861}{16} + \frac{441}{4}$,

Or - - - - - $\frac{169x^6}{25} = 103861 + 1764$,

Or - - - - - - - - $x^6 = (105625 \times \frac{25}{169} =)\ 5^6$;

Th. - - - - - - - - $x = 5$.

QUEST. CXXXVIII. What 2 numbers are those, whose difference is 2; and the product of their cubes 42875?

If the lesser number be x;
Then the greater is $x+2$;
And - - $x^3 \times \overline{x+2}^3 = 42875$ by question;
Th. - - - $x \times \overline{x+2} = \sqrt[3]{42875} = 35$,
That is - - $xx+2x=35$:
But - - $xx+2x+1 = (35+1=)\ 36$;
Th. $x+1=6$; And $x = (6-1=)\ 5$.

QUEST:

QUEST. CXXXIX. A draper sold a piece of cloth for 24*l.* and gained as much *per Cent.* as the cloth cost him: What was the price of that cloth?

Suppose the cloth cost x pounds,
Then $24-x$ was gained by selling it for 24*l.*
But $100 : x :: x : 24-x$ by question;
Th. $(100 \times \overline{24-x} =)\ 2400-100x = xx$,
Th. - - - - - - - - - $2400 = xx+100x$:
* But - $(2400+2500=)\ 4900 = xx+100x+50\times 50$;
Th. - - - - - - - - $70 = x+50$,
And - - - $(70-50=)\ 20 = x$.

QUEST. CXL There are four numbers (x, y, u, z); the continual product of x, y, and u, is 252; of x, u, and z, is 756; of u, y, and z, 336; and of y, x, and z, 432: What are those numbers?

$$\left.\begin{aligned} \text{Now } xyu &= (252 = 4\times 63 =)4\times 9\times 7, \\ xuz &= (756 = 4\times 189 =)4\times 9\times 7\times 3, \\ uyz &= (336 = 4\times 84 =)4\times 4\times 7\times 3, \\ yxz &= (432 = 4\times 108 =)4\times 4\times 9\times 3, \end{aligned}\right\}\text{by quest.}$$

$$\left.\begin{array}{l}\text{The prod. of which}\\ \text{4 equations is}\end{array}\right\} x^3y^3u^3z^3 = 4^6\times 9^3\times 7^3\times 3^3;$$

Th. - - - - - - - $xyuz = 4^2\times 9\times 7\times 3$:

$$\text{Whence}\begin{cases} \left(\dfrac{xyuz}{xyu}=\right) z = \left(\dfrac{4^2\times 9\times 7\times 3}{4\times 9\times 7} = 4\times 3 =\right) 12, \\ \left(\dfrac{xyuz}{xuz}=\right) y = \left(\dfrac{4^2\times 9\times 7\times 3}{4\times 9\times 7\times 3}=\right) 4, \\ \left(\dfrac{xyuz}{uyz}=\right) x = \left(\dfrac{4^2\times 9\times 7\times 3}{4\times 4\times 7\times 3}=\right) 9, \\ \left(\dfrac{xyuz}{yxz}=\right) u = \left(\dfrac{4^2\times 9\times 7\times 3}{4\times 4\times 9\times 3}=\right) 7. \end{cases}$$

---

* Remark. The square of a Binomial $x\pm a$, is $xx\pm 2ax+aa$, where the 3d term $aa = \frac{2a}{2}$ squared.

A quadratic equation $xx\pm 2ax$, wants the 3d term aa, to make it completely the square of $x\pm a$.

And the usual way of fitting such equations for solution, is by an operation called COMPLETING the SQUARE. Which is done by adding to both sides of the equation the square of half the coefficient of the second term.

QUEST. CXLI. Suppoſe that out of a caſk holding 81 gallons of wine when full, a certain quantity was drawn, and the caſk filled up with water; and that the ſame quantity of the mixture was afterwards drawn, and ſupplied by water three ſeveral times; and then it appeared that (beſides water) there were but 16 gallons of wine left in the caſk: How much wine was drawn each time?

If the content of the caſk ($81=$) c; the wine remaining ($16=$) r; the number of times the liquor was drawn out $=t$; and x the quantity of liquor drawn out at each time?

Drawing.

	The quantity of wine
Then $c-x=$	left after the 1ſt,
But $(c : c-x :: x :)\ \dfrac{\overline{c-x}\times x}{c}=$	drawn out at 2d,
And $\left(c-x-\dfrac{\overline{c-x}\times x}{c}=\right)\ \dfrac{\overline{c-x}^{2}}{c}=$	left after the 2d,
Alſo $\left(c : \dfrac{\overline{c-x}^{2}}{c} :: x :\right)\ \dfrac{\overline{c-x}^{2}\times x}{cc}=$	drawn out at 3d,
And $\left(\dfrac{\overline{c-x}^{2}}{c}-\dfrac{\overline{c-x}^{2}\times x}{cc}=\right)\ \dfrac{\overline{c-x}^{3}}{cc}=$	left after the 3d,
Th. $\left(c : \dfrac{\overline{c-x}\vert^{t-1}}{c^{t-2}} :: x :\right)\ \dfrac{\overline{c-x}^{t-1}\times x}{c^{t-1}}=$	dra. out at the t,
And $\left(\dfrac{\overline{c-x}\vert^{t-1}}{c^{t-2}}-\dfrac{\overline{c-x}\vert^{t-1}\times x}{c^{t-1}}=\right)\ \dfrac{\overline{c-x}\vert^{t}}{c^{t-1}}=$	left after the t.

Now $\dfrac{\overline{c-x}|^{t}}{c^{t-1}}=r$, by queſt. Or $\overline{c-x}^{t}=rc^{t-1}$;

Th. $c-x=\overline{rc^{t-1}}|^{\frac{1}{t}}$. Th. $x=c-\overline{rc^{t-1}}|^{\frac{1}{t}}$.

In this example $81-\sqrt[4]{\overline{16\times 81}|^{3}}=(81-2\times 3^{3}=27=)x$.

QUEST.

QUEST. CXLII. It is required to find two numbers x and y in proportion as r to s, so that x^m may be equal to y^n; (where if $r \sqsubset s$, then m must be $\sqsupset n$; but if $r \sqsupset s$, then $m \sqsubset n$)?

Now $r : s :: x : y$; Th. $x=\frac{ry}{s}$,

And - - $\overline{\frac{ry}{s}}\Big|^{m}=y^n$ by question;

That is - $\frac{r^m y^m}{s^m}=y^n$,

Or - - - - $\frac{r^m}{s^m}=\left(\frac{y^n}{y^m}=\right)y^{n-m}$,

Or - - - - $\overline{\frac{r}{s}}\Big|^{m}=y^{n-m}$;

Th. - - $\overline{\frac{r}{s}}\Big|^{\frac{m}{n-m}}=y$.

If R, S and Y represent the logarithms of r, s and y

Then - $\frac{m}{n-m}\times\overline{R-S}=Y$.

QUEST. CXLIII. There are two numbers whose difference is 3; and the difference of their cubes is 117: What are those numbers?

If the lesser required number $=x$,
Then the greater - - - $=x+3$,
And - - $9x^2+27x+27=\overline{x+3}^3-x^3$:
But - - $9x^2+27x+27=117$ by question,
Or - - - - $9xx+27x=90$,
Th. - - - - $xx+3x=10$:
But - - - $xx+3x+\frac{9}{4}=\left(10+\frac{9}{4}=\right)\frac{49}{4}$;
Th. - $x+\frac{3}{2}=\frac{7}{2}$; and $x=\left(\frac{7-3}{2}=\right)2$.

QUEST. CXLIV. One bought ſome oxen for 80*l.*; now had he bought four more for the ſame money, he would have paid 1*l.* leſs for each: How many did he buy?

Suppoſe he bought x oxen,

Then - - - - $\frac{80}{x}$ = the price of an ox,

And - - - $\frac{80}{x+4}$ = the price of one, if $\overline{x+4}$ coſt 80*l.*

But - - - - $\frac{80}{x}=\frac{80}{x+4}+1$;

Th. - $80x+320=80x+xx+4x$,

Th. - - - $320=xx+4x$;

But $(320+4=)\ 324=xx+4x+2^2$, comp^{g}. ſquare,

Th. - - - $18=x+2$;

And - - - $16=x$.

QUEST. CXLV. Bought two remnants of cloth, one of which was ſix yards longer than the other, for 3*l.* 8*s.*; and each of them coſt as many ſhillings a yard as there were yards therein: What was the length of each?

Suppoſe - - - - - - $x=$ the length of leſſer,

Then - - - - - - $\overline{x+6}=$ length of the greater;

And - - - - - - - $xx=$ value of the leſſer,

Alſo $(\overline{x+6}^2=)\ xx+12x+36=$ value of the greater;

Now (their ſum) $2xx+12x+36=68$ by queſtion,

Or - - - - - $2xx+12x=32$,

Th. - - - - - - - $xx+6x=16$:

But - - - - - $xx+6x+9=(16+9=)\ 25$;

Th. - - . $x+3=5$; And $x=2$.

QUEST.

QUEST. CXLVI. One having ſold a piece of cloth which coſt him 30*l.* found that if the price he ſold it for, was multiplied by his gain, the product would be equal to the cube of his gain: What was it?

Suppoſe he had $x\,l.$;
Then he ſold the cloth for $30+x$:
But - $(\overline{30+x}\times x=)\ 30x+xx=x^3$ by queſt.
Or - - $30+x=xx$;
Th. - - - $30=xx-x$:
But $\left(30+\frac{1}{4}=\right)\frac{121}{4}=xx-x+\frac{1}{4}$;
Th. - - - $\frac{11}{2}=x-\frac{1}{2}$,
And $\left(\frac{11+1}{2}=\right)\ 6=x$.

QUEST. CXLVII. A company at a tavern had 8*l.* 15*s.* to pay, but before the bill was paid, two of them ſneaked away, then thoſe who remained had each 10*s.* more to pay, than before: How many were in company?

Suppoſe there were x perſons in company;
Then - - $\frac{175}{x}=$ each man's ſhare of the bill;
Alſo - - $\frac{175}{x-2}=$ {the ſhare of each that remained after the two were gone:
Now - - $\frac{175}{x-2}=\frac{175}{x}+10$ by queſtion.
Th. - - $175x=175x-350+10x^2-20x$,
Or - - $350=10xx-20x$,
Th. - - $35=x^2-2x$:
But $(35+1=)\ 36=x^2-2x+1$;
Th. - - - $6=x-1$,
And - - - $x=7$.

QUEST. CXLVIII. A grazier bought as many sheep as cost him 60*l.*; out of which he reserved 15, and sold the remainder for 54*l.* gaining two shillings a head by them: How many sheep did he buy?

Suppose he bought x sheep at y shillings each;
Then he sold $\overline{x-15}$ sheep at $\overline{y+2}$ shillings each:

Now - $\left\{\begin{array}{r} xy=60\times20=1200 \\ \overline{x-15}\times\overline{y+2}=54\times20=1080 \end{array}\right\}$ by quest.

Or - $xy-15y+2x-30=1080$,

Or $1200-15y+2x-30=1080$;

Th. - - $-15y+2x=(1080+30-1200=)-90$:

But - - - - - - $x=\dfrac{1200}{y}$; Th. $2x=\dfrac{2400}{y}$,

And - $-15y+\dfrac{2400}{y}=-90$,

Or - - $15yy+2400=-90y$,

Or - - - - $2400=15yy-90y$,

Th. - - - - $160=yy-6y$:

But - $(160+9=)\ 169=yy-6y+9$;

Th. - - - - - $13=y-3$; And $16=y$:

Also - - - - - - $x=\left(\dfrac{1200}{16}=\right)75$.

QUEST. CXLIX. It is required to divide the number 48, into two such parts, that the sum of their alternate quotients may be $5\frac{1}{5}$?

If x be one of those parts,
Then $48-x$ will be the other;

Now - - $\dfrac{x}{48-x}+\dfrac{48-x}{x}=\left(5\dfrac{1}{5}=\right)\dfrac{26}{5}$ by question;

Or - - $5xx+5\times\overline{48-x}^2=\overline{48-x}\times26x$:

That is $11520-480x+10xx=1248x-26xx$,

Or - - - $36xx-1728x=11520$;

Th. - - - - $xx-48x=320$:

But - $xx-48x+24\times24=(576-320=)\ 256$;

Th. - $x-24=16$; Th. $x=(16+24=)\ 40$.

QUEST.

QUEST. CL. It is required to divide 37 into two ſuch parts, that the product of the ſquares of thoſe parts may be 116964?

Suppoſe - - - $x=$ one of thoſe parts;
Then - - $37-x=$ the other part,
And $\overline{37-x}|^2\times x^2=116964$ by queſtion:
Th. - $\overline{37-x}\times x=(\sqrt{116964}=)\ 342$;
That is - $37x-xx=342$:
Th. - - $xx-37x=-342$:
But $\left.xx-37x+\frac{37}{2}\right|^2=\left(\frac{1369}{4}-342=\right)\frac{1}{4}$;
Th. - - - $x-\frac{37}{2}=\frac{1}{2}$,
And - - - - $x=\left(\frac{37+1}{2}=\right)\ 19$.

QUEST. CLI. There are two numbers whoſe difference is 15; and ½ their product is the cube of the leſſer number: What are thoſe numbers?

If the leſſer of thoſe numbers be $=x$;
Then the greater - - - - - $=x+15$,
And - $(\overline{x+15}\times x=)\ xx+15x=$ their product:
But - - - - - - $\frac{xx+15x}{2}=xxx$ by queſtion,
And (dividing by $\frac{x}{2}$) - $x+15=2xx$;
Th. - - - - - - - - $\frac{15}{2}=xx-\frac{1}{2}x$:
But $\left.\left(\frac{15}{2}+\frac{1}{4}\right|^2=\frac{120+1}{16}=\right)\ \frac{121}{16}=xx-\frac{x}{2}+\frac{1}{16}$;
Th. - - - - - - - - $\frac{11}{4}=x-\frac{1}{4}$;
And - - - - - $\left(\frac{11+1}{4}=\right)\ 3=x$.

QUEST. CLII. What two numbers are those, whose sum, product, and difference of their squares, are equal among themselves?

Let x represent the greater, and y the lesser of those numbers,

Then - - $x+y=xy=xx-yy$ by question;

Whence - $1-\left(\frac{xx-yy}{x+y}=\right) x-y$;

And - - - - $1+y=x$:

But $(\overline{1+y}+y=)\ 1+2y=(\overline{1+y}\times y=)\ y+yy$;

That is - - - - $1=yy-y$:

But - - $(1+\frac{1}{4})=\frac{5}{4}=yy-y+\frac{1}{4}$;

And - - - - - $\frac{\sqrt{5}}{2}=y-\frac{1}{2}$;

Th. - - $\frac{\sqrt{5}+1}{2}=y$;

And - - $\frac{\sqrt{5}+3}{2}=x$.

QUEST. CLIII. It is required to divide the number 24 into two such parts, that their product may be thirty-five times their difference?

If x be the greater, and y the lesser of the parts required;

Then - - - - - - $x+y=24$ } by question;
And $(\overline{x-y}\times 35=)\ 35x-35y=xy$ }

But - - - - $35x+35y=(24\times 35=)\ 840$;

Th. - - - - - - - - $70x=xy+840$:

But - - - - - - - - - $y=24-x$,

Th. - - - - - - - $70x=24x-xx+840$,

Th. - - - - - $xx+46x=840$:

But - - $xx+46x+529=(840+529=)\ 1369$;

Th. - $x+23=37$; And $x=(37-23=)\ 14$.

QUEST.

QUEST. CLIV. *A* and *B* (who were 120 miles distant) set out to meet each other; *A* travelled 5 miles a day; and the number of days at the end of which they met was greater by 3 than the number of miles which *B* went in a day: How many miles did each go?

Suppose *A* travelled - - - - - x } miles;
Then *B* went - - - - - $120-x$ }

And they met at the end of - - $\frac{x}{5}$ days; { for *A* went 5 m. a day;

Th. *B* went - - - - - - - - - - $\frac{x}{5}-3$ miles a day;

And *B* went in all - - - - $\frac{x}{5}\times\overline{\frac{x}{5}-3}$ miles;

Th. - - - - - - $120-x=\left(\frac{x}{5}\times\frac{x}{5}-3=\right)\frac{xx}{25}-\frac{3x}{5}$,

Or $(\overline{120-x}\times 25=)\ 3000-25x=xx-15x$,
Th. - - - - - - - $3000=xx+10x$:
But - $(3000+25=)\ 3025=xx+10x+25$;
Th. - - - - - - - - $55=x+5$,
And - - - - - - - - $50=x$.

QUEST. CLV. There are two numbers whose difference is 7; also their sum multiplied by the greater produces 345: What are those numbers?

Let the greater number - $=x$,
Then the lesser - - - $=x-7$;
And their sum - - - $=2x-7$:
But $(\overline{2x-7}\times x)=2x^2-7x=345$ by question,

Th. - - - - - $xx-\frac{7}{2}x=\frac{345}{2}$:

But - - $\overline{xx-\frac{7}{2}x+\frac{7}{4}}\Big|^2=\left(\frac{345}{2}+\frac{49}{16}=\right)\frac{2809}{16}$;

Th. $x-\frac{7}{4}=\frac{53}{4}$; And $x=\left(\frac{53+7}{4}=\right)15$.

QUEST. CLVI. To find 4 numbers, the first of which may be to the second as the third to the fourth; that the first may be to the fourth as 1 to 5; that the second may be to the third as 5 to 9; and the sum of the second and fourth may be 20?

If x, y, u, and z be the numbers required,

Then $x : y :: u : z$; or $xz = yu$:

$x : z :: 1 : 5$; or $5x = z$:

$y : u :: 5 : 9$; or $9y = 5u$:

And - $y + z = 20$; or $z = 20 - y$:

Now - - - - - - - $5x = 20 - y$,

Th. - - - - - - - - $y = 20 - 5x$:

But $(\overline{20 - 5x} \times 9 =)\ 180 - 45x = 5u$,

Th. - - - - - - $36 - 9x = u$;

But - $(xz = yu$ that is$)\ 5x \times x = \overline{20 - 5x} \times \overline{36 - 9x}$,

Or - - - - - - - - $5xx = 720 - 360\,x + 45\,x^2$;

Or - - - - $40\,x^2 - 360x = -720$;

Th. - - - - - - $x^2 - 9x = -18$:

But - - - - $x^2 - 9x + \frac{81}{4} = (\frac{81}{4} - 18 = \frac{81 - 72}{4} =)\frac{9}{4}$;

Th. - - - - - - $\frac{9}{2} - x = \frac{3}{2}$;

And - - - - $(\frac{9 - 3}{2} =)\ 3 = x$.

QUEST. CLVII. It is required to find two numbers, the first of which may be to the second as the second is to 20; and the sum of the squares of the numbers sought may be 125?

If x and y represent the numbers required,

$$\left.\begin{array}{l}\text{Then } xx + yy = 125;\ \text{or } yy = 125 - xx \\ \text{And } x : y :: y : 20;\ \text{or } yy = 20x,\end{array}\right\}\text{by quest.}$$

Th. - - - - $125 - xx = 20x$;

Th. - - - - $xx + 20x = 125$:

But - $xx + 20x + 100 = (125 + 100 =)\ 225$;

Th. - - - - - $x + 10 = 15$;

And - - - - - - $x = 5$:

But - - - - - - $yy = (20 \times 5 =)\ 100$;

Th. - - - - - - $y = 10$.

QUEST.

QUEST. CLVIII. It is required to find the values of x, y, u and z, in the following equations;

$$\text{viz.}\left\{\begin{array}{r} x+y=10 \\ u+z=20 \\ xy+uz=120 \\ \overline{x+y+u}\times u+\overline{y+u+z}\times y=360 \end{array}\right\}?$$

$$\text{Now}\left\{\begin{array}{ll} \text{(by 1st)} & x=10-y, \\ \text{(by 2d)} & z=20-u, \\ \text{(by 3d)} & \overline{10-y}\times y+\overline{20-u}\times u=120, \\ \text{(by 4th)} & \overline{10+u}\times u+\overline{y+20}\times y=360: \end{array}\right.$$

But by adding the two last - $30u+30y=480$;

Th. - - $u+y=\left(\frac{480}{30}=\right)16$; and $y=16-u$:

Now - - - - $x=(10-y=10-\overline{16+u}=)\ u-6$:

But $(xy+uz=)\ \overline{u-6}\times\overline{16-u}+u\times\overline{20-u}=120$ (by 3d),

That is - - $22u-uu-96+20u-uu=120$,

Or - - - - - - - $42u-2u^2-96=120$;

Th. - - - - - - - - - $uu-21u=-108$:

But - - - - - - $uu-21u+\frac{21\times21}{2\times2}=\frac{441}{4}-108$

$$=\frac{9}{4};$$

Th. - - - - $\left(u-\frac{21}{2};\text{ or}\right)\frac{21}{2}-u=\frac{3}{2}$;

And - - - - - - $\left(\frac{21-3}{2}=\right)9=u$,

$$11=z,$$
$$7=y,$$
$$3=x.$$

QUEST. CLIX. What two numbers are those, whose sum is 140; and their product 3136?

If the given sum (140) $=s$; the given product (3136) $=p$; $x=$ the greater, and $y=$ the lesser number required;

$$\left.\begin{array}{lll}\text{Then} & x+y=s; & \text{or } x=s-y \\ \text{And} & xy=p; & \text{or } x=\frac{p}{y}\end{array}\right\}\text{by question,}$$

Th. $s-y=\frac{p}{y}$; or $sy-y^2=p$,

Or $y^2-sy=-p$:

But $y^2-sy+\frac{ss}{4}=\left(\frac{ss}{4}-p=\right)\frac{ss-4p}{4}$;

$$\left.\begin{array}{ll}\text{Th.} & y-\frac{s}{2} \\ \text{Or} & \frac{s}{2}-y\end{array}\right\}=\tfrac{1}{2}\sqrt{ss-4p}.$$

If y be exterminated instead of x, the equation arising will be $sx-xx=p$, which reduced as above will give

$$\left.\begin{array}{l}x-\frac{s}{2} \\ \frac{s}{2}-x\end{array}\right\}=\tfrac{1}{2}\sqrt{ss-4p}.$$

But by the assumption (viz. $x \sqsubset y$) $x \sqsubset \frac{s}{2}$, and $y \sqsupset \frac{s}{2}$;

$$\text{Th.}\left\{\begin{array}{l}x-\frac{s}{2} \\ \frac{s}{2}-y\end{array}\right\}=\tfrac{1}{2}\sqrt{ss-4p},$$

$$\text{And}\quad\left\{\begin{array}{l}x=\frac{s+\sqrt{ss-4p}}{2}, \\ y=\frac{s-\sqrt{ss-4p}}{2}.\end{array}\right.$$

QUEST.

QUEST. CLX. One buys cloth for 33*l.* 15*s.* which he sells again at 48 shillings per piece, and gained by the bargain as much as 1 piece cost: Required the number of pieces?

Suppose he bought x pieces;

Then each piece cost $\frac{675}{x}$ shillings,

And he sold them for $48x$ shillings,

And gained thereby $48x-675$ shillings;

But - - - $48x-675=\frac{675}{x}$ by question,

Or - - - $48x^2-675x=675$;

Th. - - - $x^2-\frac{225}{16}x=\frac{225}{16}$:

But - $xx-\frac{225}{16}x+\overline{\frac{225}{32}}\Big|^2=\left(\frac{225}{16}+\frac{50625}{1024}=\right)\frac{65025}{1024}$;

Th. $x-\frac{225}{32}=\frac{255}{32}$; and $x=\left(\frac{255+225}{32}=\right)$ 15.

QUEST. CLXI. What two numbers are those, whose sum taken from the sum of their squares leaves 78; and their sum added to their product makes 39?

If x and y represent the numbers required;

Then - - - $xx+yy-\overline{x+y}=78,$ } by question;
And - - - - - - $xy+\overline{x+y}=39,$ }

Or doubling the 2d $2xy+2\times\overline{x+y}=78$:

The sum of the first and last is - } $\overline{x+y}^2+x+y=156$:

But - - - $\overline{x+y}^2+\overline{x+y}+\frac{1}{4}=(156+\frac{1}{4}=)\ \frac{625}{4}$;

Th. $\overline{x+y}+\frac{1}{2}=\frac{25}{2}$; and $x+y=\frac{25}{2}-\frac{1}{2}=)$ 12:

Now - - - - - $xy+12=39$ by second;

Th. - - - - - - - - - $xy=(39-12=)$ 27:

And (by quest. 159) $\begin{cases} x=12+\sqrt{\overline{144-4\times 27}}\times\frac{1}{2}=9, \\ y=12-\sqrt{\overline{144-4\times 27}}\times\frac{1}{2}=3. \end{cases}$

QUEST.

QUEST. CLXII. What two numbers are those, whose difference multiplied by the difference of their squares will produce 576; and whose sum multiplied by the sum of their squares is 2336?

If x the greater, and y the lesser, of those numbers;

Then - - $\overline{x-y}\times\overline{xx-yy}=576$,

And - - $\overline{x+y}\times\overline{xx+yy}=2336$;

That is $\begin{cases} x^3-x^2y-xy^2+y^3=576, \\ x^3+x^2y+xy^2+y^3=2336; \end{cases}$

(By subtr.) - $2x^2y+2xy^2=1760$;

And - $x^3+3x^2y+3xy^2+y^3=4096$ by addition;

Th. - - - - - $x+y=\sqrt[3]{4096}=16$:

But - - - $2x^2y+2xy^2=2xy\times\overline{x+y}$;

Th. - $(16\times 2xy=)\ 32xy=1760$ by fifth,

And - - - - - - - $xy=55$:

Now the sum and prod. being given $\Big\}$ $x=(16+\sqrt{256-4\times 55}\times\frac{1}{2}=)\ 11$.

Then by quest. 159 $y=(16-\sqrt{256-4\times 55}\times\frac{1}{2}=)\ 5$.

QUEST. CLXIII. What two numbers are those, whose product is 6; and if their sum be added to the sum of their squares, the number resulting will be 18?

Let the required numbers be denoted by x and y;

Then $\overline{x+y}+\overline{xx+yy}=18$ by question:

And (since $xy=6$) $2xy=12$;

Th. - $\overline{x+y}+\overline{x+y}^2=30$ by addition:

But $\frac{1}{4}+\overline{x+y}+\overline{x+y}^2=(30+\frac{1}{4}=)\ \frac{121}{4}$;

Th. - - $\frac{1}{2}+\overline{x+y}=\frac{11}{2}$,

And - - - - $x+y=\left(\frac{11-1}{2}=\right)\ 5$:

Here the sum and product are given $\Big\}$ $x=5+\sqrt{25-4\times 6}\times\frac{1}{2}=3$,

Then (By quest. 159) $y=5-\sqrt{25-4\times 6}\times\frac{1}{2}=2$.

QUEST.

QUEST. CLXIV. It is required to divide the number 7 into two such parts, that the square of thrice the lesser may be 17 more than the square of twice the greater?

If the greater part - $=x$;
Then the lesser - $=7-x$:
The square of $\overline{2\times x}=4xx$,
The square of $3\times\overline{7-x}=441-126x+9xx$:
Now $441-126x+9xx=4xx+17$ by question:
Or - $5xx-126x=(17-441=)-424$,
Or - - $xx-\frac{126}{5}x=-\frac{424}{5}$;
But $\overline{xx-\frac{126}{5}x+\frac{63}{5}}\Big|^2=\left(\frac{3969}{25}-\frac{424}{5}=\right)\frac{1849}{25}$;
Th. $(x-\frac{63}{5}$, or$)\ \frac{63}{5}-x=\frac{43}{5}$, and $x=\left(\frac{63-43}{5}=\right)4$.

QUEST. CLXV. *A* sets out from *C*, towards *D*, and travels 8 miles a day, after he had gone 27 miles, *B* sets out from *D*, towards *C*, and goes every day $\frac{1}{20}$ of the whole journey; and after he had travelled as many days as he goes miles in 1 day, he met *A*: Required the distance of those places?

Suppose *C* was x miles distant from *D*;

Then *B* went $\frac{x}{20}$ miles a day,

And *B* went $\frac{xx}{400}$ miles in all;

Also *A* went $27+8\times\frac{x}{20}$ miles in all $=27+\frac{2x}{5}$:

Now - $\frac{xx}{400}+27+\frac{2x}{5}=x$ by question;

Or - $xx+10800+160x=400x$,
Or - - - $xx-240x=-10800$:
But - $xx-240x+\overline{120}^2=(14400-10800=)\ 3600$;
Th. $x-120=60$; and $x=(120+60=)\ 180$;
Or $120-x=60$; and $x=(120-60=)\ 60$.

QUEST.

QUEST. CLXVI. Two merchants after a succefsful voyage fhared 854*l.* between them; the firft had laid out 420*l* in goods, and the fecond had gained 52*l.* by what he had laid out: How much did the firft gain, and the fecond advance at firft?

Suppofe the fecond laid out x *l.*

Then $(x : 52 :: 420 :) \frac{420 \times 52}{x}$ = the firft's gain;

But the fum at firft laid out by both, added to the fum of their gains, is to be fhared between them;

That is $420 + x + 52 + \frac{420 \times 52}{x} = 854$,

Or $420x + xx + 52x + 420 \times 52 = 854x$,

Th. - - - - $xx - 382x = -21840$:

But - - $xx - 382x + \overline{191}^2 = 36481 - 21840 = 14641$;

Th. - $x - 191 = 121$; and $x = (121 + 191 =)\ 312$.

QUEST. CLXVII. Three merchants, *A*, *B*, *C*, join ftocks; *A*'s ftock was 13*l.* lefs than *B*'s: the fum of *B*'s and *C*'s ftock was 175*l.* their gain exceeded the whole ftock by 48*l.* and *A*'s fhare of that gain was 78*l.* What was each man's part of the ftock and gain?

Suppofe *A*'s ftock was x *l.*
Then *B*'s - - was $x + 13$,
And *C*'s - - was $162 - x (= 175 - x - 13)$:
Then the joint ftock was $175 + x$,
And the gain - - - $223 + x (= 48 + 175 + x)$;
Now $175 + x : 223 + x :: x : 78$ by queftion,
That is - $13650 + 78x = 223x + xx$;
Th. - - - $xx + 145x = 13650$.

But - $xx + 145x + \overline{\frac{145}{2}}|^2 = (13650 + \frac{21025}{4} =) \frac{75625}{4}$;

Th. $x + \frac{145}{2} = \frac{275}{2}$; and $x = (\frac{275 - 145}{2} =)\ 65$.

QUEST.

QUEST. CLXVIII. Towards the expence of building a ſhip, *A* paid 2000*l.* more than *B*, and 9000 more than *C*; alſo the ſquare of what *A* paid is equal to the ſum of the ſquares of *B*'s and *C*'s payments: How much did each pay?

Suppoſe *A* paid - - - - x pounds,
Then *B* paid - - - - - $x-2000$,
And *C* - - - - - - - $x-9000$:
B's payment ſqu. - - - $=xx-4000x+4000000$,
C's ditto - - - - - - $=xx-18000x+81000000$;
Their ſum is - - - $=2x^2-22000x+85000000$:
Now
$2x^2-22000x+85000000=xx$ by queſtion;
Th. - - - $x^2-22000x=-85000000$;
But $x^2-22000x+\overline{11000}^2=36000000$;
Th. - - - - $x-11000=6000$;
And - - - - - - $x=(11000+6000=)\ 17000$.

QUEST. CLXIX. A draper bought ſome pieces of two ſorts of cloth for 47*l.* 7*s.* 11*d.*; there were as many pieces bought of each ſort, and as many pence *per* yard paid for them, as a piece of that ſort contained yards; now two pieces, one of each ſort, meaſured together, were 35 yards: How many yards were in each?

If a piece of one ſort contained x yards,
Then a piece of the other ſort contained $35-x$ yards,

Now $\begin{cases} x \text{ pieces, each of } x \text{ yard} & =xx \text{ yards;} \\ 35-x \text{ pieces, each } 35-x \text{ yds.} & =\overline{35-x}^2 \text{ yards;} \end{cases}$

And $\begin{cases} xx \text{ yards, at } x \text{ pence a yard} & =x^3 \text{ pence,} \\ \overline{35-x}^2 \text{ yds. at } 35-x\ d. \text{ a yd.} & =\overline{35-x}^3 \text{ pence;} \end{cases}$

Alſo - - - - - 47*l.* 7*s.* 11*d.* $=11375$ pence:
Now - - - - - $(\overline{35-x}^3+x^3=11375$ by queſt.
Or - - - - - $105x^2-3675x=-31500$,
Or - - - - - - - - $x^2-35x=-300$;

But - - - $xx-35x+\frac{35\times35}{2\times2}=(\frac{1225}{4}-300=)\frac{25}{4}$,

Th. - - - $x-\frac{35}{2}=\frac{5}{2}$; and $x=(\frac{35+5}{2}=)\ 20$.

QUEST.

QUEST. CLXX. Three merchants, *A*, *B*, and *C*, on comparing their gains, find that between them they have gained 3037*l.* that *B*'s gain, added to the square root of *A*'s made 871*l.* but if added to the square root of *C*'s, made 877*l.* What were there several gains?

Suppose *A* had gained xx; *B*, yy; and *C*, zz pounds;

Then - - - - $\left.\begin{array}{r} xx+yy+zz=3037, \\ yy+x=\ \ 871, \\ yy+z=\ \ 877, \end{array}\right\}$ by question;

The $\left\{\begin{array}{l}\text{sum}\\ \text{diff.}\end{array}\right\}$ of the 2 last $\left\{\begin{array}{r} 2y^2+x+z=1748; \\ z-x=\quad 6; \end{array}\right.$

And - - - - - $zz-2zx+xx=\ \ 36$:

The diff. of first and last $yy+2xz=3001$;

Th. - - - - - - $2yy+4xz=6002$:

The diff. of last and 4th - $4xz-x-z=4254$;

By 5th - - - - - - - $z=x+6$;

Th. - $4x\times\overline{x+6}-x-x-6=4254$,

That is - - $4x^2+22x-6=4254$,

Or - - - - - - $4x^2+22x=4260$;

Th. - - - - - - - $x^2+\frac{11}{2}x=1065$:

But - - - $\overline{xx+\frac{11}{2}x+\frac{11}{4}}\Big|^2=1065+\frac{121}{16}$

$=\frac{17161}{16}$;

Th. - $x+\frac{11}{4}=\frac{131}{4}$; and $x=\left(\frac{131-11}{4}=\right)30$;

And - - - - - - - - - $z=(30+6=)\ 36$;

But by second - - $yy+30=871$,

Hence - - - - - - - - $yy=841$;

And - - - - - - - - - $zz=1296$.

QUEST.

QUEST. CLXXI. Two partners A, and B, had gained 18*l.* 15*s.* by trade; A's money was in trade 12 months, and he received for his principal and gain 26*l.* also B's money (viz. 30*l.*) was in trade 17 months: What money did A put into trade?

Suppose A advanced x pounds;

Then the product of A's stock and time $\Big\} = 12x$,

And of B's - - - - - $=(30\times 17=)\ 510$;

And their sum - - - - $=12x+510$:

Now $(12x+510 : 18\tfrac{3}{4} :: 12x :)\ \dfrac{225x}{12x+510}$ A's gain;

But - - $x+\dfrac{225x}{12x+510}=26$ by question;

Or - $12xx+510x+225x=312x+13260$,

Or - - - $12xx+423x=13260$,

Or - - - - $xx+\dfrac{141}{4}x=\dfrac{4420}{4}$:

But $\overline{xx+\dfrac{141}{4}x+\dfrac{141}{8}}\Big|^2=\left(\dfrac{4420}{4}+\dfrac{19881}{64}=\right)\dfrac{90601}{64}$;

Th. $x+\dfrac{141}{8}=\dfrac{301}{8}$; and $x=\left(\dfrac{301-141}{8}=\right)20$.

QUEST. CLXXII. There are three numbers, the difference of whose differences is 4; their sum 40; and continual product 1764: What are those numbers?

If the second number - - - - $=z$,

And diff. of the second and least $=x$;

Then the numbers - - - - $=\begin{cases} z-x, \\ z, \\ z+x+4; \end{cases}$

And (their sum) - - - - - $40=3z+4$ by quest.

Th. - - - - $36=3z$; and $12=z$:

But - - - - $\overline{z-x}\times z\times\overline{z+x+4}=z^3+4z^2-zx^2-4zx$,

That is $1728+4\times 144-12x^2-48x=1764$ by quest.

Or - - - - - - - - $540=12x^2+48x$;

Th. - - - - - - - - $45=x^2+4x$:

But - - - - $(45+4=)\ 49=x^2+4x+4$;

Th. - - - - - - - - $7=x+2$,

And - - - - - - - - $5=x$.

QUEST.

QUEST. CLXXIII. The joint ſtock of 2 partners, *A B*, was 165*l.* *A*'s money was in trade 12, and *B*'s 8 months; when they ſhared ſtock and gain, *A* received 67, and *B* 126*l.* What was each man's ſtock?

Suppoſe *A* advanced x pounds;
Then *B* advanced $165-x$ pounds;
And the product of *A*'s ſtock and time $=12x$,
Ditto of *B*'s - - - $(\overline{165-x}\times 8)=1320-8x$;
And their ſum is - - - - - $4x+1320$:
Alſo their gain - $=(67+126-165=)\ 28$:
But $(4x+1320 : 28 :: 12x :)\ \dfrac{336x}{1320+4x}=$ *A*'s gain,
And (*A*'s ſtock and gain$=$)$x+\dfrac{336x}{1320+4x}=67$ by queſtion:
Or - - - - - $1320x+4x^2+336x=88440+268x$;
Th. - - - - - $x^2+347x=22110$:
But - - - $x^2+347x+\overline{\dfrac{347}{2}}\Big|^2=22110+\dfrac{120409}{4}$,
Or - - - $xx+347x+\overline{\dfrac{347}{2}}\Big|^2=\dfrac{208849}{4}$;
Th. - - - $x+\dfrac{347}{2}=\dfrac{457}{2}$; and $x=\left(\dfrac{457-347}{2}=\right)55$.

The

The root of the equation $xx+347x=22110$, may be obtained by the tables of the logarithms of numbers and tangents, according to the method given by the excellent mathematician Dr. *Edmund Halley*, in his geometrical lectures, published as an appendix to the last edition of *Kersey's* algebra; as follows,

The logarithm of	$22110 = 4,3445887;$
From half thereof	$+10 = 12,1722943,$
Take the logarithm of	$\frac{347}{2} = 2,2392995;$
Remains the log. tang. of . . . . $40^{\circ}\ 35'\ 52''$	$= 9,9329948:$
To the log. tang. of $\frac{1}{2}$ that arc, viz. of $20\ 17\ 56$	$= 9,5680714.$
Add $\frac{1}{2}$ the logarithm of	$22110 = 2,1722943;$
The sum (—10) is the log. of	$x = 1,7403657.$
Therefore $x=55$.	

QUEST.

QUEST. CLXXIV. Two partners, *A* and *B*, had gained 140*l.* by trade; *A*'s money was three months in trade, and his gain was 60*l.* less than his stock: also *B*'s money, which was 50*l.* more than *A*'s, was in trade 5 months: What was *A*'s stock?

Suppose *A*'s stock - - - $=x\ l.$
Then *B*'s stock - - - $=x+50,$
And *A*'s gain - - - $=x-60,$
Also *B*'s gain - - - $=200-x\ (=140-\overline{x-60})$:
Now $3x : x-60 :: \overline{x+50}\times5 : \overline{200-x}$, by quest.
That is - $3x\times\overline{200-x}=\overline{x-60}\times\overline{5x+250},$
Or - - - $600x-3x^2=5xx-50x-15000,$

Th. - - - $xx-\frac{650}{8}x=\frac{15000}{8}$:

But - $\overline{xx-\frac{650}{8}x+\frac{325}{8}}\Big|^2=\frac{15000}{8}+\frac{105625}{64}$
$=\frac{225625}{64}$;

Th. $x-\frac{325}{8}=\frac{475}{8}$; And $x=\left(\frac{475+325}{8}=\right)100.$

QUEST. CLXXV. There are three numbers the difference of whose differences is 6; their sum is 42; and the sum of their squares 794: What are those numbers?

Let the numbers required $=\left\{\begin{array}{l}z-x,\\ z,\\ z+x+6;\end{array}\right\}$ as in quest. 172.

Then - - - $42=3z+6$;
Th. - $36=3z$; and $12=z$:

Now their squares $\left\{\begin{array}{l}\overline{12-x}^2=144-24x+xx,\\ 12\times12=144,\\ \overline{18+x}^2=324+36x+xx;\end{array}\right.$

Th· (their sum) - $794=612+12x+2xx,$
Or - - - - - $182=2xx+12x$;
Th. - - - - - $91=xx+6x$:
But - - $(91+9=)\ 100=xx+6x+9$;
Th. - - - - - $10=x+3,$
And - - - - - $7=x.$

QUEST.

QUEST. CLXXVI. It is required to divide the number 63 into 5 such parts; that the first *more* 2, the second *less* 2, the third multiplied by 2, the fourth divided by 2, and the square root of the fifth, may be equal among themselves?

If x, y, u, z, and e represent the numbers required;

Then - - $x+2=y-2=2u=\frac{z}{2}=\sqrt{e}$ by question;

Therefore
$$\left\{\begin{array}{l} x=x, \\ x+4=y, \\ \frac{x+2}{2}=u, \\ (\overline{x+2}\times 2=)\ 2x+4=z, \\ xx+4x+4=e: \end{array}\right.$$

And $xx+8x+12+\frac{x+2}{2}=(x+y+u+z+e=)\ 63$,

Or $2xx+16x+24+x+2=(63\times 2=)\ 126$,

That is $2x^2+17x+26=126$;

Th. - - $xx+\frac{17}{2}x=50$:

But - $\overline{xx+\frac{17}{2}x+\frac{17}{4}}|^2=(50+\frac{289}{16}=)\ \frac{1089}{16}$;

Th. $x+\frac{17}{4}=\frac{33}{4}$; and $x=(\frac{33-17}{4}=)\ 4$.

QUEST. CLXXVII. What two numbers are those, whose product is 15; and the sum of their square is 34?

If the greater number $=x$;

Then the lesser - - $=\frac{15}{x}$,

And - $xx+\frac{15\times 15}{xx}=34$ by question;

Or - - - $x^4+225=34xx$;

Th. - - $x^4-34x^2=-225$:

But $x^4-34x^2+17\times 17=(289-225=)\ 64$;

Th. $x^2-17=8$; and $x^2=(17+8=)\ 25$:

Whence - - - $x=5$.

F QUEST.

QUEST. CLXXVIII. A grocer sold 80 lb. of mace, and 100 lb. of cloves for 65 *l.* but he sold 60 lb. more of cloves for 20 *l.* than he did of mace for 10 *l.* What prices did he sell each at?

Suppose he sold the mace at x, And the cloves at y, } shillings a lb.

Then (x : 1 lb. :: (10 *l.* =) 200*s.* :) $\frac{200}{x}$ = lbs. of m. for 10 *l.*

And (y : 1 lb. :: (20 *l.* =) 400*s.* :) $\frac{400}{y}$ = lbs. of c. for 20 *l.*

$$\left.\begin{array}{lr} \text{Now - -} & (65\,l.=)\ 1300=80x+100y \\ \text{And - - - - -} & \frac{200}{x}+60=\frac{400}{y} \end{array}\right\}\text{by quest.}$$

By first - - - $\frac{130-8x}{10}=y$,

By second - - - - - - $y=\frac{20x}{10+3x}$;

Th. - - - - - $\frac{130-8x}{10}=\frac{20x}{10+3x}$

Or $1300-80x+390x-24x^2=200x$;

That is - - $24x^2-110x=1300$;

Th. - - - - - $x^2-\frac{110}{24}x=\frac{1300}{24}$:

But - - $\left.x^2-\frac{110}{24}x+\frac{55}{24}\right)^2=\frac{1300}{24}+\frac{3025}{570}$

$=\frac{34225}{576}$;

Th. $x-\frac{55}{24}=\frac{185}{24}$; and $x=\left(\frac{185+55}{24}=\right)10$.

Or

Or the root of $xx - \frac{110}{24}x = \frac{1300}{24}$ may be found as follows, by Dr. *Halley*'s method before quoted:

The Logarithm of - - -	$1300=$	$3,1139434,$
The Logarithm of - - - -	$24=$	$1,3802112;$
Therefore the Logarithm of -	$\frac{1300}{24}=$	$1,7337322;$
Also Log. of (half $110=$) - -	55	$1,7403627,$
And Log. of - - - -	24	$1,3802112,$
Th. Log. of - - - -	$\frac{55}{24}=$	$0,3601515:$

From half the Log. of - $\frac{1300}{24}$, $+10=10,8668661,$

Take the Log. of - $\frac{55}{24}$ $=0,3601515;$

Remains the Log. Tang. of $72^{\circ}:42':16''=10,5067146:$

From half the Log. of - $\frac{1300}{24}$, $+10=10,8668661,$

Take the Log. Tang. of half the Arc, above found } $36:21:8=9,8668617;$

Remains the Log. of x - - - $=1,0000044.$

Th. $x=10.$

QUEST. CLXXIX. There are three numbers whose differences are equal, the sum of their squares is 93; and if the first be multiplied by 3, the second by 4, and the third by 5, the sum of those products will be 66: What are those numbers?

If $z=$ the second number, and $x=$ their difference;

Then the numb. are $\left\{\begin{array}{l} z-x \\ z \\ z+x \end{array}\right\}$ and their squares $= \left\{\begin{array}{l} zz-2xz+xx, \\ zz, \\ zz+2xz+xx; \end{array}\right.$

Therefore (by question) $\quad 93=3z^2+2xx:$

But - - - $\left\{\begin{array}{l} z-x\times 3=3z-3x, \\ z \times 4=4z, \\ z+x\times 5=5z+5x; \end{array}\right.$

Therefore (by question) $\quad 66=12z+2x,$

And - - - $\quad 33-6z=x,$

Also - $\quad 1089-396z+36z^2=xx:$

Now - - - - - $\quad 93=3z^2+2178-792z+72zz,$

Or, - - - $\quad 75z^2-792z=-2085;$

Th. - - - $\quad z^2-\frac{264}{25}z=-\frac{695}{25}:$

But - - $\quad z^2-\frac{264}{25}z+\left.\frac{132}{25}\right|^2=\frac{17424}{625}-\frac{695}{25}=\frac{49}{625};$

Th. $\left(z-\frac{132}{25}=\frac{7}{25}\text{ Or,}\right)\frac{132}{25}-z=\frac{7}{25},$

And - - $\quad z=\left(\frac{132-7}{25}=\right)5=z:$

Hence - $\quad x=3.$

And the numbers are 2, 5, 8.

Other-

Otherwise, by *Dr. Halley's* method:

If $zz - \frac{264}{25}z = -\frac{695}{25}$

Then from Log. of	- -	695	= 2,8419848,
Take Log. of	- - -	25	= 1,3979400;
Remains Log. of	- - -	$\frac{695}{25}$	= 1,4440448:
And from Log. of		$\left(\frac{264}{2}=\right)$ 132	= 2,1205739,
Take Log. of	- - - -	25	= 1,3979400,
Remains Log. of	- -	$\frac{132}{25}$	= 0,7226339:
Now from ½ the Log. of	-	$\frac{695}{25}+10$	= 10,7220224,
Take Log. of	- - -	$\frac{132}{25}$	= 0,7226339;
Remains Log. Sine of		86° : 57′ : 45″	= 9,9993885:
To and from half the Log. of		$\frac{695}{25}$	= 0,7220224,
Add or subtract the Log. Tang. of ½ the above found Arc,		43° : 28′ ; 52″	= 9,9769646;
The Log. of $z=$		Sum −10	= 0,6989870,
		Differ. +10	= 0,7450578:

Th. $z=5$.

QUEST. CLXXX. What number is that, which divided by the product of its two digits quotes 3; and if 18 be added to it, the digits will be inverted?

If the digits be represented by x and y;

Then the number will be - - - - - $10x+y$,

And (by question) - $\begin{cases} 3=\dfrac{10x+y}{xy}, \\ 10y+x=10x+y+18; \end{cases}$

But by the last - - - - - - $9y=9x+18$;

And - - - - - - - $y=x+2$:

Now (writing $x+2$ for y in the 1st) $3=\dfrac{10x+x+2}{x\times\overline{x+2}}$,

Or - - $(3x\times\overline{x+2}=)\ 3xx+6x=11x+2$;

Th. - - - - - - $xx-\frac{5}{3}x=\frac{2}{3}$:

But - - - - - $\overline{xx-\frac{5}{3}x+\frac{5}{6}}\Big|^2=\left(\frac{2}{3}+\frac{25}{36}=\right)\frac{49}{36}$;

Th. - - - - - $x-\frac{5}{6}=\frac{7}{6}$; and $x=\left(\frac{7+5}{6}=\right)$ 2;

And the number is 24.

QUEST. CLXXXI. What two numbers are those whose product is 35; and the difference of their cubes is 218?

If the lesser number be x;

Then the greater is $\dfrac{35}{x}$:

Now by quest. $218=\left(\overline{\dfrac{35}{x}}\Big|^3-x^3=\right)\dfrac{42875}{x^3}-x^3$,

Or - - - $218x^3=42875-x^6$;

Th. - $x^6+218x^3=42875$:

But $x^6+218x^3+\overline{109}^2=(42875+11881=)\ 54756$;

Th. $x^3+109=234$; and $x^3=(234-109=)\ 125$;

Whence - - - $x=(\sqrt[3]{125}=)\ 5$.

QUEST.

QUEST. CLXXXII. There is a number consisting of three digits, the first of which is to the second as the second to the third; the number itself is to the sum of its digits as 124 to 7; and if 594 be added to it, the digits will be inverted: What is that number?

Suppose x, y, and z, were the digits in their order,
Then $100x+10y+z$ will be the number;
Now 1st $100x+10y+z : x+y+z :: 124 : 7$ by quest.
That is $700x+70y+7z=124x+124y+124z$,
Or - - - $576x=54y+117z$;
Th. - - - $x=\left(\frac{54y+117z}{576}=\right)\frac{6y+13z}{46}$:

And 2d $100z+10y+x=100x+10y+z+594$,
That is - - $99z-524=99x$,
Th. - - - $z-6=(x:=)\frac{6y+13z}{64}$,
Or - - $64z-384=6y+13z$,
Or - - $51z-384=6y$; And $y=\frac{17z-128}{2}$;
But $x:y::y:z$; Th. $xz=yy$,
That is - - $\overline{z-6}\times z=\frac{17z-128}{2}\times\frac{17z-128}{2}$,
Or - - - $4zz-24z=289z^2-4352z+16384$,
Or - $285zz-4328z=-16384$;
Th. - - $zz-\frac{4328}{285}z=-\frac{16384}{285}$;
But $\overline{zz-\frac{4328}{285}z+\frac{2164}{285}}\Big|^2=\left(\frac{4682896}{285\times285}-\frac{16384}{285}=\right)$
$\frac{13456}{285\times285}$.
Th. $z-\frac{2164}{285}=\frac{116}{285}$; and $z=\left(\frac{2164+116}{285}=\frac{2280}{285}\right)$ 8.

Quest. CLXXXIII. There are three numbers whose squares have equal differences; their sum is 13; and the sum of their squares 75: What are those numbers?

Let x, y, and z, be the numbers required,
And d the difference of their squares;

$$\text{Then} \quad \left\{\begin{array}{l} yy-d=xx, \\ yy \quad\;\; =yy, \\ \underline{yy+d=zz;} \end{array}\right.$$

And (their sum) - $3yy=75$ by quest.
Th. - $yy=25$; and $y=5$:
Now $(x+y+z=)\ x+5+z=13$,
And - - - - $x+z=8$,
Th. - - $xx+2xz+zz=64$;
Also - - - $xx+zz=(y^2-d+y^2+d=)\ 2yy$,
That is - - - $xx+zz=(25\times2=)\ 50$,
And (by substr.) - $2xz=(64-50=)\ 14$;
Th. - - - - - $xz=7$:

The sum and prod. giv. Then by (quest. 159) $\left\{\begin{array}{l} z=(8+\sqrt{64-4\times7}\times\frac{1}{2}=)\ 7, \\ x=(8-\sqrt{64-4\times7}\times\frac{1}{2}=)\ 1. \end{array}\right.$

Quest. CLXXXIV. There are three numbers whose differences are equal, their sum is 9; and the sum of their fourth powers 707: What are those numbers?

If $\left\{\begin{array}{l} z-x \\ z \\ \underline{z+x} \end{array}\right\}$ be the numbers required;

Then $3z \quad = $ (their sum $=$) 9; and $z=3$:

$$\text{Now} \quad \left\{\begin{array}{l} \overline{3-x}^4=81-108x^3+54x^2-12x^3+x^4, \\ 3^4 \quad\;\; =81, \\ \underline{\overline{3+x}^4=81+108x^3+54x^2+12x^3+x^4;} \end{array}\right.$$

Whence $707=243+108x^2+2x^4$ by quest.
Or - $464=108x^2+2x^4$,
Th. - $232=x^4+54xx$:
But - $961=x^4+54xx+729$;
Th. - - $31=x^2+27$; or $4=xx$; and $2=x$.

Quest.

QUEST. CLXXXV. What two numbers are those, whose sum multiplied by the greater produces 77; and whose difference multiplied by the lesser gives 12?

Let x = greater, and y = lesser number;

Then $(\overline{x+y} \times x =)\ xx+xy=77$ } by quest.
And $(\overline{x-y} \times y =)\ xy-yy=12$ }

By first - - - $xy=77-xx$,
By second - - - $xy=12+yy$;

Th. - - - - $77-xx=12+yy$,

Or - - - $65-yy=xx$;

Th. - - $\sqrt{65-yy}=x$:

But - $y\sqrt{65-yy}-yy=12$ by second,

Or - - - $\sqrt{65-yy}=\frac{12+yy}{y}$;

Th. - - - $65-yy=\frac{144+24yy+y^4}{yy}$,

Or - - - $65y^2-y^4=144+24yy+y^4$,

Or - - - - $-144=2y^4-41yy$;

Th. - - - $-\frac{144}{2}=y^4-\frac{41}{2}yy$:

But $\left(\frac{1681}{16}-\frac{144}{2}=\right)\ \frac{529}{16}=\overline{y^4-\frac{41}{2}yy+\frac{41}{4}}\Big|^2$;

Th. - - - $\frac{23}{4}=y^2-\frac{41}{4}$,

And - $\left(\frac{41+23}{4}=\right)\ 16=y^2$;

Whence - - - - $4=y$.

QUEST. CLXXXVI. What two numbers are those, whose sum is 5; and the sum of their 5th powers 275?
If x and y represent the numbers required,
Then $x+y=5$; And $x^5+y^5=275$ by quest.
Now

$$\overline{x+y}^5=x^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+y^5=3125$$

But $x^5 \qquad\qquad +y^5=275$

The diff. $5x^4y+10x^3y^2+10x^2y^3+5xy^4=2850$

Or - - $5xy\times x^3+2x^2y+2xy^2+y^3=2850$

But $\overline{x+y}^3$, Or $x^3+3x^2y+3xy^2+y^3=125$

The difference - - - $x^2y+xy^2=125-\frac{2850}{5xy}$

Or - - - - $(\overline{x+y}\times xy=)\ 5xy=125-\frac{2850}{5xy}$

Also by 1st - - - - - $25x^2y^2=625xy-2850$,

Or - - - - - - - - - $x^2y^2=25xy-114$,

Th. - - - - - - $x^2y^2-25xy=-114$:

But - - - - - $\overline{x^2y^2-25xy+\frac{25}{2}}\Big|^2=\frac{625}{4}-114$

$$=\frac{169}{4};$$

Th. - - - $(xy-\frac{25}{2}$ or$)\ \frac{25}{2}-xy=\frac{13}{2}$;

And - - - - - - $(\frac{25-13}{2}=)\ 6=xy$.

Now (by quest. 159) $\begin{cases} x=(5+\sqrt{25-4\times6}\times\frac{1}{2}=)\ 3. \\ y=(5-\sqrt{25-4\times6}\times\frac{1}{2}=)\ 2. \end{cases}$

QUEST.

QUEST. CLXXXVII. The reckoning of some men and boys at a tavern came to 53 shillings; of which the men paid each as many shillings as there were men in company; and the boys paid each 1 shilling; now if the number of men and boys were interchanged, the expence (regulated as before) would have been but 23 shillings: How many men and boys were there?

Suppose there were x men, and y boys;

$$\left.\begin{array}{l}\text{Then } xx+y=53:\ \text{Or } xx=53-y,\\ \text{And } yy+x=23:\ \text{Or } x=23-yy,\end{array}\right\}\text{per quest.}$$

Whence $529-46y^2+y^4=53-y$;

Th. $y^4-46y^2+y+476=0$.

$$\left.\begin{array}{l}\text{If } y=-1;\ \text{Then } 430=0;\\ y=\ \ 0;\ \text{Then } 476=0;\\ y=\ \ 1;\ \text{Then } 432=0;\end{array}\right\}\text{of which}\left\{\begin{array}{l}1,2,5,10,43,\\ 1,2,4,7,14,17,\\ 1,2,3,4,6,9,\end{array}\right\}\text{are divisors}$$

Where 5, 4, and 3, differ by unity;

And $\dfrac{y^4-46y^2+y+476}{y-4}=y^3+4y^2-30y-119$;

Th. - - - - - $y=4$.

QUEST. CLXXXVIII. It is required to divide the number 35, into two such parts; that the square of the lesser, multiplied by the greater, may produce 750?

Let - - $x=$ the lesser part,

Th. $35-x=$ the greater part;

And $(35-x\times xx=)\ 35xx-x^3=750$ per question;

Th. $x^3-35xx+750=0$:

$$\left.\begin{array}{l}\text{If } x=-1;\ \text{Then } 714=0;\\ x=\ \ 0;\ \text{Then } 750=0;\\ x=\ \ 1;\ \text{Then } 716=0;\end{array}\right\}\text{of which}\left\{\begin{array}{l}1,2,3,6,7,14,17,\\ 1,2,3,5,6,10,25,\\ 1,2,4,179.\end{array}\right\}\text{are div.}$$

Now 6, 5, and 4 differ by unity;

And $\dfrac{x^3-35xx+750}{x-5}=xx-30x-150$;

Th. - - - - - $x=5$.

QUEST. CLXXXIX. There are three numbers which have equal differences; if the square of the least be added to the product of the two greater, the sum will be 576; but if the square of the greatest be added to the product of the two lesser, the sum will be 792: What are these numbers?

If $\left.\begin{cases} z-x \\ z \\ z+x \end{cases}\right\}$ represent the numbers required;

$$\left.\begin{array}{ll}\text{Then} & \overline{z-x}^2+\overline{z+x}\times z=576 \\ \text{And} & \overline{z+x}^2+\overline{z-x}\times z=792\end{array}\right\}\text{ by quest.}$$

$$\left.\begin{array}{l}\text{Now} \\ \text{their} \\ \text{diff.}\end{array}\right\}(4zx-2zx=)\ 2zx=(792-576=)\ 216,$$

$$\text{Or} \quad - \quad xz=108\text{; and } z=\frac{108}{x}:$$

But

$$\overline{\frac{108}{x}-x}^2+\overline{\frac{108}{x}+x}\times\frac{108}{x}=576 \text{ by first; and}$$

$$\text{since} \quad \frac{11664}{xx}-216+xx=\overline{\frac{108}{x}-x}^2,$$

$$\text{and} \quad \frac{11664}{xx}+108 \quad =\overline{\frac{108}{x}+x}\times\frac{108}{x}\Big):$$

$$\text{Th.} \quad \frac{23328}{xx}-108+xx=576;$$

$$\text{Or} \quad - \quad - \quad - \quad \frac{23328}{xx}+xx=(576+108=)\ 684,$$

$$\text{Or} \quad - \quad - \quad 23328+x^4=684xx;$$

$$\text{Or} \quad - \quad - \quad x^4-684xx=-23328\text{: and compl}^{g}\text{. the sq.}$$

$$x^4-684xx+\overline{342}^2=(116964-23328=)\ 93636;$$

Whence

$(x^2-342$; Or) $342-xx=306$,

Th. - $(342-306=)$ $36=xx$; and $6=x$.

QUEST.

QUEST. CXC. There are two numbers the sum of whose squares is 89; and their sum multiplied by the greater produces 104: What are those numbers?

Suppose $x=$ greater, and $y=$ lesser;

$$\left.\begin{array}{ll}\text{Then} - - - & xx+yy=89\\ \text{And } (\overline{x+y}\times x=) & xx+xy=104\end{array}\right\}\text{ by quest.}$$

$$\text{By}\left\{\begin{array}{ll}\text{first} - - - & xx=89-yy,\\ \text{second} - - & xx=104-xy;\end{array}\right.$$

Th. - - - - $89-yy=104-xy$,

And - - - - - - - $x=\dfrac{15+yy}{y}$;

Also - - - - - - $xx=\dfrac{225+30yy+y^4}{yy}$:

But $\dfrac{225+30yy+y^4}{yy}+yy=89$ by first,

Or $225+30yy+y^4+y^4=89yy$,

Th. - - $2y^4-59y^2=-225$:

But $y^4-\dfrac{59}{2}y^2+\left.\dfrac{59}{4}\right|^2=\left(\dfrac{3481}{16}-\dfrac{225}{2}=\right)\dfrac{1681}{16}$;

Th. $y^2-\dfrac{59}{4}=\dfrac{41}{4}$; and $y^2=\left(\dfrac{59+41}{4}=\right)25$;

Whence - - - - $y=5$.

QUEST. CXCI. What two numbers are as 3 to 2; the sum of whose cubes is 280?

If $x=$ the greater of those numbers,

Then - $(3:2::x:)\ \dfrac{2x}{3}=$ the lesser;

Whence $\left(x^3+\dfrac{8x^3}{27}=\right)\dfrac{35x^3}{27}=280$, by quest.

Or (by Division) - $\dfrac{x^3}{27}=(8=)\ 2^3$,

Or (by Multiplication) $x^3=(2^3\times 27=)\ 2^3\times 3^3$;

Th. - - - - - $x=(2\times 3=)\ 6$.

QUEST.

QUEST. CXCII. What two numbers are those, whose sum is 14; and if the sum of their squares be multiplied by the sum of their cubes the product will be 72800?

If the diff. of the 2 numb. $=2x$;

Then (by qu.) $\left\{\begin{array}{l}\dfrac{14+2x}{2}=)\ 7+x=\sqsubset \\ \dfrac{14-2x}{2}=)\ 7-x=\sqsupset\end{array}\right\}$ number:

And - - - $\left\{\begin{array}{l}49+14x+x^2= \\ 49-14x+x^2=\end{array}\right\}$ square of $\left\{\begin{array}{l}\sqsubset \\ \sqsupset\end{array}\right.$

Th. - - - - - - $98+2x^2=$ sum of sqrs.

Also $\left\{\begin{array}{l}343+147x+21x^2+x^3= \\ 243-147x+21x^2+x^3=\end{array}\right\}$ cu. of $\left\{\begin{array}{l}\sqsubset \\ \sqsupset\end{array}\right.$

Th. - - - - - - $686+42x^2=$ sum of cu.

But - - $\overline{98+2xx}\times\overline{686+42xx}=72800$ by quest.

That is $67228+5488x^2+84x^4=72800$,

Or - - - - $84x^4+5488x^2=5572$,

Or - - - - - - $3x^4+196x^2=199$;

Th. - - - - - - $x^4+\dfrac{196}{3}xx=\dfrac{199}{3}$:

But - - $x^4+\dfrac{196}{3}xx+\left|\dfrac{98}{3}\right|^2=\dfrac{199}{3}+\dfrac{9604}{9}$

$=\dfrac{10201}{9}$;

Th. - $x^2+\dfrac{98}{3}=\dfrac{101}{3}$; and $x^2=\left(\dfrac{101-98}{3}=\right)1$;

Whence - - - - - - - $x=1$:

And - - - - $\left\{\begin{array}{l}(7+1=)\ 8= \\ (7-1=)\ 6=\end{array}\right\}$ the numbers required.

QUEST.

QUEST. CXCIII. *A* bought 7 sheep more than *B* for 15*l.*; now the square of *B*'s sheep is equal to the number of shillings that *A* paid for each sheep: How many did each buy?

If *B* bought x sheep, then *A* bought $x+7$;
And $\overline{x+7}$ sheep at xx shill. *per* sheep cost x^3+7xx shill.
But $x^3+7xx=(15\times 20=)\ 300$ *per* quest.
Th. $x^3+7xx-300=0$.

$$\left.\begin{array}{l}\text{If } x=-1;\ 294=0:\\ \quad x=\ \ 0;\ 300=0:\\ \quad x=\ \ 1;\ 292=0:\end{array}\right\}\text{of which}\left\{\begin{array}{l}1,2,3,6,7,14.\\ 1,2,3,4,5,6,10,\\ 1,2,4,73,\end{array}\right\}\text{are divisors.}$$

Whence 6, 5, and 4, differ by unity;

And $\dfrac{x^3+7xx-300}{x-5}=xx+2x+60$;

Th. - - - - - $x=5$.

QUEST. CXCIV. It is required to divide 16*l.* so, among two persons, that the cube of the one's share, may exceed the cube of the others, by 386?

If the greater share be x*l.* then the lesser will be $16-x$;
Now - - - - - $x^3-\overline{16-x}^3=386$ by question,
And $4096-768x+48x^2-x^3=\overline{16-x}^3$,
Th. $2x^3-48x^2+768x-4096=386$,
Or $x^3-24x^2+384x-2048=193$;
Th. $x^3-24x^2+384x-2241=0$:

$$\left.\begin{array}{l}\text{If } x=-1;\ 2650=0:\\ \quad x=\ \ 0;\ 2241=0:\\ \quad x=\ \ 1;\ 1880=0:\end{array}\right\}\text{of which}\left\{\begin{array}{l}1,2,5,10,25,53\\ 1,3,9,27,83\\ 1,2,4,5,8,10,20\end{array}\right\}\text{are divisions.}$$

Where 10, 9, and 8 differ by unity;

And $\dfrac{x^3-24x^2+384x-2241}{x-9}=xx-15x+249$:

Th. - - - - - - - - - $x=9$.

QUEST. CXCV. A man playing at hazard, won the first throw as much money as he had in his pocket; at the second throw he won 5 shillings more than the square root of what he then had; at the third throw he won the square of all he then had; and then he had 112*l.* 16*s.*: What had he at first?

Suppose that at first he had x shillings;
Then after winning x he had $2x$;
And winning $\sqrt{2x}+5$ he had $2x+\sqrt{2x}+5$:
Substitute - - - - - - - $y=2x+\sqrt{2x}+5$,
Then winning yy he had $y+yy$:
Now - - - - - - - $yy+y=(112l.\ 16s.=)\ 2256$ sh. by quest.
But - - - - - - - $yy+y+\frac{1}{4}=(2256+\frac{1}{4}=)\ \frac{9025}{4}$;
Th. - - $y+\frac{1}{2}=\frac{95}{2}$; And $y=\frac{95-1}{2}=47$:
But (by restitution) - $2x+\sqrt{2x}+5=47$,
Or - - - - - - - $2x+\sqrt{2x}=(47-5=)\ 42$,
Th. - - - - - - $x+\frac{\sqrt{2}}{2}x^{\frac{1}{2}}=\left(\frac{42}{2}=\right)\ 21$:
But - - - - - - $x+\frac{\sqrt{2}}{2}x^{\frac{1}{2}}+\frac{2}{16}=\left(21+\frac{2}{16}=\frac{338}{16}=\right)\ \frac{169\times 2}{16}$;
Th. - $x^{\frac{1}{2}}+\frac{\sqrt{2}}{4}=\frac{13\sqrt{2}}{4}$; And $x^{\frac{1}{2}}=\frac{\overline{13-1}\times\sqrt{2}}{4}=\frac{12\sqrt{2}}{4}=3\sqrt{2}$:
Whence - - - - - - $x=(3\sqrt{2}\times 3\sqrt{2}=9\times 2=)\ 18$.

QUEST. CXCVI. What two numbers are thoſe the ſum of whoſe ſquares is 58; and their product multiplied by the greater gives 147?

If x be the greater, and y the leſſer of thoſe numbers.

Then - $xx+yy=58$; Or $xx=58-yy$:

And $(xy\times x=)\ xxy=147$; Or $xx=\frac{147}{y}$:

Th. - - $58-yy=\frac{147}{y}$;

Or - - $58y-y^3=147$,

Or $y^3-58y+147=0$.

$$\left.\begin{array}{l} {}^{*}\text{If } y=-1;\ 204=0: \\ \quad\ y=\ \ 0;\ 147=0: \\ \quad\ y=\ \ 1;\ \ 90=0: \end{array}\right\}\text{of which}\left\{\begin{array}{l} 1,2,3,4,6,12,17, \\ 1,3,7,21,49, \\ 1,2,3,5,6,9,10,15,18, \end{array}\right\}\text{are div.}$$

Now 4, 3, 2, (one of which numbers is taken from each of thoſe ranks of diviſors.) differ by unity;

Alſo $y-3$) will divide $y^3-58+147$, and leave no remainder: See the work.

$$\begin{array}{r} y-3)\,y^3-58y+147\,(y^2+3y-49 \\ \underline{y^3-3y^2\qquad\qquad} \\ +3y^2-58y\qquad \\ \underline{3y^2-\ 9y\qquad} \\ -49y+147 \\ \underline{-49y+147} \\ 0\ .\ 0 \end{array}$$

Th. $y=3$; and $xx=\frac{147}{3}=59$. Or $x=7$.

* *By Sir* Iſaac Newton'*s method of finding diviſors.*

QUEST.

QUEST. CXCVII. What two numbers are as 5 to 6; and the sum of their cube roots is 6?

Let - - - $y=$ the lesser of the numb. requir.

Th. - $(5:6::y:\frac{6y}{5}=$ the greater number;

But - - $\overline{\frac{6y}{5}}\Big|^{\frac{1}{3}}+y^{\frac{1}{3}}=6$ by question,

That is $\overline{\frac{6}{5}}\Big|^{\frac{1}{3}}\times y^{\frac{1}{3}}+y^{\frac{1}{3}}=6$; put $a=\overline{\frac{6}{5}}\Big|^{\frac{1}{3}}$

Then - - - $y^{\frac{1}{3}}=\frac{6}{a+1}$:

And - - - - $y=\overline{\frac{6}{a+1}}\Big|^{3}$

But $(a=)\ \overline{\frac{6}{5}}\Big|^{\frac{1}{3}}=\sqrt[3]{1,2,}=1,06273$ (see the operation)

* L. $a=\frac{1}{3}$ L. $\frac{6}{5}$, And L. $y=3$ L. $\frac{6}{a+1}$:

0,7781513 is L. 6	0,7781513 is L. 6
0,6989700 is L. 5	0,3144275 is L. $a+1$
0,0791813 diff.	0,4637256
0,0263938 is L. $a=1,062651$	1,3911768 is L. y
	Th. $y=24,6137$.

* Here L. stands for Logarithm.

Other-

Otherwise, by extracting the cube root in a manner like that of Dr. *Halley*'s Irrational Theorem.

Thus, let $\sqrt[3]{1,2,}=1+r,$
Then - $1,2,=1+3r+3r^2+r^3;$
Now r^3 being small, relative to r and r^2,

Th. $0,2=3r+3r^2$, Or $\frac{0,2}{3}=r+r^2$,

And $r^2+r+\frac{1}{4}=\frac{1}{4}+\frac{0,2}{3}=\frac{15}{60}+\frac{4}{60}=)\ \frac{19}{60};$

Then $r+\frac{1}{2}=(\sqrt{\frac{19}{15\times4}}=\frac{1}{2}\sqrt{\frac{19}{15}}=\frac{1}{2}\sqrt{1,266}$ &c. $=)$ $[0,56273;$
And $r=0,06273;$

Whence $\sqrt[3]{1,2,}=(1+r=1,06278;$ And $y=24,612.$

QUEST.

QUEST. CXCVIII. It is required to divide the number 56, into two such parts that the sum of their cubes may be to the cube of the greater as 245 to 238?

Let x be the greater, and y the lesser part;
Then $x^3+y^3 : x^3 :: 245 : 238$ by quest.
That is $238x^3+238y^3=245x^3$,
Or - - - - - $238y^3= \ 7x^3$;
Th. - - - - - $34y^3= \ x^3$,
And - - $\sqrt[3]{34}\times y= \ x$:
But - $\sqrt[3]{34}\times y+y= 56$ by question:
Th. - - - - - - $y=\frac{56}{\sqrt[3]{34}+1}$:
But - - - $\sqrt[3]{34}=3{,}239609$; see the operation;
Let - - - $\sqrt[3]{34}=3+r$;
Then - - - - $34=27+27r+9r^2+r^3$,
And - - - - - $7=27r+9r^2+r^3$,
Or - - - - $7-r^3=27r+9r^2$,
Th. - $\frac{7}{9}-\frac{r^3}{9}=r^2+3r$:
But - - $\frac{9}{4}+\frac{7}{9}-\frac{r^3}{9}r^2+3r+\frac{9}{4}$,
Or - - - $\frac{109}{36}-\frac{r^3}{9}=r^2+3r+\frac{4}{9}$;
Th. $\sqrt{\frac{109}{36}-\frac{r^3}{9}}=r+\frac{3}{2}$:

Or

Or - - $\sqrt{\frac{109}{36}}=r+\frac{3}{2}$ (rejecting $\frac{r^3}{9}$ as inconsiderable)

Th. - - $\frac{1}{6}\sqrt{109}=r+\frac{3}{2}$;

Or $1{,}74005$, &c. $=r+\frac{3}{2}$,

Or $0{,}24005$, &c. $=r$:

Now $0{,}001536$, &c. $=\frac{r^3}{9}$;

And $\frac{0{,}001536,\ \&c.}{1{,}74005\times 2}=0{,}000441=\sqrt{\frac{109}{36}}-\sqrt{\frac{109}{36}-\frac{r^3}{9}}$;

Th. $0{,}239609$, &c. $=r=(0{,}24005$, &c. $-0{,}000441)$:

And $3{,}239609$, &c. $=(3+r=)\sqrt[3]{34}$;

Therefore - - $y=\left(\frac{56}{3{,}239609+1}=\right)$ $13{,}2088$, &c.

QUEST.

QUEST. CCII. Two masons A and B jointly perform a piece of work in 12 days; now if the sum of the days in which they could each have separately performed the same, be multiplied by the days in which A alone (he working quicker than B) could have done it, the product will be 1000: In what time could each do it?

Suppose A could do it alone in x days,
And B in - - - - - y days;

$$\left.\begin{array}{ll}\text{Then} & (x : 1 :: 12 :) \dfrac{12}{x} = \text{the work done by } A, \\ \text{And} & (y : 1 :: 12 :) \dfrac{12}{y} = \text{the work done by } B,\end{array}\right\} \text{in 12 days;}$$

Th. - - - $\dfrac{12}{y} + \dfrac{12}{x} = 1,$

Or - - - - $12x + 12y = xy,$

Or - - - - - $12x = xy - 12y$:

Th. - - - - $\dfrac{12x}{x-12} = y.$

And - - - - $\dfrac{12xx}{x-12} = xy$:

But $(\overline{x+y} \times x =)\ xx + xy = 1000$ by quest.

That is - $xx + \dfrac{12xx}{x-12} = 1000,$

Or $x^3 - 12xx + 12x^2 = 1000x - 1200$;

Th. $x^3 - 1000x + 12000 = 0.$

$$\begin{array}{l|l|l|l|l} \text{If } x = -1; & & 12999 = 0: & & 1,3,7,21,619, \\ \phantom{\text{If }} x = \ \ 0; & \text{Then} & 12000 = 0: & \text{of which} & 1,2,3,4,5,6,8,10,12,16,20, \\ \phantom{\text{If }} x = \ \ 1; & & 11001 = 0: & & 1,3,19,57,193. \end{array} \Bigg\} \text{are div.}$$

Now 21, 20, 19, differ by unity;

And $\dfrac{x^3 - 1000x + 12000}{x-20} = x^2 + 20x - 600$;

Th. - - - - - - $x = 20$;

And $\left(\dfrac{12 \times 20}{20-12} =\right) 30 = y.$

QUEST.

QUEST. CCIII. *A*, *B*, and *C*, who among them had 2000 shillings, went to play, and *B* lost to *A* the square root of what *A* began with, and had 341 shillings left; but if he had lost to *C* the cube root of what *C* began with, he would have had 362 shillings left: What sum had each at first?

Suppose *A* had x; *B*, y; and *C*, z shillings;

$$\left.\begin{array}{l} \text{Then } x+y+z=2000;\ \text{or } y=2000-x-z, \\ \qquad y-x^{\frac{1}{2}}=341;\ \text{or } y=341+x^{\frac{1}{2}}, \\ \qquad y-z^{\frac{1}{3}}=362;\ \text{or } y=362+z^{\frac{1}{3}}, \end{array}\right\}\text{by quest.}$$

Whence - - $341+x^{\frac{1}{2}}=362+z^{\frac{1}{3}}$,

Or - - - - - - - - $x^{\frac{1}{2}}=21+z^{\frac{1}{3}}$;

Th. - - - - - - - - $x=441+42z^{\frac{1}{3}}+z^{\frac{2}{3}}$:

$$\left.\begin{array}{l}\text{Now (by}\\ \text{1st and 3d}\end{array}\right\}\ 362+z^{\frac{1}{3}}=2000-441-42z^{\frac{1}{3}}-z^{\frac{2}{3}}-z;$$

Or - $z+z^{\frac{2}{3}}+43z^{\frac{1}{3}}-1197=0$.

If $z^{\frac{1}{3}}=-2$;	Then	$-1287=0$:	of which	1,3,9,11,13,33,39,	are divisors;
If $z^{\frac{1}{3}}=-1$;		$-1240=0$:		1,2,4,5,8,10,20,31,40,	
If $z^{\frac{1}{3}}=0$;		$-1197=0$:		1,3,7,9,19,21,57,	
If $z^{\frac{1}{3}}=1$;		$1152=0$:		1,2,3,4,6,8,12,16,	
If $z^{\frac{1}{3}}=2$;		$1099=0$:		1,7,157,	

Where 11, 10, 9, 8, 7, differ by unity;

And $\dfrac{z+z^{\frac{2}{3}}+43z^{\frac{1}{3}}-1197}{z^{\frac{1}{3}}-9}=z^{\frac{2}{3}}+10z^{\frac{1}{3}}+133$;

Th. - - - - - - $z^{\frac{1}{3}}=9$:

And - - - - - - $z=(9^3=)\ 729$.

QUEST. CCIV. A charitable person gave 20 shillings among some poor persons, men and women, and to each as many pence as there were poor men; now if he had given a like sum to a number of persons equal to the square of the number of poor women, it would have cost him 64 shillings: How many men and women did he give to?

Suppose to x men and y women,

$$\left.\begin{array}{l}\text{Then } (\overline{x+y}\times x=)\ xx+xy=(20\times 12=)\ 240\\ \text{And}\ -\ yy\times x=\quad yyx=(64\times 12=)\ 768\end{array}\right\}\text{ by quest.}$$

Now - - $240-xx=(xy=)\ \dfrac{768}{y}$;

Th. - - $240-\dfrac{768}{y}=(xx=)\ \overline{\dfrac{768}{yy}}\Big|^2$,

That is - - $240-\dfrac{768}{y}=\dfrac{589824}{yyyy}$, which divides by 12×4:

Or - - - - $5-\dfrac{16}{y}=\dfrac{12288}{yyyy}$;

Th. $5y^4-16y^3-12288=0$.

$$\left.\begin{array}{l}\text{If } y=-1;\ \text{Then } 12267=0:\\ \quad y=\ \ 0;\ \text{Then } 12288=0:\\ \quad y=\ \ 1;\ \text{Then } 12299=0:\end{array}\right\}\text{of which}\left\{\begin{array}{l}1,3,9,29,47,\\ 1,2,3,4,6,8,12,16,48,\\ 1,7,49,251,\end{array}\right\}\text{are divi.}$$

Where 9, 8, 7. differ by (unity) a divisor of (5) the coeficient of (y^4) the highest power of (y) the unknown quantity;

And $\dfrac{5y^4-16y^3-12288}{y-8}=5y^3+24y^2+192y+1536$;

Th. - - - - - - - $y=8$.

QUEST.

QUEST. CCV. If the product of the solidities of two cubes, whose sides differ by 4, be multiplied by the solidity of the greater, it will produce 3176523: What are their sides?

If $x=$ the side of the gr. and $y=$ side of the less. cube;

Then $x-y=4$; Or $x=y+4$,

And $x^3\times x^3y^3=3176523$; Or $x^6=\frac{3176523}{y^3}$, } by Qu.

Whence $\overline{y+4}^6=\frac{3176523}{y^3}$,

Or $\overline{y+4}^6\times y^3=(3176523=)\ \overline{147}^3$;

Th. $\overline{y+4}^2\times y=(147=)\ 49\times 3$,

And $\overline{y+4}\times y^{\frac{1}{2}}=7\sqrt{3}$; Th. $y^{\frac{3}{2}}+4y^{\frac{1}{2}}=7\sqrt{3}$

Substitute $z-x$ for $y^{\frac{1}{2}}$; and z^3-x^3 for $7\sqrt{3}$:

Then $z^3-3z^2x+3zx^2-x^3+4z-4x=z^3-x^3$,

Th. $4\times\overline{z-x}=3zx\times\overline{z-x}$; And $\frac{4}{3z}=x$:

But $(z^3-x^3=)\ z^3-\frac{4^3}{3^3z^3}=7\sqrt{3}$,

Th. $z^6-7\sqrt{3}\times z^3=\frac{4^3}{27}$:

But $z^6-7\sqrt{3}\times z^3+\frac{49\times 3}{4}=\left(\frac{64}{27}+\frac{147}{4}=\right)\frac{4225}{36\times 3^3}$.

Th. $z^3-\frac{7\sqrt{3}}{2}=\frac{65}{6\sqrt{3}}$; and $z^3=\frac{65}{6\sqrt{3}}+\frac{7\sqrt{3}}{2}$

Or $z^3=\frac{65}{6\sqrt{3}}+\frac{21\times 3}{6\sqrt{3}}=\frac{64}{3\sqrt{3}}$:

Whence $z=4\sqrt{\frac{1}{3}}$:

But $(z^3-x^3=)\ \frac{64}{3\sqrt{3}}-x^3=7\sqrt{3}$,

Th. $\frac{1}{\sqrt{3}}=x$,

Now $\left(z-x=\frac{4}{\sqrt{3}}-\frac{1}{\sqrt{3}}=\right)\frac{3}{\sqrt{3}}=y^{\frac{1}{2}}$,

And $\left(\frac{9}{3}=\right)\ 3=y$.

QUEST. CCVI. What two numbers are those, whose sum, and product, being severally multiplied by the lesser will produce 175, and 250?

If $x=$ the greater, and $y=$ the lesser of those numbers;

Then $(\overline{x+y} \times y =)\ xy+yy=175$ } *per* quest.
And - - $(xy \times y=)\ xyy=250$ }

Th. - - $\frac{175-yy}{y}=(x=)\ \frac{250}{yy}$;

And - - $175y-y^3=270$: here $y \sqsubset 1$, and $\sqsupset 2$.

Th. - $y=1,4458$, &c. See the operation below *.

Suppose $y=\frac{3}{2}-r$;

Th. $175y=\frac{525}{2}-175r$,

And - $y^3=\frac{27}{8}-\frac{27}{4}r+\frac{9}{2}r^2-r^3$;

By sub. $250=\frac{2073}{8}-\frac{673}{4}r-\frac{9}{2}r^2+r^3$,

Or $2000=2073-1346r-36r^2+8r^3$,

(By transp.) $1346r+36r^2-8r^3=73$:

Now - - - 1st, let $1346r'=73$;

Then - - - - - - - - $r'=(\frac{73}{1346}=)\ 0,0542$,

2d Let - - $1346r''+36r'r''=73$,

That is $1346r''+36 \times ,0542r''=73$.

Or - - $1346r''+1,8512r''=73$;

Th. - $(\frac{73}{1347,96}=)\ 0,054156=r''$:

3d - $1346r+36r''r-7r''''r\ r=73$,

That is $1346r+1,949616r-0,023456r=73$;

Th. $r=\frac{73}{1347,926160}=0,0541572$, &c.

Now - - - - $\frac{3}{2}-r=y$;

That is $1,5-0,0541572=1,4458429$, &c. $=y$.

* *This operation is conformable to Dr. Halley's rational theorem.*

QUEST.

QUEST. CCVII. What two numbers are thoſe, the difference of whoſe ſquares is 9; and the ſum of their cubes 189?

$$\left.\begin{array}{lll}\text{Now} & xx-yy=9; & \text{Or } xx=9+yy, \\ \text{And} & x^3+y^3=189; & \text{Or } x^3=189-y^3,\end{array}\right\}\text{ by queſtion;}$$

$$\left.\begin{array}{l}\text{Alſo the 2d equation being}\\ \text{divided by the 1ſt, gives}\end{array}\right\} x=\frac{189-y^3}{9+yy};$$

$$\text{Th.} \quad xx=\frac{35721-378y^3+y^6}{81+18yy+y^4}:$$

$$\text{(But by firſt)} \quad \frac{35721-378y^3+y^6}{81+18yy+y^4}=9+yy,$$

$$\text{Whence} \quad 35721-378y^3=729+243y^2+27y^4;$$

$$\text{Th.} \quad y^4+14y^3+9y^2-1296=0.$$

$$\left.\begin{array}{l}\text{If } y=-2;\\ y=-1;\\ y=\ \ 0;\\ y=\ \ 1;\\ y=\ \ 2;\end{array}\right\}\text{ Then }\left\{\begin{array}{l}1356=0\\ 1300=0\\ 1296=0\\ 1272=0\\ 1132=0\end{array}\right\}\text{ of which }\left\{\begin{array}{l}1,2,3,4,6,12,113,\\ 1,2,4,5,10,13,20,25,\\ 1,2,3,4,6,8,9,12,\\ 1,2,3,4,6,8,12,24,\\ 1,2,4,283,\end{array}\right\}\text{ are diviſors;}$$

Where 6, 5, 4, 3, 2, differ by unity;

$$\text{And} \quad \frac{y^4+14y^3+9y^2-1296}{y-4}=y^3+18yy+81y+324;$$

$$\text{Th.} \quad y=4, \text{ and } x=5.$$

QUEST. CCVIII. What two numbers are those whose sum is 5; and the sum of their cubes being multiplied by the cube of the lesser will produce 280?

If x be the greater, and y the lesser of those numbers,

$$\left.\begin{array}{ll}\text{Then} - - - - & x+y=5\text{; Or } x=5-y \\ \text{And} - - & (\overline{x^3+y^3}\times y^3=x^3y^3+y^6=280\end{array}\right\}\text{per qu.}$$

$$\begin{array}{ll}\text{But } (\overline{5-y}^3=) & 125-75y+15y^2-y^3=x^3 \\ \text{And} - & 125y^3-75y^4+15y^5-y^6=x^3y^3\end{array}$$

$$\begin{array}{ll}\text{Th.} & 125y^3-75y^4+15y^5-y^6+y^6=280 \text{ by 2d step,} \\ \text{Or} - & 15y^5-75y^4+125y^3-280=0 \\ \text{Or} - - & 3y^5-15y^4+25y^3-56=0\end{array}$$

$$\left.\begin{array}{l}\text{If } y=-1\text{; Then } -99=0\text{; and } 1,3,9,11 \\ \quad y=\ 0\text{; Then } -56=0\text{; and } 1,2,4,7,8 \\ \quad y=\ 0\text{; Then } -45=0\text{; and } 1,3,5,9\end{array}\right\}\text{are divisors:}$$

Where 3, 2, and 1, differ by unity,

$$\text{And } \frac{3y^5-15y^4+25y^3-56}{y-2}=3y^4-9y^3+7y^2+14y+28$$

Th: $y=2$; and $x=(5-2=)\ 3$.

QUEST.

QUEST. CCIX. The value of x, y, and z, in the following equations, is required;

$$\text{Viz.} \quad \left\{ \begin{array}{r} x^3+y^3+z^3=50 \\ y^3+x=18 \\ y^3+z=17 \end{array} \right\} ?$$

Now (the diff. two last) $x-z=1$;
Th. - - - - - - - - $x=1+z$,
And - - - - - - - - $x^3=1+3z+3z^2+z^3$;

But from third - - - - $y^3=\underline{17-z}$;
Th. (by 1st) $1+3z+3z^2+z^3+17-z+z^3=50$,
That is - - - - - - - $2z^3+3z^2+2z=32$;
Th. - - - - - - - $2z^3+3z^2+2z-32=0$.

$$\left.\begin{array}{l} \text{If } z=-1;\ \text{Then } 33=0: \\ \phantom{\text{If }} z=\ \ 0;\ \text{Then } 32=0: \\ \phantom{\text{If }} z=\ \ 1;\ \text{Then } 25=0: \end{array}\right\} \text{which} \left\{\begin{array}{l} 1,3,11, \\ 1,2,4,8,16, \\ 1,5,25, \end{array}\right\} \text{are divisors.}$$

Where 3, 2, 1, differ by (unity) a divisor of (2) the coefficient of (z^3) the highest power of (z) the unknown quantity;

And $\dfrac{2z^3+3z^2+2z-32}{z-2}=2z^2+7z+16$;
Th. - - - - - $z=2$.

QUEST. CCX. What are the values of x and y in the following equations:

Viz. - - - - $\left\{\begin{array}{l} xxxyy+xyyyy=156 \\ xxyyy+xxxxy=234 \end{array}\right\}$?

Let - - - - - - - - - - - $zy=x$;

Then - - - - - - - $z^3y^5+zy^5=156$ } by quest.

And - - - - - - - $z^2y^5+z^4y^5=234$ } by quest.

Th. - - - - - - - - - - $y^5=\frac{156}{z^3+z}$,

And - - - - - - - - - - $y^5=\frac{234}{z^2+z^4}$;

Whence $\frac{156}{z^3+z}=\frac{234}{z^2+z^4}$; Or $\frac{2}{z^2+1}=\frac{3}{z+z^3}$,

Or - - - - - - - - $2z+2z^3=3z^2+3$;

Th. - - - - - $2z^3-3z^2+2z-3=0$:

If $z=-1$; Then $10=0$: } of which { 1, 2, 5, 10, } are divisors

$z=\ \ 0$; Then $3=0$: } of which { 1, 3, } are divisors

$z=\ \ 1$; Then $2=0$: } of which { 1, 2, } are divisors

Where 5, 3, 1, have 2 for a common difference.

And - $\frac{2z^3-3z^2+2z-3}{2z-3}=z^2+1$;

Th. - - - - - - $2z=3$;

And - - - - - - - - $z=\frac{3}{2}$:

But (by first) $\overline{\tfrac{3}{2}}|^3\times y^5+\tfrac{3}{2}y^5=156$,

Or (dividing by $\frac{3}{2}$) $\frac{9}{4}y^5+y^5=\left(\frac{156\times 2}{3}=\right)$ 104,

Hence (dividing by $3\frac{1}{4}$) $y^5=\left(\frac{104\times 4}{13}=\right)$ 32;

Th. - - - - - - - - $y=2$:

And - - - - - - - - $x=(\frac{3}{2}\times 2=)$ 3.

QUEST.

QUEST. CCXI. What two numbers are those, whose product multiplied by the greater will produce 405; and their difference multiplied by the lesser, 20?

If x represent the greater, and y the lesser;

$$\left.\begin{array}{ll}\text{Then} & (xy \times x =)\ x^2 y = 405, \\ \text{And} & (\overline{x-y} \times y =)\ xy - yy = 20,\end{array}\right\}\text{ by question;}$$

$$\text{Now by}\ \left\{\begin{array}{ll}\text{first} & xy = \frac{405}{x}, \\ \text{second} & xy = 20 + yy;\end{array}\right.$$

$$\text{Th.}\quad \frac{405}{x} = 20 + yy;$$

$$\text{And}\quad \frac{405}{20+yy} = x;$$

$$\text{Also}\quad \frac{405y}{20+yy} = xy;$$

$$\text{But (by second)}\ \frac{405y}{20+yy} - yy = 20;$$

$$\text{Th.}\quad 405y - 20y^2 - y^4 = 400 + 20y^2,$$

$$\text{Or}\quad y^4 + 40y^2 - 405y + 400 = 0.$$

$$\text{If}\ \left.\begin{array}{r}y = -1; \\ y = 0; \\ y = 1;\end{array}\right\}\text{Then}\left\{\begin{array}{r}846 = 0: \\ 400 = 0: \\ 36 = 0:\end{array}\right\}\text{of which}\left\{\begin{array}{l}1,2,3,6,9,47, \\ 1,2,4,5,8,10,20, \\ 1,2,3,4,9,12,18,\end{array}\right\}\text{are div.}$$

Where 6, 5, and 4, differ by unity;

$$\text{And}\quad \frac{y^4 + 40y^2 - 405y + 400}{y-5} = y^3 + 5y^2 + 65y - 80,$$

$$\text{Th.}\quad y = 5.$$

QUEST. CCXII. What two numbers are those, whose product multiplied by the lesser will produce 225; but if their difference be multiplied by the greater, 36?

If $x=$ the greater, and $y=$ the lesser of those numbers;

$$\left.\begin{array}{lll} \text{Then} & - & (xy\times y=)\ xyy=225, \\ \text{And} & & (\overline{x-y}\times x=)\ xx-xy=36, \end{array}\right\}\text{by question;}$$

Now (by first) - - - $xy=\frac{225}{y}$,

And (by second) $xx-36=xy$;

Whence - - - - $xx-36=\frac{225}{y}$,

Th. - - - - - - $xx=\frac{225}{y}+36$;

But (by first) - - - $x=\frac{225}{yy}$,

And - - - - - $xx=\left(\frac{225\times 225}{yyyy}=\right)\frac{50625}{yyyy}$;

Whence - - - $\frac{225}{y}+36=\frac{50625}{yyyy}$,

Or - - - - - $\frac{25}{y}+4=\frac{5625}{y^4}$,

Or - - - $25y^3+4y^4=5625$;

Th. - $4y^4+25y^3-5625=0$.

$$\left.\begin{array}{l} \text{If } y=-1; \\ y=0; \\ y=1; \end{array}\right\}\text{Then}\left\{\begin{array}{l} 5604=0; \\ 5625=0; \\ 5596=0; \end{array}\right\}\text{of which}\left\{\begin{array}{l} 1,2,3,4,6,12,467, \\ 1,3,5,9,15,25,45, \\ 1,2,4,1399, \end{array}\right\}\text{are div.}$$

Where 6, 5, 4, differ by (1) a divisor of (4) the coefficient of (y^4) the highest power of y;

And $\frac{4y^4+25y^3-5625}{y-5}$
$=4y^3+45y^2+225y+1125$.

Th. $y=5$.

QUEST.

QUEST. CCXIII. There are two numbers, the difference of whose cubes is 604; and if their difference be multiplied by the greater, the product will be 36: What are those numbers?

If $x=$ the greater, and $y=$ the lesser of those numbers;

$$\left.\begin{array}{ll}\text{Then } \overline{x-y}\times x=36;\ \text{Or } & x-y=\dfrac{36}{x}, \\ \text{Also - - - - - -} & x^3-y^3=604,\end{array}\right\}\text{by question:}$$

$$\left.\begin{array}{l}\text{And divid. the 2d}\\ \text{equat. by the 1st,}\end{array}\right\} x^2+xy+y^2=\left(\frac{604x}{36}=\right)\frac{151x}{9};$$

$$\text{The 1st squared}\quad x^2-2xy+y^2=\left(\frac{36\times 36}{xx}=\right)\frac{1296}{xx};$$

$$\text{The diff. of 2 last is -}\quad 3xy=\frac{151x}{9}-\frac{1296}{xx},$$

$$\text{Th. - - - - -}\quad \frac{11664}{151-27y}=x^3:$$

$$\text{But by 2d.}\quad \frac{11664}{151-27y}-y^3=604.$$

$$\text{Th. } 27y^4-151y^3+16308y-79540=0,$$

If $y=-2$;	Then	$110516=0$:	of which	1, 2, 4, 7, 14, 28, 3947,	are divis.
$y=-1$;		$95670=0$:		1, 2, 3, 5, 6, 9, 10, 15,	
$y=\ \ 0$;		$79540=0$:		1, 2, 4, 5, 10, 20, 41, 97,	
$y=\ \ 1$;		$63356=0$:		1, 2, 4, 15839,	
$y=\ \ 2$;		$47700=0$:		1, 2, 3, 4, 5, 6, 9, 10,	

Where 7, 6, 5, 4, 3, differ by (1) a divisor of (27) the coefficient of (y^4) the highest power of y;

$$\text{And } \frac{27y^4-151y^3+16308y-79540}{y-5}$$
$$=27y^3-16y^2-80y+15908;$$

Th. $y=5$.

QUEST. CCXIV. What two numbers are those, the sum of whose cubes is 468; and the square of the greater added to their product, makes 84?

If $x=$ the greater, and $y=$ the lesser of those numbers;

$$\left.\begin{array}{l} \text{Then } xx+xy=84\text{; Or } x+y=\dfrac{84}{x}, \\ \text{And - - - - -} \quad x^3+y^3=468, \end{array}\right\} \text{by quest.}$$

$$\left.\begin{array}{l}\text{Now 2d equat.}\\ \text{div. by 1st gives}\end{array}\right\} x^2-xy+yy=\left(\frac{468x}{84}=\right)\frac{39x}{7},$$

And (add. $3xy$) $x^2+2xy+yy=\dfrac{39x}{7}+3xy$;

Now (from 1st) $x^2+2xy+yy=\left(\dfrac{84}{x}=\dfrac{84}{x}=\right)\dfrac{7056}{xx}$;

Th. - - - - $\dfrac{39x}{7}+3xy=\dfrac{7056}{xx}$,

Or - - - - $39x^3+21yx^3=(7056\times 7=)\ 49392$;

Th. - - - - - - - $x^3=\left(\dfrac{49392}{39+21y}=\right)\dfrac{16464}{13+7y}$:

Now (by 2d) $\dfrac{16464}{13+7y}+y^3=468$;

Th. $7y^4+13y^3-3276y+10380=0$.

$$\left.\begin{array}{l} \text{If } y=-1\text{;} \\ \quad y=\ \ 0\text{;} \\ \quad y=\ \ 1\text{;} \end{array}\right\} \text{Then} \left\{\begin{array}{r} 13650=0: \\ 10380=0: \\ 7126=0: \end{array}\right\} \text{of which} \left\{\begin{array}{l} 1,2,3,5,6,10,15, \\ 1,2,3,4,5,6,9, \\ 1,2,4,13,\ \&c. \end{array}\right| \text{are div.}$$

Where 6, 5, 4, differ by unity.

And $\dfrac{7y^4+13y^3-3276y+10380}{y-5}$

$=7y^3+48y^2+240y-2076$;

Th. $y=5$.

QUEST.

QUEST. CCXV. What two numbers are those, the difference of whose squares is 16; and the sum of their cubes 152?

If $x=$ the greater, and $y=$ the lesser of those numbers;

Then $x^2-y^2=16$; Or $x^2=16+y^2$, } by question;
And $x^3+y^3=152$; Or $x^3=152-y^3$, }

But $(\overline{x^2}|^3=)\ x^6=4096+768y^2+48y^4+y^6$,
And $(\overline{x^3}|^2=)\ x^6=23104-304y^3+y^6$;
Th. $4096+768y^2+48y^4+y^6=23104-304y^3+y^6$,
Or - $4096+768y^2+48y^4=23104-304y^3$,
Or - $48y^4+304y^3+768y^2=19008$;
Th. $3y^4+19y^3+48y^2-1188=0$.

If $y=-1$;	Then	$-1156=0$;	of which	1, 2, 4, 17,	are divisors,
$y=0$;		$-1188=0$;		1, 2, 3, 4, 6, 9, 11, 12,	
$y=1$;		$-1118=0$;		1, 2, 13, 43,	

Where 4, 3, and 2, differ by unity;

And $\dfrac{3y^4+19y^3+48y^2-1188}{y-3}$
$=3y^3+28y^2+132y+396$;
Th. $y=3$.

QUEST.

QUEST. CCXVI. What two numbers are those, the sum of whose square roots is 4; and the difference of their cube roots is 1?

Suppose $x=$ the gr. and $y=$ the less, of those numbers;

$$\left.\begin{array}{l}\text{Then } x^{\frac{1}{2}}+y^{\frac{1}{2}}=4;\ \text{Or } (x^{\frac{1}{2}}=)\ x^{\frac{3}{6}}=4-y^{\frac{3}{6}},\\ \text{And } x^{\frac{1}{3}}-y^{\frac{1}{3}}=1;\ \text{Or } (x^{\frac{1}{3}}=)\ x^{\frac{2}{6}}=1+y^{\frac{2}{6}},\end{array}\right\}\text{by quest.}$$

Th. - - $\overline{4-y^{\frac{3}{6}}}\,^{2}=(x=)\ \overline{1+y^{\frac{2}{6}}}\,^{3}$,

That is $16-8y^{\frac{3}{6}}+y=1+3y^{\frac{2}{6}}+3y^{\frac{4}{6}}+y$.

Th. - - - - - $15=3y^{\frac{4}{6}}+8y^{\frac{3}{6}}+3y^{\frac{2}{6}}$:

Let - - - - - $y^{\frac{1}{6}}=1+z$;

Then - - - $3y^{\frac{4}{6}}=3+12z+18z^2+12z^3$, &c.

And - - - $8y^{\frac{3}{6}}=8+24z+24z^2+8z^3$.

Also - - - $3y^{\frac{2}{6}}=3+6z+3z^2$;

Th. - - - - - $15=14+42z+45z^2+20z^3$, &c.

Th. - - $\dfrac{1-20z^3}{45}=z^2+\frac{14}{15}z$;

Or rejecting $20z^3$ as small; $\frac{1}{45}=z^2+\frac{14}{15}z$:

But - - $(\frac{1}{45}+\frac{49}{225}=)\ \frac{54}{225}=z^2+\frac{14}{15}z+\frac{49}{225}$.

Th. - - - - - - $\dfrac{\sqrt{54}}{15}=z+\frac{7}{15}$.

And - - - - - $\dfrac{\sqrt{54}-7}{15}=\left(\dfrac{7,348469-7}{15}\right.$

$=0,023231=)\ z$.

$$\left.\begin{array}{l}\text{Now by restoring } 20z^3=\\ 20\times\overline{,023231}\,^{3} \text{ and re-}\\ \text{newing the operation}\end{array}\right\}\ z=0,0232196;$$

And - - - - - - - - $y^{\frac{1}{6}}=1,0232196$;

Th. - - - - - - - - - $y=1,1476592$,

And - - - - - - - - $x=8,5773504$.

QUEST.

QUEST. CCXVII. When the days were 16 hours long, a person being asked what hour it was, replied; the cube root of the hours to come till night, being added to the square root of those past since morning, their sum will be the hour already past: Required the time of the day?

If yy be the hours past; and x those to come;

Then - - - - $yy+x=16$; Or $x=16-yy$ } per qu.

And - - - - $y+\sqrt[3]{x}=y^2$; Or $x=\overline{yy-y}|^3$ }

Th - - - $y^6-3y^5+3y^4-y^3=16-yy$;

Or - - - $y^6-3y^5+3y^4-y^3+y^2=16$.

Let - - - - - - - - - $2+z=y$;

Then $64+192z+240z^2+160z^3$, &c. $=y^6$

$48+96z+72z^2+24z^3$, &c. $=3y^4$

$4+4z+z^2$ - - - $=y^2$

The sum

$116+292z+313z^2+184z^3$, &c. $=y^6+3y^4+y^2$

Also $96+240z+240z^2+120z^3$, &c. $=3y^5$

$8+12z+6z^2+z^3$, &c. $=y^3$;

Th. $12+40z+67z^2+63z^3$, &c. $=16$,

Or - - - - - - $40z+67z^2+63z^3=(16-12=)4$.

1. If - - - - - - $40z'=4$; $z'=\frac{4}{40}=\frac{1}{10}$,

2. If $40z''+67z'z''=4$: $z''=\dfrac{4}{40+\frac{67}{10}}=\dfrac{40}{467}$

3. If - - $40z+67z''z+63z''z''z=4$;

Then $40z+\dfrac{67\times40}{467}z+\dfrac{63\times40\times40}{467\times467}z=4$,

Or $8723560z+1251560z+100800z=872356$,

Th. $z=\frac{872356}{10075920}=0,086578$, &c. And $y=2,086578$.

QUEST.

QUEST. CCXVIII. A company of men and women ſpent 4*l.* 16*s.* each man paid the ſquare of the ſhillings that each women paid; and by that means, he paid as many ſhillings more than her, as there were women in company; now if there had been no more women than men, they would have collected among them but 3*l.* How many men and women were there?

Suppoſe each women paid x ſhillings,
Then each man paid xx ſhillings,
And - - - $xx-x=$ the women in company;
Then - - $\overline{x^3-x^2}=$ ſhill. paid by all the women,
And - $96-x^3-x^2=$ ditto paid by all the men;
Th. - $\frac{96+x^2-x^3}{xx}=$ the number of men:

Now $\left(\frac{96+x^2-x^3}{xx}\times x=\right)\frac{96+x^2-x^3}{x}=$ the ſum that would have been paid by the women if the number were no greater than the number of men;

But $96x+x^2-x^3+\frac{96+x^2-x^3}{x}=60$ *per* queſt.

Or $96x+x^3-x^4+96+x^2-x^3=60x$,
Or - - $96x-x^4+96+x^2=60x$,
Or - - - $x^4-xx-36x-96=0$;
But - - $\frac{x^4-xx-36x-96}{x-4}=x^3+4x^2+15x+24$;
T- - - - - - - - - - - - - $x=4$.
Hence there were 12 women and 3 men.

QUEST.

QUEST. CCXIX. The product of the areas of 2 squares, the sum of whose sides is 15, being multiplied by the area of the lesser, will produce 153664: What are the areas of those squares?

Let $x=$ the side of the lesser, and $y=$ the side of the greater square.

Then $(x^2y^2 \times x^2=)\ x^4y^2=153664$,

Or - - - - - - $x^2y=\sqrt{153664}=392$,

Or - - - - - - $xx=\frac{392}{y}$;

Th. - - - - - - $x=\left(\sqrt{\frac{392}{y}}=\right)\ \frac{14\sqrt{2}}{\sqrt{y}}$;

But $(x+y=15$; Th.$)\ x=15-y$,

Th. - - - - - $15-y=\frac{14\sqrt{2}}{\sqrt{y}}$;

Or - - - $15y^{\frac{1}{2}}-y^{\frac{3}{2}}=14\sqrt{2}$,

Th. $y^{\frac{3}{2}}-15y^{\frac{1}{2}}+14\sqrt{2}=0$.

$$\begin{array}{l|c|l|c|l|l} \text{If } y^{\frac{1}{2}}=-\sqrt{2}; & & 27\sqrt{2}=0: & & \sqrt{2},3\sqrt{2},9\sqrt{2}; & \text{are di-} \\ \phantom{\text{If }} y^{\frac{1}{2}}=\phantom{-\sqrt{2}}0; & \text{Then} & 13\sqrt{2}=0: & \text{And} & \sqrt{2},2\sqrt{2},7\sqrt{2}, & \text{visors.} \\ \phantom{\text{If }} y^{\frac{1}{2}}=\sqrt{2}; & & 1\sqrt{2}=0: & & \sqrt{2}, & \end{array}$$

Of which $3\sqrt{2}$, $2\sqrt{2}$, and $\sqrt{2}$, differ by $1\sqrt{2}$;

Th. $2\sqrt{2}=y^{\frac{1}{2}}$.

Th. $y=(2\sqrt{2}\times 2\sqrt{2}=)\ 8$; and $yy=64$.

Also $x=(15-x=)\ 7$; and $xx=49$.

QUEST.

QUEST. CCXXVI. *B* asked *A*, who sat with a basket of apples to sell, how many he had? *A* replied, That he could not tell; but remembered that when he told them into his basket by twos, threes, fours, fives, and sixes, there always remained 1 apple over, but that telling them by sevens none remained: How many had he?

The numbers 2, 3, 4, 5, and 6, are divisors of the number 60.
The question therefore is to find a number that divided by 60 will leave +1, and by 7 no remainder.

Suppose x the number, y and z any whole number;

Then $\frac{x}{60}=y+\frac{1}{60}$; Or $x=60y+1$,

$\frac{x}{7}=z$; Or $x=7z$;

Th. $60y+1=7z$; and $z=8y+\frac{4y+1}{7}$

Now $\frac{4y+1}{7}$ being a whole number; $y=5$;

Th. $\frac{4y+1}{7}=3$; And $z=43$;

Hence $x=(60y+1=7z=)$ 301.

QUEST. CCXXVII. A higler's servant, who was sent with a basket of eggs, had the misfortune to break them, and being called upon to pay for them, his master forgot the number; but the mistress remembered, that when she told them by twos, 1 remained; by threes, 2 remained; by fours, 3; by fives, 4; by sixes, 5; and by sevens, none remained; What number of eggs were there?

This

This question differs from the former only in this, that the number required divided by 60 will leave -1.

That is $\frac{1}{60}x = y - \frac{1}{60}$; Or $x = 60y - 1$,
And - $\frac{1}{7}x = z$; Or $x = 7z$;

Th. $60y - 1 = 7z$; And $z = 8y + \frac{4y-1}{7}$;

Now $\frac{4y-1}{7}$ being a whole number, $y = 2$;

Then $\frac{4y-1}{7} = 1$; And $z = 17$;

Hence - $x = (60y - 1 = 7z =)\ 119$.

QUEST. CCXXVIII. Required the values of x, and y, in the equation $71x + 17y = 1005$?

Substitute - $z - 4x = y$;
Then $71x + 17z - 68x = 1005$ by question;
Th. - - - $17z = \underline{1005 - 3x}$,
Or - - - - $17z = 335 - x \times 3$;

Now $\left\{ \begin{array}{l} x = 3,\ 6,\ 9,\ 12,\ \&c. \\ 335 - x = 17,\ 34,\ 51,\ 68,\ \&c. \end{array} \right\}$ by qu. 220.

Or $\left\{ \begin{array}{l} x = 3,\ 3+3,\ 3+2\times 3,\ \&c.\ 3+3n \\ x = 318,\ 318-17,\ 318-2\times 17,\ \&c.\ 318-17n \end{array} \right.$

But ($z - 4x = y$ that is) $3 + 3n - 1272 + 68n = y$,

Th. - - - - - - - - - $n \sqsubset \left(\frac{1269}{71} =\right)\ 17\frac{62}{71}$;

And (because $x = 318 - 17n$) $n \sqsupset \left(\frac{318}{17} =\right)\ 18\frac{12}{17}$;

Th. $n = 18$; $x = 12$; and $y = 9$.

QUEST.

QUEST. CCXXIX. What are the values of x and y in the equations, $xy+666=x^3$; And $xx+yy-95=5y$?

Now (from 1st) $xy=x^3-666$; And $y=\dfrac{x^3-666}{x}$;

Th. - - $yy=\left(\dfrac{x^3-666}{x}\right)^2$; And $5y=\dfrac{5x^3-3330}{x}$;

Whence (by 2d) $\left(\dfrac{x^3-666}{x}\right)^2+xx-95=\dfrac{5x^3-3330}{x}$;

Or $x^6-4x^4-1332x^3-95x^2+3330x+443556=0$.

And by the method of divisors, there will be found the divisors 10, 9, 8, differing by unity;

And as $x-9$ will divide the equ. Th. $x=9$; And $y=7$.

QUEST. CCXXX. A mason had two cubical pieces of marble, containing between them 1072 solid inches, and being severally placed in his yard, they stand upon 130 square inches of the ground: What are the sides of those cubes?

Suppose $x=$ side of ⊏; and $y=$ side of ⊐, cube;

Then $x^3+y^3=1072$; Or $x^3=1072-y^3$,

And $xx+yy=130$; And $x^2=130-yy$;

But - $x^6=\left(1072-y^3\right)^2=\left(130-yy\right)^3$:

Hence $y^6-195y^4-1072y^3+25350y^2-523908=0$.

And by the method of divisors, there will be found the divisors 9, 8, 7, 6, 5, differing by unity.

But $y-7$ will divide the eqat. Th. $y=7$; And $x=9$.

QUEST.

QUEST. CCXXXI. What are the values of x and y in the eq. $5x+8y=198$?

Let $x+y=s$. Then $\begin{cases} 5x+5y=5s; \\ 8x+8y=8s; \end{cases}$

The diff. between these eq. and the given one, are $\begin{cases} 3y=198-5s; \\ 3x=8s-198; \end{cases}$

Hence, when $\begin{cases} y \sqsubset 0; \text{ Then } s \sqsupset \left(\frac{198}{5}=\right) 39\frac{3}{5}; \\ x \sqsubset 0; \text{ Then } s \sqsubset \left(\frac{198}{8}=\right) 24\frac{6}{8}; \end{cases}$

Also, when x and y are integers, Then $\frac{1}{3}s$ must be int.
Th. between $24\frac{6}{8}$ and $30\frac{3}{5}$; $s=27, 30, 33, 36, 39$.

Now when $s=27$, $y=(\overline{198-135}\times\frac{1}{3}=)$ 21; and $x=6$.
If $s=30, 33, 36, 39$, Then $y=16, 11, 6, 1$.
And $x=14, 22, 30, 38$.
Hence in the eq. of this form, when x increases by the coeffi. of y, y decreases by the coeffi. of x.

QUEST. CCXXXII. What are the values of x and y in the equat. $5x+8y=104$?

Suppose the values of x and y to be equal;

Then $\overline{5+8}\times x=104$;
Th. $x\ (=y) = (\frac{104}{13}=)$ 8:
Now by quest. last, $x=8, 16$: and $y=8, 3$.

QUEST. CCXXXIII. It is required to mix brandy at $15s.\ 10d.$ per gallon, with rum at $21s.\ 7d.$ per gallon, and arrack at $22s.\ 10d.$ per gallon, to make 429 gallons worth $16s.\ 8d.$ per gallon?

Suppose x gallons of brandy, y of rum, and z of arrack;

$$\left.\begin{array}{ll}\text{Then} & x+y+z=429 \\ \text{And} & 190x+259y+274z=(429\times 200=)\ 85800\end{array}\right\}\text{by qu.}$$

$$\begin{array}{ll}\text{But} & 190x+190y+190z=(429\times 190=)\ 81510: \\ \text{(By substract.)} & 69y+84z=4290, \\ \text{Or} & 23y+28z=1430;\end{array}$$

$$\text{Substitute}\quad y+z=n,$$

$$\text{Then}\quad \left\{\begin{array}{l}23y+23z=23n \\ 28y+28z=28n\end{array}\right\}\text{by multipl.}$$

$$\text{And by subtr.}\quad \left\{\begin{array}{l}5y=28n-1430 \\ 5z=1430-23n\end{array}\right.$$

$$\text{Th.}\quad \left\{\begin{array}{l}n\sqsubset\frac{1430}{28}=58\frac{2}{28} \\ n\sqsupset\frac{1430}{23}=62\frac{4}{23}\end{array}\right.$$

And when y and z are integers $\frac{n}{5}$ must be an integer,

$$\begin{array}{ll}\text{Whence} & n=55,\ 60, \\ & y=\left(\frac{28n}{5}-286=\right)\ 22,\ 50 \\ & z=\left(286-\frac{23n}{5}=\right)\ 33,\ 10 \\ & x=(429-y-z=)\ 374,\ 369.\end{array}$$

QUEST.

QUEST. CCXXXIV. A clock has 2 indices A and B; A goes round the circumference once in 12 hours, and B goes round once in 1 hour: How often, and at what times, are they together in every 12 hours?

Suppose $x=$ the time when the two indices will be together;

H. Cir. H. Cir.

Then $12:1::x:\frac{1}{12}x=$ the space A moves through,
And $1:1::x:x=$ the space B moves through,
But in x time, B has moved 1 circumference more than A;

Th. $x-\frac{1}{12}x=1$. Hence $x=1^{h.}\ 5'\ 27''$, &c.

Therefore the two indices will be together 11 times in 12 hours; and at the distance of $1\frac{1}{11}$ hours.

QUEST. CCXXXV. A clock has three indices A, B, and C, A goes round the circumference once in 12 hours, B goes round the circumference once in 1 hour, and C goes round the circumference once in 1 minute: How often, and at what times are they together in every 12 hours?

Suppose x the time when A and B are together,
And y - - - - B and C are together;

Then $x=\frac{12}{11}, \frac{2\times 12}{11}, \frac{3\times 12}{11}$, &c. to $\frac{12n}{11}$, by quest. last.

But $1:1::y:y$ = space B moves through,
And $\frac{1}{60}:1::y:60y=$ space C movs through;
And because C has moved 1 circumference more than B;

Th. $60y-y=1$. Th. $y=(\frac{1}{59}\text{h.}=)\ 1^{m.}\ 1''\ 1'''$, &c.

Or - - $y=\frac{1}{59}, \frac{2}{59}, \frac{3}{59}, \frac{m}{59}$, &c.

Now find the val. of m and n in $\frac{m}{59}=\frac{12n}{11}$. That $x=y$.

Th. $11m=708n$; Hence $n=11, 22$, &c.
And $m=708, 1416$, &c.

But $\frac{12\times 11}{11}=12$; And $\frac{708}{59}=12$.

Therefore the 3 hands are together only at 12 o'clock.

QUEST. CCXXXVI. Required the least number, that being divided by 28, will leave n; and by 19, will leave m, for remainders?

Let x= number sought, and y, z, any whole number,

$$\left.\begin{array}{ll}\text{Then} & \frac{x}{28}=y+\frac{n}{28}\text{; Or } x=28y+n\\ \text{And} & \frac{x}{19}=z+\frac{m}{19}\text{; Or } x=19z+m\end{array}\right\}\text{by question;}$$

Assume $\left\{\begin{array}{l}A=19a=28b+1\\B=28c=19d+1\end{array}\right\}$ where a, b, c, and d, are unknown,

Or - $\left\{\begin{array}{l}19a-28b=1,\\28c-19d=1:\end{array}\right.$

Now 1st. let $b'-1=b$;

Then $19a-28b'+28=1$; Or $19a-28b'=-27$,

Where if - - - $a=b'$; $a=(b'=)\ \frac{-27}{-9}=3$;

Th. - $(3-1=)\ 2=b$; And $A=(19\times 3=)\ 57$:

Secondly, let $d'+8=d$;

Then $28c-19d'-152=1$; Or $28c-19d'=153$,

Where if - - - $c=d'$; $c=(d'=)\ \frac{153}{9}=17$;

Th. - $(17+8=)\ 25=d$; And $B=(17\times 28=)\ 476$:

But because $\left\{\begin{array}{l}A=21b+1\\B=19d+1\end{array}\right\}$; Th. $\left\{\begin{array}{l}nA=28nb+n,\\mB=19ma+m;\end{array}\right.$

From which equations, compared with the 2 first, it will appear,

That $\left\{\begin{array}{l}nA\\mB\end{array}\right\}$ will divide by $\left\{\begin{array}{l}28\\19\end{array}\right\}$ and leave $\left\{\begin{array}{l}n\\m\end{array}\right\}$ for rem.

And because $\left\{\begin{array}{l}A=19a,\\B=28c;\end{array}\right.$

Th.

Th. $\left\{\begin{matrix} nA \\ mB \end{matrix}\right\}$ will divide by $\left\{\begin{matrix} 19 \\ 28 \end{matrix}\right\}$ and leave no remainder:

Th. $nA+mB$ being divided by 28, will leave n; and by 19, will leave m for remainders:

But the numbers, which being divided by 28, and 19, will leave m and n for remainders, differ by $(28\times 19 =)$ 532.

Th. $\left(\dfrac{nA+mB}{532}=\right)\dfrac{57n+476m}{532}$ will leave x for a remainder.

EXAMPLE.

The cycle of the sun 17; and the cycle of the moon 13, being given; to find the year of the Dionysian period?

Here $n=17$, $m=13$,
$A=57$, $B=476$:

Now $57\times 17=969$, $=nA$,
And $476\times 13=8188$; $=mB$,

$$28\times 19=532)7152(13$$
$$\begin{array}{r} 532 \\ \hline 1837 \\ 1596 \\ \hline 241 \end{array}$$

ANSWER. The 241st year thereof.

For $\dfrac{241}{28}=8\dfrac{17}{28}$; And $\dfrac{241}{19}=12\dfrac{13}{19}$.

QUEST. CCXXXVII. Required the least number (x) which being severally divided by 28, 19, and 15, will leave the remainders, n, m, and p?

$$\text{Assume}\left|\begin{array}{l} A=(19\times 15a=)\ 285a=28b+1, \\ B=(28\times 15c=)\ 420c=19d+1, \\ C=(28\times 19e=)\ 532e=15f+1; \end{array}\right| \text{Or} \left|\begin{array}{l} 285a-28b=1, \\ 420c-19d=1, \\ 532e-15f=1. \end{array}\right.$$

First; Let $9a+3+b'=b$;
Then $33a-28b'=85$;
Th. (if $a=b'$) $a=\frac{85}{5}=17$; And $A=285\times 17=4854$.

Secondly; Let $21c+1+d'=d$;
Then $21c-19d'=20$;
Th. (if $c=d'$) $c=\frac{20}{2}=10$; And $B=420\times 10=4200$.

Thirdly; Let $34e+6+f'=f$;
Then $22e-15f'=91$;
Th. (if $e=f'$) $e=\frac{91}{7}=13$; And $C=(532\times 13=)\ 6916$.

$$\text{Now because}\left\{\begin{array}{l} A=28b+1 \\ B=19d+1 \\ C=15f+1 \end{array}\right\}\text{Theref.}\left\{\begin{array}{l} nA=28nb+n, \\ mB=19md+m, \\ pC=15pf+p; \end{array}\right.$$

$$\text{That is}\left|\begin{array}{l} nA \\ mB \\ pC \end{array}\right|\text{will divide by}\left|\begin{array}{l} 28 \\ 19 \\ 15 \end{array}\right|\text{and leave the rem.}\left|\begin{array}{l} n, \\ m, \\ p. \end{array}\right.$$

$$\text{And because}\left|\begin{array}{l} A=19\times 15a \\ B=28\times 15b \\ C=28\times 19c \end{array}\right|\text{Ther.}\left|\begin{array}{l} nA \\ mB \\ pC \end{array}\right|\text{will divide by}\left|\begin{array}{l} 19 \text{ and } 15 \\ 28 \text{ and } 15 \\ 28 \text{ and } 19 \end{array}\right|\text{without rem.}$$

Th.

Th. $nA+mB+pC$ will divide by 28, 19, and 15, and leave the remainders n, m, and p;

Th. $\left(\frac{nA+mB+pC}{28\times19\times15}=\right)\frac{nA+mB+pC}{7980}$ will leave x for for a remainder;

That is $\frac{4845n+4200m+6916p}{7980}$ will leave x for a remainder.

EXAMPLE.

In what year of the Julian period, was the cycle of the sun 12, the cycle of the moon 13, and the Roman indiction 14?

Now $\left\{\begin{array}{l}4845\times12=58140\\4200\times13=54600\\6916\times14=96824\end{array}\right\}$ their sum is 209564;

Then $\frac{209564}{7980}=26$; And remainder 2084 is the answ.

CoROL. If it be required to find the least number (x) that being divided by M, will leave m; by N, n; by P, p; by Q, q; by R, r; &c. as remainders (M, N, P, Q, R, &c. being prime to each other)?

Assume $A=\overline{NPQR, \&c.}\times a=Mb+1$,

$B=\overline{MPQR, \&c.}\times c=Nd+1$,

$C=\overline{MNQR, \&c.}\times e=Pf+1$,

$D=\overline{MNPR, \&c.}\times g=Qh+1$;

And having feverally found the values of A, B, C, D, &c. as above;

Then $\frac{Am+Bn+Cp+Dq+Er, \&c.}{M\times N\times P\times Q\times R, \&c.}$ will leave x for a remainder.

EXAMPLE.

The leaft number that can be divided by 2, 3, 5, 7, and 11, feverally; and leave 1, 2, 3, 4, and 5, for remainders, is required?

Here $M=2$; $N=3$; $P=5$; $Q=7$; $R=11$.
And $m=1$; $n=2$; $p=3$; $q=4$; $r=5$.

$$\text{Here}\begin{cases} A=(3.5.7.11a=)\ 1155a=2b+1, \\ B=(2.5.7.11c=)\ 770c=3d+1, \\ C=(2.3.7.11e=)\ 462e=5f+1, \\ D=(2.3.5.11g=)\ 330g=7h+1, \\ E=(2.3.5.7i=)\ 210i=11k+1; \end{cases}$$

$$\text{Or}\begin{cases} 1155a-2b=1, \\ 770c-3d=1, \\ 462e-5f=1, \\ 330g-7h=1, \\ 210i-11k=1: \end{cases}$$

First,

First; Let $578a-b'=b$;
Then $1155a-1156a+2b'=1$; Or $2b'-a=1$,
Th. $(b'=)\ a=\frac{1}{1}=1$; And $A=(1155\times1=)\ 1155$:

Secondly; Let $255c+1+d'=d$;
Then $770c-765c-3-3d'=1$; Or $5c-3d'=4$;
Th. $c=(d'=)\ \frac{4}{2}=2$; And $B=(770\times2=)\ 1540$:

Thirdly; Let $91e+1+f'=f$;
Then $462e-455e-5-5f'=1$; Or $7e-5f'=6$;
Th. $e=(f'=)\ \frac{6}{2}=3$; And $C=(462\times3=)\ 1386$:

Fourthly; Let $46g+h'=h$;
Then $330g-322g-7h'=1$; Or $8g-7h'=1$;
Th. $g=(h'=)\ \frac{1}{1}=1$; And $D=(330\times1=)\ 330$:

Fifthly; Let $18i+k'=k$;
Then $210i-198i-11k'=1$; Or $12i-11k'=1$;
Th. $i=(k'=)\ \frac{1}{1}=1$; And $E=(210\times1=)\ 210$:

$$\text{Then }\frac{1155\times1+1540\times2+1386\times3+330\times4+210\times5}{2\times3\times5\times7\times11}$$

$$=\frac{1155+3080+4158+1320+1050}{2310}$$

$$=\frac{10763}{2310}=4+\frac{1523}{2310};$$

Then 1523 is the number required.

QUEST. CCXXXVIII. Forty-one persons, men, women, and children, spent 40*s.* whereof each man paid 4*s.* each woman 3*s.* and each child 4*d.* How many of each were there?

Suppose there were x men; y women; and z children;

$$\left.\begin{array}{ll}\text{Then} & x+y+z=41, \\ \text{And} & 4x+3y+\tfrac{1}{3}z=40,\end{array}\right\} \textit{per} \text{ quest.}$$

Now $12x+9y+z=120$, by second,

$$\left.\begin{array}{ll}\text{Or - - -} & x+y=41-z, \\ \text{And -} & 12x+9y=120-z,\end{array}\right\} \text{by transposition;}$$

$$\text{Also} \left\{\begin{array}{l}12x+12y=492-12z \\ 9x+9y=369-9z\end{array}\right\} \text{by multipl.}$$

$$\text{Again -} \left\{\begin{array}{l}3x=8z-249 \\ 3y=372-11z\end{array}\right\} \text{by subtraction;}$$

$$\text{Th. - -} \left\{\begin{array}{l}z>\left(\dfrac{249}{8}=\right) 31\tfrac{1}{8} \\ z<\left(\dfrac{372}{11}=\right) 33\tfrac{9}{11}\end{array}\right.$$

Also, when x and y are whole numbers, z must be a multiple of 3:
But the only multiple of 3, between $31\frac{1}{8}$, and $33\frac{9}{11}$, is 33;

Th. $z=33$; $x=\left(\dfrac{8\times33-249}{3}=\right) 5$; And $y=3$.

QUEST.

QUEST. CCXXXIX. If the product of the solidities of two cubes, the sum of whose sides is 10, be multiplied by the solidity of the greater, it will produce 3176523: What are their sides?

If x be the side of the cube,

And y the side of the lesser;

Then $(x^3x^3y^3=)\ x^6y^3=3176523$,

And - - - - - $x^6=\frac{3176523}{y^3}$:

But $(x+y=10;$ Th.$)\ x=10-y$,

And - - - - - $x^6=\overline{10-y}^6$;

Th. - - - $\overline{10-y}^6=\frac{3176523}{y^3}$,

Or - - - $\overline{10-y}^6\times y^3=3176523$;

Th. - - - $\overline{10-y}\times y^{\frac{1}{2}}=\sqrt[6]{3176523}$

Or - - - - - - - $=(\sqrt[2]{49\times3}=)\ 7\sqrt{3}$;

Th. $y^{\frac{3}{2}}-10y^{\frac{1}{2}}+7\sqrt{3}=0$.

QUEST. CCXL. How many ways may 4 forts of wine, whose prices are 16, 10, 8, and 6*d.* per quart, be mixed; so as to make 100 quarts in all, and be worth 12*d. per* quart?

Let the required quantities of the four several sorts of wine be represented by x, y, u, and z;

Then $\left\{\begin{array}{l} x+y+u+z=100 \\ 16x+10y+8u+6z=(100\times12=)\,1200 \end{array}\right\}$ by q.

Or - - - $\left\{\begin{array}{l} y+u+z=100-x \\ 10y+8u+6z=1200-16x \end{array}\right\}$ by tranf.

And - - $\left\{\begin{array}{l} 10y+10u+10z=1000-10x \\ 6y+6u+6z=600-6x \end{array}\right\}$ by mult.

Alfo by fub. - $\left\{\begin{array}{l} 2u+4z=6x-200, \\ 4y+2u=600-10x; \end{array}\right.$

Th. - - - - - - - $\left\{\begin{array}{l} x \sqsubset \left(\dfrac{200}{6}=\right) 33\tfrac{1}{3}, \\ x \sqsupset \left(\dfrac{600}{10}=\right) 60: \end{array}\right.$

But by tranfp. - $\left\{\begin{array}{l} u+z=100-x-y, \\ 8u+6z=1200-16x-10y, \end{array}\right.$

And by multip. - $\left\{\begin{array}{l} 8u+8z=800-8x-8y, \\ 6u+6z=600-6x-6y, \end{array}\right.$

Whence - - - - - $\left\{\begin{array}{l} z=4x+y-200, \\ u=300-5x-2y; \end{array}\right.$

Th. - - - - - - - $\left\{\begin{array}{l} y \sqsubset 200-4x, \\ y \sqsupset \dfrac{300-5x}{2}: \end{array}\right.$

From

From which limits it will appear, that

If $x=59$; $y \sqsupset 2\frac{1}{2}$; $y \sqsubset -36$; Th. y has 2 values,
If $x=58$; $y \sqsupset 5$; $y \sqsubset -32$; Th. y has 4 values,
If $x=57$; $y \sqsupset 7\frac{1}{2}$; $y \sqsubset -28$; Th. y has 7 values,
&c. &c. &c. &c.

If $x=51$; $y \sqsupset 22\frac{1}{2}$; $y \sqsubset -4$; Th. y has 22 values,

---

If $x=50$; $y \sqsupset 25$; $y \sqsubset 0$; Th. y has 24 values,
If $x=49$; $y \sqsupset 27\frac{1}{2}$; $y \sqsubset 4$; Th. y has 23 values,
If $x=48$; $y \sqsupset 30$; $y \sqsubset 8$; Th. y has 21 values,
&c. &c. &c. &c.

If $x=34$; $y \sqsupset 65$; $y \sqsubset 64$; Th. y has 0 value.

Hence the number of the values of y may be obtained by summing two arithmetical progressions;

In the first; The least term is 2, greatest term 22, and number of terms 9:

Th. the sum is ($\overline{22+2}\times\frac{9}{2}=$) 108.

In the second; The least term is 0, greatest term 24; and number of terms 17;

Th. the sum is ($\overline{24+0}\times\frac{17}{2}=$) 204:

Th. $108+204=312$ is the required number of answers.

See QUEST. VI. PART II.

QUEST. CCXLI. To find a number, the products of which by two given numbers 32 and 8, may be square numbers?

If x be the number required;
Then $32x$ and $8x$ are square numbers,
And their roots are $4\sqrt{2x}$ and $2\sqrt{2x}$;
Th. $x=\frac{1}{2}$ a square number:
But 4. 16. 36. 64. 100. 144, &c. are square numbers;
Th. 2. 8. 18. 32. 50. 72, &c. $=x$.

QUEST. CCXLII. Required two such square numbers, that their difference may be a square number?

Let $x=$ root of $\sqsupset$ squ. $x+d=$ root of $\sqsubset$ squ.
And $z=$ root of their difference;

Then $(\overline{x+d}^2-xx=)$ $2dx+dd=zz$, by quest.

That is - - - $\overline{2x+d}\times d=zz\times 1$;

Then by qu. 220 $\left.\begin{array}{l} 2x+d=1, 2, 3, 4, 5, 6, \&c. \text{ to } n, \\ zz=d, 2d, 3d, 4d, 5d, 6d, \&c. \text{ to } nd; \end{array}\right.$

Now $(zz=)$ nd, is a rational square, when n and d are square numbers;

And because $2x+d=n$; Th. $x=\frac{1}{2}\times\overline{n-d}$;

And - - $x+d=\frac{1}{2}\times\overline{n+d}$:

Now let - $m^2=n$; And $r^2=d$;

Then $\frac{1}{4}\times\overline{m^2+r^2}\big|^2-\frac{1}{4}\times\overline{m^2-r^2}\big|^2=r^2m^2$,

Th. - - - $\overline{m^2+r^2}^2-\overline{m^2-r^2}^2=4r^2m^2$;

Where m and r, may be any numbers whatsoever; only $m\sqsubset r$.

COROL. I. m^2-r^2, and $2rm$, are the roots of two square numbers whose sum will be a square number, *viz.* $\overline{m^2+r^2}^2$.

COROL. II. The squares of all numbers that are in proportion, as m^2-r^2, $2rm$, and m^2+r^2, will have the property required in the question.

EX-

EXAMPLES.

If		Then			For
m	r	$2rm$	m^2-r^2	m^2+r^2	$\overline{2rm}^2+\overline{m^2-r^2}^2=\overline{m^2+r^2}^2$
2	1	4	3	5	16+ 9= 25
3	1	6	8	10	36+ 64= 100
4	1	8	15	17	64+ 225= 289
5	1	10	24	26	100+ 576= 676
6	1	12	35	37	144+1225= 1369
7	1	14	48	50	196+2304= 2500
8	1	16	63	65	256+3969= 4225
9	1	18	80	82	324+6400= 6724
3	2	12	5	13	144+ 25= 169
4	2	16	12	20	256+ 144= 400
5	2	20	21	29	400+ 441= 841
6	2	24	32	40	576+1024= 1600
7	2	28	45	53	784+2025= 2809
8	2	32	60	68	1024+3600= 4624
9	2	36	77	85	1296+5929= 7225
4	3	24	7	25	576+ 49= 625
5	3	30	16	34	900+ 256= 1156
6	3	36	27	45	1296+ 729= 2025
7	3	42	40	58	1764+1600= 3364
8	3	48	55	73	2304+3025= 5329
9	3	54	72	90	2916+5184= 8100
5	4	40	9	41	1600+ 81= 1681
6	4	48	20	52	2304+ 400= 2704
7	4	56	33	65	3136+1089= 4225
8	4	64	48	80	4096+2304= 6400
9	4	72	65	97	5184+4225= 9409
6	5	60	11	61	3600+ 121= 3721
7	5	70	24	74	4900+ 576= 5476
8	5	80	39	89	6400+1521= 7921
9	5	90	56	106	8100+3136=11236
7	6	84	13	85	7056+ 169= 7225
8	6	96	28	100	9216+ 784=10000
9	6	108	45	117	11664+2025=13689
8	7	112	15	113	12544+ 225=12769
9	7	126	32	130	15876+1024=16900
9	8	144	17	145	20736+ 289=21025

QUEST. CCXLIII. Two numbers in proportion as n to p are required, so that the sum of their squares, may be a square number?

By COROL. I. QUEST. 242. The sum of the squares of m^2-r^2, and $2rm$; is a square number:

But by quest. $n:p::m^2-r^2:2rm$;

Th. - - - - - $2nrm=pm^2-pr^2$,

Or - - $pr^2+2nrm=pm^2$;

Th. - - $r^2+\frac{2nm}{p}r=m^2$:

But $\overline{r^2+\frac{2nm}{p}r+\frac{nm}{p}}\Big|^2=\frac{\overline{pp+nn}\times mm}{pp}$;

Th. - - - - $r+\frac{nm}{p}=\sqrt{\overline{pp+nn}}\times\frac{m}{p}$:

And - - - - - - $r=\overline{\sqrt{\overline{pp+nn}}-n}\times\frac{m}{p}$:

Therefore for m, some multiple of p must be assumed.

EXAMPLE.

To find two numbers in proportion as 8 to 15, the sum of whose squares may be a square number?

Let $m=(2\times 15=)\ 30$;

Then $r=(\overline{\sqrt{225+64}-8}\times\frac{30}{15}=)\ 18$.

Then $(\overline{30}^2-\overline{18}^2=)\ 576$, and $(2\times 30\times 18=)\ 1080$, are the numbers required.

For $(\overline{576}^2+\overline{1080}^2=)\ 331776+1166400=1498176$

$=\overline{1224}^2$.

QUEST.

QUEST. CCXLIV. It is required to divide a given square number into two square numbers?

Let zz be the square to be divided:

Now $\overline{m^2+r^2\times z}|^2=\overline{m^2-r^2\times z}|^2+\overline{2rmz}|^2$

COROL. II. QUEST. 242.

Or $\overline{m^2+r^2}|^2\times zz=\overline{m^2-r^2\times z}|^2+\overline{2rmz}|^2$

Th. - - - $zz=\overline{\frac{m^2-r^2}{m^2+r^2}z}|^2+\overline{\frac{2rm}{m^2+r^2}z}|^2$

Where m and r may be taken at pleasure; so $m\sqsubset r$.

EXAMPLE.

Divide 25, into 2 square numbers?

Assume $m=2$; and $r=1$;

Then $25=(\overline{\frac{4-1}{4+1}\times 5}|^2+\overline{\frac{2\times 2}{4+1}\times 5}|^2=)\ 9+16$:

Assume $m=3$; and $r=1$;

Then $25=\overline{\frac{9-1}{9+1}\times 5}|^2+\overline{\frac{2\times 3}{9+1}\times 5}|^2=)\ 16+9$:

Assume $m=3$; and $r=2$;

Then $25=\overline{\frac{9-4}{9+4}\times 5}|^2+\overline{\frac{2\times 2\times 3}{9+4}\times 5}|^2$

$=\overline{\tfrac{25}{13}}|^2+\overline{\tfrac{60}{13}}|^2$.

QUEST.

QUEST. CCXLIII. Two numbers in proportion as n to p are required, so that the sum of their squares, may be a square number?

By COROL. I. QUEST. 242. The sum of the squares of m^2-r^2, and $2rm$; is a square number:

But by quest. $n:p::m^2-r^2:2rm$;

Th. - - - - $2nrm=pm^2-pr^2$,

Or - - $pr^2+2nrm=pm^2$;

Th. - - $r^2+\frac{2nm}{p}r=m^2$:

But $\overline{r^2+\frac{2nm}{p}r+\frac{nm}{p}}\Big|^2=\frac{\overline{pp+nn}\times mm}{pp}$;

Th. - - - - $r+\frac{nm}{p}=\sqrt{pp+nn}\times\frac{m}{p}$:

And - - - - - - $r=\overline{\sqrt{pp+nn}-n}\times\frac{m}{p}$:

Therefore for m, some multiple of p must be assumed.

EXAMPLE.

To find two numbers in proportion as 8 to 15, the sum of whose squares may be a square number?

Let $m=(2\times15=)\ 30$;

Then $r=(\overline{\sqrt{225+64}-8}\times\frac{30}{15}=)\ 18$.

Then $(\overline{30}^2-\overline{18}^2=)\ 576$, and $(2\times30\times18=)\ 1080$, are the numbers required.

For $(\overline{576}^2+\overline{1080}^2=)\ 331776+1166400=1498176$

$=\overline{1224}^2$.

QUEST.

QUEST. CCXLIV. It is required to divide a given square number into two square numbers?

Let zz be the square to be divided:

Now $\overline{m^2+r^2\times z}\,\big|^2=\overline{m^2-r^2\times z}\,\big|^2+\overline{2rmz}\,\big|^2$

COROL. II. QUEST. 242.

Or $\overline{m^2+r^2}\,\big|^2\times zz=\overline{m^2-r^2\times z}\,\big|^2+\overline{2rmz}\,\big|^2$

Th. - - - $zz=\overline{\frac{m^2-r^2}{m^2+r^2}z}\,\Big|^2+\overline{\frac{2rm}{m^2+r^2}z}\,\Big|^2$

Where m and r may be taken at pleasure; so $m\sqsubset r$.

EXAMPLE.

Divide 25, into 2 square numbers?

Assume $m=2$; and $r=1$;

Then $25=\left(\overline{\frac{4-1}{4+1}\times 5}\,\Big|^2+\overline{\frac{2\times 2}{4+1}\times 5}\,\Big|^2=\right) 9+16$:

Assume $m=3$; and $r=1$;

Then $25=\overline{\frac{9-1}{9+1}\times 5}\,\Big|^2+\overline{\frac{2\times 3}{9+1}\times 5}\,\Big|^2=)\ 16+9$:

Assume $m=3$; and $r=2$;

Then $25=\overline{\frac{9-4}{9+4}\times 5}\,\Big|^2+\overline{\frac{2\times 2\times 3}{9+4}\times 5}\,\Big|^2$

$=\overline{\frac{25}{13}}\,\big|^2+\overline{\frac{60}{13}}\,\big|^2$.

QUEST.

QUEST. CCXLV. It is required to divide a given number, which is the sum of two square numbers, into two other square numbers?

If the roots of the two squares that compose the given number be a, and d; also $a \sqsupset d$;

Then $aa+dd=$ the given number:

Let $\left\{\begin{array}{l} mx+a= \\ nx-d= \end{array}\right\}$ the root of the $\left\{\begin{array}{c} \text{greater} \\ \text{lesser} \end{array}\right\}$ square required;

Then $aa+dd= \left\{\begin{array}{l} mmxx+2amx+aa+ \\ nnxx-2dnx+dd, \end{array}\right.$

Or - $2dnx=mmxx+2amx+nnxx$,

Or - $2dn=mmx+2am+nnx$;

Th. $\dfrac{\overline{dn-am}\times 2=}{mm+nn}=x$;

Where $n \sqsubset m$; but n must not be in proportion to m, as $d+a$ to $d-a$; because then $dn=am$.

EXAMPLE.

Let it be required to divide the number 1450, which is the sum of $\overline{35}^2$ and $\overline{15}^2$, into two other square numbers?

Assume $n=2$, and $m=1$; Then

$$\left(\frac{\overline{35\times 2-15\times 1}\times 2}{1\times 1+2\times 2}=\frac{\overline{70-15}\times 2}{1+4}=\frac{55\times 2}{5}=\right) 22=x,$$

And $\left\{\begin{array}{l} (1\times 22+15=22+15=)\ 37 \\ (2\times 22-35=44-35=)\ \ 9 \end{array}\right\}$ are the roots of the required squares.

QUEST. CCXLVI. It is required to find two such square numbers, that if any given number be added to their product, the sum may be a square number?

Let x and m, be the roots of the required squares;

And n the given number;

Then $mmxx+n$, must be a square number:

Let $mx+d$, be the root thereof;

Then $mmxx+2mdx+dd=mmxx+n$,

Or - - - $2mdx+dd=n$,

Or - - - - $2mdx=n-dd$;

Th. - - - - - - $x=\frac{n-dd}{2md}$.

Where dd may be any square number less than n; and m any number whatsoever.

EXAMPLE.

If the given number n be 28;

Then $dd=25,\ 16,\ 9,\ 4,\ 1$;

$n-dd=\ 3,\ 12,\ 19,\ 24,\ 27$;

If $m=1$; $x=\frac{3}{10},\ \frac{12}{8},\ \frac{19}{6},\ \frac{24}{4},\ \frac{27}{2}$;

If $m=2$; $x=\frac{3}{10},\ \frac{12}{16},\ \frac{19}{12},\ \frac{24}{8},\ \frac{27}{4}$;

If $m=3$; $x=\frac{3}{30},\ \frac{12}{24},\ \frac{19}{18},\ \frac{24}{12},\ \frac{27}{6}$;

&c. &c. &c.

QUEST.

QUEST. CCXLIX. What are the values of x, y, u, and z, in the equation $nx+nny=nu+nnz$?

By division $x+ny=u+nz$,

And by transp. $x-u=\overline{z-y}\times n$,

Then (q. 220.) $n:1::x-u:z-y$.

Whence (the value of n being fixed) x, y, u and z, have innumerable values,

But if $u=x$; Then $y=z$.

QUEST. CCL. What are the values of x, y, e, u, z, and i, in the equation $nx+n^2y+n^3e=nu+n^2z+n^3i$?

By division $x+ny+n^2e=u+nz+n^2i$;

Where if $x=u$; $ny+n^2e=nz+n^2i$;

In which (by qu. last) If $y=z$,

Then - - - - - - $e=i$.

COROL. Hence if there be never so many terms in an equation of the above form (the number of terms on each side being equal) *viz.*

$an+bn^2+cn^3+dn^4$, &c. $=\alpha n+\beta n^2+\gamma n^3+\delta n^4$, &c.

One value of each of the unknown coeficients, α, β, γ, δ, &c. will be found by making,

$\alpha=a$,

$\beta=b$,

$\gamma=c$, &c.

THE

THE

MATHEMATICAL REPOSITORY.

PART II.

QUESTION I.

IT is required to find the sum (A) of n terms of the series of numbers 0, 1, 2, 3, 4, &c. to $\overline{n-1}$?

Since this is required by the given number n only; the sum A must be equal to n, or its powers, multiplied by some, yet unknown, coefficients x and y.

And because n is greater than $\overline{n-1}$ the last term, of that part of the series, whose sum is required;

Th. (n taken n times $=$) nn must be greater than A:
Let therefore $xnn-yn=A$;

Now if n, the next greater term of the series be added thereto, the number of terms will be $n+1$; and the sum of them $A+n$:

Then (because x and y (though unknown) are constant quantities)

$$x\times\overline{n+1}^2-y\times\overline{n+1}=A+n,$$

That is $xn^2+2xn+x-yn-y=A+n$;

And - - - - $2xn+x-y=n$ $\left\{\text{by subtracting the value of } A,\right.$

Or - - - - - - $2xn+x=n+y$:

Now (by quest. 249.) if - $x=y$,

Then - - - - - - $2x=1$;

Th. - - - - - - $x\,(=y)=\frac{1}{2}$,

And $\left(\frac{nn}{2}-\frac{n}{2}=\right)\overline{n-1}\times\frac{1}{2}n=A=\frac{1}{2}\overline{n-1}\times n$:

Corol.

COROL. The sum of n terms of $1+2+3+4$, &c. ($=\overline{n+1}$ terms of $0+1+2+3$, &c.) will be found to be $\frac{\overline{n+1}\times n}{2}$, by writing $\overline{n+1}$ for n, in the above expression.

In an Arithmetical progression (i. e. a rank or series of numbers whose differences are equal) if,

$$\left.\begin{array}{l} a \\ z \\ d \\ n \\ s \end{array}\right\} \text{represent} \left\{\begin{array}{l} \text{the least (number or) term,} \\ \text{the greatest term,} \\ \text{the difference of any two adjacent terms,} \\ \text{the number of terms,} \\ \text{the sum of all the terms:} \end{array}\right.$$

Then, QUEST. II. and III. a, d, and n, are given; to find z, and s?

$$\text{Now}\quad \left.\begin{array}{ll} a & = \text{the least term} \\ a+d & = \text{the second} \\ a+2d & = \text{the third} \\ a+3d & = \text{the fourth} \\ a+4d & = \text{the fifth} \\ \text{\&c.} & \quad \text{\&c.} \end{array}\right\} \begin{array}{l} \text{because } d \text{ is the} \\ \text{difference of any} \\ \text{two adjacent terms.} \end{array}$$

Then $a+\overline{n-1}\times d=$ (the nth term $=$) z,

And $\quad s=\left\{\begin{array}{l}\overline{n \text{ terms of } 1+1+1+1 \text{ \&c.}}\times a+ \\ \overline{n \text{ terms of } 0+1+2+3 \text{ \&c.}}\times d:\end{array}\right.$

But $\quad na=n$ terms of $1+1+1+1$, &c.

And $\frac{n\times\overline{n-1}}{2}=n$ terms of $0+1+2+3$, &c. by quest. I.

Th. $\quad s=na+\frac{n\times\overline{n-1}}{2}d. =\overline{a+\tfrac{1}{2}\overline{n-1}\times d}\times n.$

EXAMPLE.

It is required to find the sum of n terms, of the series of odd numbers, 1, 3, 5, 7, &c.

Here $a=1$; and $d=2$;

Th. ($n+\frac{n\times\overline{n-1}}{2}2=n+nn-n=$) nn is the sum required.

COROL.

COROL. Hence every square number (nn) is the sum of n terms of the series of odd numbers, beginning with unity?

QUEST. IV. How many terms of the series of odd numbers, 1. 3. 5, &c. must be added together, to produce the 6th power of 12?

If $6=2m$, $12=r$; and $n=$ the number of terms required;

Then $1+3+5$, &c. to n terms $=\overline{12}|^6=r^{2m}$;
But n terms of $1+3+5$, &c. $=nn$ (by corol. qu. last);
Th. - - - $n^2=r^{2m}$; or $n=r^m$:
In this example - $n=(12^3=)\ 1728$.

QUEST. V. It is required to find 13 terms of the indefinite series, 3. 5. 7. 9, &c. whose sum, may be the 3d power of 13?

Let $x=$ the first of the 13 terms where the series is to begin; $d=2$; $m=3$; $n=13$.

Then $(nx+\frac{nn-n}{2}2=)\ nx+nn-n$ will be the sum of n terms by question 2d:
But $nx+nn-n=(\overline{13}|^3=)\ n^m$ by question;
And - $x+n-1=n^{m-1}$ by division;
Th. - - - $x=n^{m-1}-n+1$.
That is - - $x=(13^2-12=)\ 157$.

QUEST. VI. and VII. In an arithmetical progression are given a, z, and n, to find d, and s?

By quest. 2; $a+\overline{n-1}\times d=z$,

Or - - - - - $\overline{n-1}\times d=z-a$;

Th. - - - $d=\frac{z-a}{n-1}$.

By quest. 3; - $na+\frac{n\times\overline{n-1}}{2}d=s$;

And (from above) $na+\frac{n\times\overline{n-1}}{2}\times\frac{z-a}{n-1}=s$,

That is - - - $na+\frac{n\times\overline{z-a}}{2}=s$,

Or $\left(\frac{2na+nz-na}{2}=\right)\frac{na+nz}{2}=s$;

Th. - - - $\frac{n\times\overline{a+z}}{2}$, $=s$.

QUEST. VIII. and IX In an arithmetical progression, are given a, d, and z; to find n and s?

By quest. 2; $a+\overline{n-1}\times d=z$,

Or - - $nd-d=z-a$;

Th. - - - $n=\left(\frac{z-a+d}{d}=\right)\frac{z-a}{d}+1$;

QUEST.

QUESTION IX.

By quest. 7; $\frac{n\times\overline{a+z}}{2}=s$;

Then - - $n=\frac{2s}{z+a}$;

And by q. 8; $\frac{z-a}{d}+1=\frac{2s}{z+a}$;

Th. $\overline{\frac{z-a}{d}+1}\times\frac{z+a}{2}=s$.

QUEST. X. and XI. In an arithmetical progression are given a, n and s; to find z, and d?

By quest. 7; $\frac{na+nz}{2}=s$,

Or - - - $na+nz=2s$;

Th. - - $z=\frac{2s-na}{n}$;

Or - - - $z=\frac{2s}{n}-a$.

By quest. 3; $na+\frac{n\times\overline{n-1}}{2}d=s$,

Or - - $\frac{n\times\overline{n-1}}{2}d=s-na$;

Th. - - - - $d=\frac{\overline{s-na}\times 2}{n\times\overline{n-1}}$.

QUEST. XII. and XIII. In an arithmetical progression are given a, z, and s; to find n, and d?

By quest. 7; - $\frac{n \times \overline{a+z}}{2} = s$;

Th. - - - $n = \frac{2s}{a+z}$.

By quest. 9; $\overline{\frac{z-a}{d}+1} \times \frac{z+a}{2} = s$,

Or - - $\frac{z-a}{d}+1 = \frac{2s}{z+a}$,

Or - - - $\frac{z-a}{d} = \frac{2s}{z+a} - 1$;

Or - - $\overline{z+a} \times \overline{z-a} = 2ds - d \times \overline{z+a}$;

Th. - - $\frac{\overline{z+a} \times \overline{z-a}}{2s - \overline{z+a}} = d$.

QUEST. XIV. and XV. In an arithmetical progression are given a, d, and s; to find z, and n?

By quest. 9; $\overline{\frac{z-a}{d}+1} \times \frac{z+a}{2} = s$,

Or - - - $\frac{zz-aa}{2d} + \frac{z+a}{2} = s$,

Or - - $zz-aa+zd+ad = 2ds$;

Th. - - - $z^2+zd = 2ds+aa-ad$:

But - - $z^2+zd+\frac{dd}{4} = 2ds+aa-ad+\frac{dd}{4}$,

Or

Or - $4z^2+4zd+dd=8ds+4a^2-4ad+dd$,

Or - - $\overline{2z+d}^2=8ds+\overline{2a+d}^2$;

Th. - - - $2z+d=\sqrt{8ds+\overline{2a+d}^2}$:

And - - - - $z=\frac{\sqrt{8ds+\overline{2a+d}^2}-d}{2}$.

By qu. 3d; $na+\frac{n\times\overline{n-1}}{2}d=s$,

That is - $2na+nnd-nd=2s$,

Or - $dn^2+\overline{2a-d}\times n=2s$;

Then - - $nn+\frac{2a-d}{d}n=\frac{2s}{d}$;

But $nn+\frac{2a-d}{d}n+\overline{\frac{2a-d}{2d}}^2=\left(\frac{2s}{d}+\overline{\frac{2a-d}{2d}}^2=\right)$ $\frac{8ds+\overline{2a-d}^2}{4dd}$,

Th. - - $n+\frac{2a-d}{2d}=\frac{\sqrt{8ds+\overline{2a-d}^2}}{2d}$;

And - - - $n=\frac{\sqrt{8ds+\overline{2a-d}^2}-\overline{2a-d}}{2d}$.

QUEST. XVI. and XVII. In an arithmetical progression are given z, d, and n; to find a, and s?

By question 2d; $a+\overline{n-1}\times d=z$;

Th. - - - $a=z-\overline{n-1}\times d$.

By question 7; $\frac{n\times\overline{a+z}}{2}=s$,

But above - - $a=z-\overline{n-1}\times d$,

Th. - - $a+z=2z-\overline{n-1}\times d$;

Th. - $\frac{2nz - n \times \overline{n-1} \times d}{2} = s,$

That is - $nz - \frac{n \times \overline{n-1}}{2} d = s.$

QUEST. XVIII. and XIX. In an arithmetical progression are given d, n, and s; to find a, and z?

By quest. 3d; $na + \frac{n \times \overline{n-1}}{2} d = s,$

Or - - - $na = s - \frac{n \times \overline{n-1}}{2} d;$

Th. - - - $a = \frac{s}{n} - \frac{n-1}{2} d.$

By quest. 17; $nz - \frac{n \times \overline{n-1}}{2} d = s;$

Th. - - - $z = \frac{s}{n} + \frac{n-1}{2} d.$

QUEST. XX. and XXI. In an arithmetical progression are given z, n, and s; to find a, and d?

By quest. 7; - $\frac{n \times \overline{a+z}}{2} = s,$

Or - - - $a + z = \frac{2s}{n};$

Th. - - - $a = \frac{2s}{n} - z.$

By

By quest. 17; $nz-\frac{n\times\overline{n-1}}{2}d=s$,

Or - - - $nz-s=\frac{n\times\overline{n-1}}{2}d$;

Th. - - . $\frac{\overline{nz-s}\times 2}{n\times\overline{n-1}}=d$.

QUEST. XXII. and XXIII. In an arithmetical progression are given z, d, and s; to find a, and n?

By quest. 9; $\overline{\frac{z-a}{d}+1}\times\frac{z+a}{d}=s$,

Or - $zz-aa+zd+ad=2ds$;

Th. - - $aa-da=zz+zd-2ds$:

But - - $aa-da+\frac{dd}{4}=zz+zd+\frac{dd}{4}-2ds$,

Or - - - $\overline{2a-d}^2=\overline{2z+d}^2-8ds$;

Th. - - - $2a-d=\sqrt{\overline{2z+d}^2-8ds}$,

And - - - - $a=\frac{\sqrt{\overline{2z+d}^2-8ds}+d}{2}$.

By quest. 17; $2nz-nnd+nd=2s$,

Or - - $dnn-\overline{2z+d}\times n=-2s$,

Th. - - $nn-\frac{2z+d}{d}n=-\frac{2s}{d}$:

But $nn-\frac{2z+d}{d}n+\left.\frac{2z+d}{2d}\right|^2=\left.\frac{2z+d}{2d}\right|^2-\frac{2s}{d}$,

That is - $\left.n-\frac{2z+d}{2d}\right|^2=\frac{2z+d-8sd}{4dd}$;

Th. - - $n-\frac{2z+d}{2d}=\frac{\sqrt{\overline{2z+d}|^2-8ds}}{2d}$,

Th. - - $n=\frac{2z+d\pm\sqrt{\overline{2z+d}|^2-8ds}}{2d}$.

The foregoing XX Cases of Arithmetick Progression are here placed together for Ready Use.

Quest.	Given.	Req.	Solution.	
II. III.	$a,\ d,\ n,$	$z,\ s,$	$z=a+\overline{n-1}\times d.$	$s=\overline{a+\frac{1}{2}\overline{n-1}\times d}\times n.$
VI. VII.	$a,\ z,\ n,$	$d,\ s,$	$d=\frac{z-a}{n-1}.$	$s=\overline{a+z}\times\frac{1}{2}n.$
VIII. IX.	$a,\ d,\ z,$	$n,\ s,$	$n=\frac{z-a}{d}+1.$	$s=\overline{\frac{z-a}{d}+1}\times\frac{z+a}{2}.$
X. XI.	$a,\ n,\ s,$	$z,\ d,$	$z=\frac{2s}{n}-a.$	$d=\frac{\overline{s-na}\times 2}{n\times\overline{n-1}}.$
XII. XIII.	$a,\ z,\ s,$	$n,\ d,$	$n=\frac{2s}{a+z}.$	$d=\frac{\overline{z+a}\times\overline{z-a}}{2s-\overline{z+a}}.$

XIV. XV.	$a,\ d,\ s,$	$z,\ n,$	$z=\frac{1}{2}\sqrt{8ds+\overline{2a+d}\vert^2}-\frac{1}{2}d.$	$n=\frac{\sqrt{8ds+\overline{2a+d}\vert^2}-2a-d}{2d}.$
XVI. XVII.	$z,\ d,\ n,$	$a,\ s,$	$a=z-\overline{n-1}\times d.$	$s=\overline{z-\frac{1}{2}\overline{n-1}\times d}\times n.$
XVIII. XIX.	$d,\ n,\ s,$	$a,\ z,$	$a=\frac{s}{n}-\frac{1}{2}\overline{n-1}\times d.$	$z=\frac{s}{n}+\frac{1}{2}\overline{n-1}\times d.$
XX. XXI.	$z,\ n,\ s,$	$a,\ d,$	$a=\frac{2s}{n}-z.$	$d=\frac{nz-s}{n\times\frac{1}{2}\overline{n-1}}.$
XXII. XXIII.	$z,\ d,\ s,$	$a,\ n,$	$a=\frac{1}{2}\sqrt{\overline{2z+d}\vert^2-8ds}+d.$	$n=\frac{\overline{2z+d}\pm\sqrt{\overline{2z+d}\vert^2-8ds}}{2d}.$

QUEST. XXIV. Two men A and B set out at the same time; A travels 8 miles a day; and B travels the first day 1 mile, the second day 2 miles, the third day 3 miles, &c. In how many days will B overtake A?

If x be the number of days required;
Then A will travel $8x$ miles;

Also the sum of an arithmetical progression whose first term is 1, common difference 1, and No. of terms x, is $\left(\frac{\overline{x+1}\times x}{2}=\right)\frac{xx+x}{2}$ by corol. to quest. 1st;

Th. B will travel $\frac{xx+x}{2}$ miles:

But $\frac{xx+x}{2}=8x$ by quest.

Th. $\frac{x+1}{2}=8$;

And $x=15$.

QUEST. XXV. A Stationer sold 7 reams of paper, the particular prices whereof were certain numbers of shillings in arithmetical progression; the price of the second ream, that is, of that next above the cheapest, was 8 shillings; and the price of the dearest ream was 23 shillings: What was the price of each ream?

If $z=$ the price of the cheapest ream,
And $x=$ the difference of their prices;
Then $z+x=8$, And $z+6x=23$, } by quest. 2d;
Th. $8-x=(z=)\ 23-6x$;
Whence $x=\left(\frac{23-8}{5}=\right)3$.

QUEST. XXVI. Two post-boys A, and B, set out at the same time, from two cities, which are 360 miles asunder, in order to meet each other: A rides 40 miles the first day, 38 the second, 36 the third, and so on, decreasing two miles every day; but B goes 20 miles the first day, 22 the second, 24 the third, &c increasing 2 miles every day; In what number of days will they meet?

If $x=$ the time of their meeting;

Then x terms of $\overline{40+38+36,\&c.}=40x-\frac{xx-x}{2}\times 2$, by q. 17

And x terms of $\overline{20+22+24,\&c.}=20x+\frac{xx-x}{2}\times 2$, by q. 3.

Now $40x-xx+x+20x+xx-x=360$ by question,
That is $(40x+20x=)\ 60x=360$;
Th. - - - $x=6$.

QUEST. XXVII. To find three numbers in arithmetical progression, whereof the sum of the squares shall be 1232, and the square of the mean shall exceed the product of the extremes by 16?

If $x=$ the mean number; and $z=$ the common difference;
Then $x-z$; x; and $x+z$, are the numbers;
But - $xx=\overline{x-z}\times\overline{x+z}+16$ by quest.
That is $xx=xx-zz+16$;
Th. $zz=16$; and $z=4$:
Now $\overline{x-4}^2=xx-8x+16=$ square 1st.
$xx=xx$ - $=$ square 2d.
And $\overline{x+4}^2=xx+8x+16=$ square 3d.
Th. $1232=3xx+32$ the sum of squares.
And $400=xx$;
Th. $20=x$.

QUEST. XXVIII. One being asked what were the several ages of his five children, answered, that the age of the eldest exceeded that of the second by 2 years; and by the same excess the second exceeded the third, the third the fourth, &c. and if the age of the eldest child were multiplied by the age of the youngest it would produce 128: The age of each child is required?

If $x=$ the age of the eldest;
Then $x-(\overline{5-1}\times 2=)\ 8=$ age of the youngest, by q. 16.
Now $(\overline{x-8}\times x=)\ xx-8x=128$ by quest.
But - $xx-8x+16=(128+16=)\ 144$;
Th. - - - $x=(12+4=)\ 16$.

QUEST. XXIX. There is a number consisting of three places, whose digits are in arithmetical progression; if this number be divided by the sum of its digits the quotient will be 26; and if to the number you add 198, the digits will be inverted: What is the number?

If x be the middle digit, and d the common difference of the digits;

$$\text{Then}\left\{\begin{matrix}x-d\\x\\x+d\end{matrix}\right\}\text{are the digits and}\left\{\begin{matrix}100x-100d\\10x\\x+d\end{matrix}\right\}\begin{matrix}\text{digits va-}\\\text{lued accord-}\\\text{ing to their}\\\text{places and}\end{matrix}\left\{\begin{matrix}100x+100d\\10x\\x-d\end{matrix}\right.$$

Th. $3x=$ sum digits; $111x-99d=$ number;
And $111x+99d=$ numb. the digits being inverted.
Now $111x-99d+198=111x+99d$ by quest.
Or - - $198=198d$;
Th. - - $1=d$.
Also - $\dfrac{111x-99}{3x}=26$ by quest.
Or - $111x-99=78x$;
Th. - - $x=\left(\dfrac{99}{111-78}=\dfrac{99}{33}=\right)\ 3$.
And the number is 234.

QUEST.

QUEST. XXX. If the ſum of 6 numbers in arithmetical progreſſion be 48, and the product of the common difference multiplied into the leaſt term be equal to the numbers of terms: What are the numbers of that progreſſion?

If $6=n$; $48=s$; $a=$ leaſt term, and $d=$ common diff.

Then $da=(n=)\ 6$; or $a=\frac{6}{d}$ (per queſtion);

Alſo - $s=na+\frac{n\times\overline{n-1}}{1\times 2}d$ by queſt. 3;

That is $48=6a+\left(\frac{6\times 5}{2}d=\right)15d$,

Or - $16=2a+5d$:

But - $16=\frac{12}{d}+5d$, (by writing $\frac{6}{d}$ for a):

Th. $16d=12+5dd$,

Or $dd-\frac{16}{5}d=-\frac{12}{5}$:

But $\overline{d-\frac{8}{5}}\Big|^2=\left(\frac{64}{25}-\frac{12}{5}=\right)\frac{4}{25}$;

Th. $d-\frac{8}{5}=\frac{2}{5}$; and $d=\left(\frac{8+2}{5}=\right)2$.

QUEST. XXXI. There are four numbers in arithmetical progreſſion, whereof the product of the extremes is 3250; and that of the means 3300: What are the numbers?

If $z=$ the firſt number, and $x=$ the common difference;

Then z; $z+x$; $z+2x$; and $z+3x$; will be the numbers;

And - $(z\times\overline{z+3x}=)\ zz+3xz=3250$ } by qu.

Alſo $(\overline{z+x}\times\overline{z+2x}=)\ zz+3xz+2xx=3300$ }

Th. (by ſubtraction) - - - $2xx=50$,

And - - - - - $x=5$.

Now $zz+3\times 5z=3250$ (by firſt);

But $\overline{zz+15z+\frac{15}{2}}\Big|^2=\left(3250+\frac{225}{4}=\right)\frac{13225}{4}$;

Th. - $z+\frac{15}{2}=\frac{115}{2}$;

And - - - - $z=\left(\frac{115-15}{2}=\right)50$.

And the numbers are 50, 55, 60, 65.

QUEST.

QUEST. XXXII. The continual product of four numbers in arithmetical progression is 945; and their common difference 2: What are those numbers?

If $x=$ the least number required;
Then x, $x+2$, $x+4$, and $x+6$ will be the numbers;
But $x\times\overline{x+2}\times\overline{x+4}\times\overline{x+6}=x^4+12x^3+44x^2+48x$;
Th. $x^4+12x^3+44x^2+48x=945$ by quest.
Where - - $x=3$.

QUEST. XXXIII. It is required to find (B) the sum of n terms of the series of square numbers, 0, 1, 4, 9, 16, &c. to $\overline{n-1}^2$?

Since n^3 is greater than B (See quest. 1st.)
Let - - $xn^3-yn^2+zn=B$;
Then (because the $\overline{n+1}$th term of the series is nn)
$x\times\overline{n+1}^3-y\times\overline{n+1}^2+z\times\overline{n+1}=B+nn$:
Now - $xn^3+3xn^2+3xn+x=x\times\overline{n+1}^3$,
$yn^2+2yn+y=y\times\overline{n+1}^2$,
$zn+z=z\times\overline{n+1}$;

Th. $xn^3+\overline{3x-y}\times n^2+\overline{3x-2y+z}\times n+\overline{x-y+z}=B+nn$;

And $3xn^2+\overline{3x-2y}\times n+\overline{x-y+z}=nn$ $\left\{\begin{array}{l}\text{by subtracting}\\ \text{the value of } B.\end{array}\right\}$

Or $3xn^2+3xn+\overline{x+z}=nn+2yn+y$;

Whence by corol. qu. 250. $\left\{\begin{array}{l}3x=1,\\ 3x=2y,\\ x+z=y;\end{array}\right.$

Th. - - $\left\{\begin{array}{l}x=\frac{1}{3},\\ y=\frac{1}{2},\\ z=(\frac{1}{2}-\frac{1}{3}=)\ \frac{1}{6}:\end{array}\right.$

Th.

Th. $\left(\frac{n^3}{3}-\frac{n^2}{2}+\frac{n}{6}=\right)\frac{n.\overline{n-1}.\overline{2n-1}}{1.\ 2.\ 3.}=B.$

COROL. the sum of n terms of 1, 4, 9, 16, &c. will be found $=\frac{n.\overline{n+1}.\overline{2n+1}}{1.\ 2.\ 3.}$; by writing $\overline{n+1}$, for n, in the value of B.

Note $\frac{n.\overline{n-1}.\overline{2n-1}}{1.\ 2.\ 3.}$ signifies $\frac{n\times\overline{n-1}\times\overline{2n-1}}{1\times2\times3}$.

QUEST. XXX. Two travellers A and B set out together, from the same place; A goes 8 miles the first day, 12 the second, 16 the third, and so on, increasing 4 miles every day; but B goes 1 mile the first day, 4 the second, 9 the third, and so on, according to the squares of the number of days: How many days must they travel before B overtakes A?

Suppose B will overtake A in x days,

Th. x terms of $8+12+16$, &c. $=8x+\frac{x\times\overline{x-1}}{2}4$ by q. 3.

And x terms of $1+4+9$, &c. $=\frac{x\times\overline{x+1}\times\overline{2x+1}}{1\times2\times3}$, by [cor. q. 33.

Now by quest. $\frac{x\times\overline{x+1}\times\overline{2x+1}}{1\times2\times3}=8x+\frac{x\times\overline{x-1}}{2}\times4.$

Or by division $\frac{\overline{x+1}\times\overline{2x+1}}{2\times3}=8+\overline{x-1}\times\frac{4}{2},$

Or - - $2x^2+3x+1=48+12x-12.$

Th. - - $x^2-\frac{9}{2}x=\frac{35}{2}$:

But - $\overline{x^2-\frac{9}{2}x+\frac{9}{4}}\Big|^2=\left(\frac{81}{16}+\frac{35}{2}=\right)\frac{361}{16}$;

Th. $x-\frac{9}{4}=\frac{19}{4}$; and $x=\left(\frac{19+9}{4}=\right)7.$

QUEST.

QUEST. XXXV. The sum (1225) of the series 0, 1, 2, 3, 4, &c. continued to an unknown number of terms, being given; to find the sum of the squares of those terms?

If $1225=A$; $0+1+4+9$, &c. $=B$; No. terms $=n$:

Then (by quest. 3.) - $\frac{n.\overline{n-1}}{1.\ 2.}=A$;

And (by qu. 33.) $\frac{n.\overline{n-1}.\overline{2n-1}}{1.\ 2.\ 3.}=B$:

The quotient arising from the division of the second by the first } $\frac{2n-1}{3}=\frac{B}{A}$;

Th. - - - - - $n=\frac{3B+A}{2A}$,

And - - - $n-1=\frac{3B-A}{2A}$;

But $\left(\frac{n}{1}\times\frac{n-1}{2}=\right)\frac{3B+A}{2A}\times\frac{3B-A}{4A}=A$, by first,

Or - - - $\frac{9B^2-A^2}{8A^2}=A$,

Or - - - $9B^2-A^2=8A^3$,

Or - - - - $9B^2=8A^3+A^2$;

Th. - - - - $B^2=\frac{\overline{8A+1}\times A^2}{9}$,

And - - - - - $B=\frac{A}{3}\times\sqrt{8A+1}$.

In this example $B=\left(\frac{1225}{3}\times\sqrt{8\times1225+1}=\right)40425$.

QUEST. XXXVI. What is the sum of 10 square numbers, whose roots are in an arithmetical progression, the least term of which is 3, and the common difference 2?

Let $10=n$; $3=a$; $2=d$; then it is required to find (S) the sum of n terms of a^2, $\overline{a+d}^2$, $\overline{a+2d}^2$, &c.

Now $a^2=aa$,

$\overline{a+d}^2 = aa+2\times 1ad+dd$,

$\overline{a+2d}^2 = aa+2\times 2ad+4dd$,

$\overline{a+3d}^2 = aa+2\times 3ad+9dd$,

$\overline{a+4d}^2 = aa+2\times 4ad+16dd$;

&c. &c. &c.

$$\text{Th. } S=\begin{cases}\text{Sum of } n \text{ terms of } \overline{1+1+1,\ \&c.} & \times aa,\\ +\text{ Ditto - of } \overline{0+1+2+3,\ \&c.} & \times 2ad,\\ +\text{ Ditto - of } \overline{0+1+4+9,\ \&c.} & \times dd:\end{cases}$$

But n terms of $\overline{1+1+1,\ \&c.} = n$,

Ditto of $\overline{0+1+2+3,\ \&c.} = \dfrac{n\times\overline{n-1}}{1\times 1}$ by quest. 1;

And Dit. of $\overline{0+1+4+9,\ \&c.} = \dfrac{n\times\overline{n-1}\times\overline{2n-1}}{1\times 2\times 3}$ by q. 33.

$$\text{Th. } S = naa+n\times\overline{n-1}\times ad+\frac{n\times\overline{n-1}\times\overline{2n-1}}{1\times 2\times 3}\times dd.$$

In this exam. $S=10.\ 3^2+10.\ 9.\ 3.\ 2+\dfrac{10.\ 9.\ 19}{1.\ 2.\ 3.}\times 2^2$

$=10.\ 9+10.\ 9.\ 6+10.\ 6.\ 19$

$=90+540+1140=1770.$

QUEST.

QUEST. XXXIX. The sum (C) of n terms of the series of cube numbers (0, 1, 8, 27, 64, &c. to $\overline{n-1}^3$ is required?

Let - - $xn^4 - yn^3 + un^2 - zn = C$;

Then $x\times\overline{n+1}^4 - y\times\overline{n+1}^3 + u\times\overline{n+1}^2 - z\times\overline{n+1} = C + n^3$,

But - $xn^4 + 4xn^3 + 6xn^2 + 4xn + x = x\times\overline{n+1}^4$,

$$yn^3 + 3yn^2 + 3yn + y = y\times\overline{n+1}^3,$$

$$un^2 + 2un + u = u\times\overline{n+1}^2,$$

$$zn + z = z\times\overline{n+1};$$

Th. $$\left.\begin{array}{lllll} xn^4 & +4xn^3 & +6xn^2 & +4xn & +x \\ & -y & -3y & -3y & -y \\ & & +u & +2u & +u \\ & & & -z & -z \end{array}\right\} = C + n^3;$$

Then by subtract. $$\left\{\begin{array}{llll} 4xn^3 & +6xn^2 & +4xn & +x \\ -3y & -3y & -y & \\ & +2u & +u & \\ & & -z & \end{array}\right\} = n^3,$$

Or $$\left\{\begin{array}{llll} 4xn^3 & +6xn^2 & +4xn & +x \\ & & +2u & +u \end{array}\right\} = \left\{\begin{array}{llll} n^3 & +3yn^2 & +3yn & +y \\ & & & +z \end{array}\right\}$$

Now by corol. to qu. 251. $$\left\{\begin{array}{l} 4x = 1, \\ 6x = 3y, \\ 4x + 2u = 3y, \\ x + u = y + z; \end{array}\right\} \text{And} \left\{\begin{array}{l} x = \frac{1}{4}, \\ y = \frac{1}{2}, \\ u = \frac{1}{4}, \\ 0 = z. \end{array}\right.$$

Th. $$\left(\frac{n^4}{4} - \frac{n^3}{2} + \frac{n^2}{4} = \right)\frac{n^2\times\overline{n-1}^2}{2\times 2} = C.$$

Corol. By writing $n+1$, for n, in the value of C, it will appear that the sum of n terms of the series 1, 8, 27, 64, 125, &c. $= \dfrac{n^2\times\overline{n+1}^2}{2\times 2}$.

QUEST. XL. It is required to find the sum of 10 cube numbers, whose roots are in an arithmetical progression; the least term of which is 3, and the common difference 2?

Let $10=n$; $3=a$; and $2=d$; then it is required to find (Σ) the sum of n terms of a^3, $\overline{a+d}^3$, $\overline{a+2d}^3$, &c.

Now
$$\begin{aligned}
a^3 &= a^3;\\
\overline{a+d}^3 &= a^3+3\times 1a^2d+3\times 1ad^2+d^3,\\
\overline{a+2d}^3 &= a^3+3\times 2a^2d+3\times 4ad^2+8d^3,\\
\overline{a+3d}^3 &= a^3+3\times 3a^2d+3\times 9ad^2+27d^3,\\
\overline{a+4d}^3 &= a^3+3\times 4a^2d+3\times 16ad^2+64d^3;\\
\&c. &\qquad \&c.
\end{aligned}$$

$$\text{Th.}\,\Sigma=\left\{\begin{array}{l}\text{Sum of } n \text{ terms of } \overline{1+1+1}, \&c.\times a^3,\\ +\text{ Dit. of } \overline{0+1+2+3}, \&c.\times 3a^2d\\ +\text{ Dit. of } \overline{0+1+4+9}, \&c.\times 3ad^2\\ +\text{ Dit. of } \overline{0+1+8+27}, \&c.\times d^3\end{array}\right\}\text{See question}\left\{\begin{array}{l}1,\\33\\39\end{array}\right.$$

That is $\Sigma=na^3+\frac{n.\overline{n-1}}{1.2.}3a^2d+\frac{n.\overline{n-1}.\overline{2n-1}}{1.2.3.}3ad^2+\frac{n^2.\overline{n-1}^2}{2.2.}d^3.$

In this exam. $\Sigma=(10\times 3^3+\frac{10.9}{1.2.}\times 3.3^2.2+\frac{10.9.19}{1.2.3.}\times 3.3.2^2+\frac{10^2.9^2}{2.2.}\times 2^3=)\ 29160.$

QUEST.

QUEST. XLI. The sum of 10 numbers in arithmetical progression is 120; and the sum of their cubes 29160: What are those numbers?

Let $a=$ least number; $d=$ common difference; $n=(10=)$ number of terms; $s=120$; and $\Sigma=29160$;

Then by quest.

$$\begin{cases} 3)\ s=na+\dfrac{n.\overline{n-1}}{1.\ 2.}d, \\ 40)\ \Sigma=na^3+\dfrac{n.\overline{n-1}}{1.\ 2.}.3a^2d+\dfrac{n.\overline{n-1}.\overline{2n-1}}{1.\ 2.\ 3}.3ad^2+\dfrac{n^2.\overline{n-1}^2}{2.\ 2.}d^3: \end{cases}$$

Now $$s^3=n^3a^3+\frac{n.\overline{n-1}}{1.\ 2}3n^2a^2d+\frac{n^2.\overline{n-1}^2}{2.\ 2.}3nad^2+\frac{n^3.\overline{n-1}^3}{2.\ 2\ 2.}d^3.$$

And $$\frac{s^3}{nn}=na^3+\frac{n.\overline{n-1}}{1\ 2}3a^2d+\frac{n.\overline{n-1}^2}{2.\ 2.}3ad^2+\frac{n.\overline{n-1}^3}{2.\ 2.\ 2.}d^3;$$

Th. $$\Sigma-\frac{s^3}{nn}=\overline{\frac{2n-1}{3}-\frac{n-1}{2}}\times\frac{n.\overline{n-1}}{1.\ 2.}.3ad^2+\overline{\frac{n}{1}-\frac{n-1}{2}}\times\frac{n.\overline{n-1}^2}{2.\ 2.}.d^3;$$

But $$\frac{n+1}{6}=\frac{2n-1}{3}-\frac{n-1}{2};\ \text{and}\ \frac{n+1}{2}=\frac{n}{1}-\frac{n-1}{2};$$

Th. $$\Sigma-\frac{s^3}{nn}=\frac{\overline{n-1}.n.\overline{n+1}}{1.\ 2.\ 2.}ad^2+\frac{\overline{n-1}^2.n.\overline{n+1}}{2.\ 2.\ 2.}d^3;$$

But $$\frac{\overline{n-1}.\overline{n+1}}{2.\ 2.}sdd=\frac{\overline{n-1}.n.\overline{n+1}}{1.\ 2.\ 2.}ad^2+\frac{\overline{n-1}^2.n.\overline{n+1}}{2.\ 2.\ 2.}d^3$$ from first;

Th.

Th. $\frac{\overline{n-1}}{2}\cdot\frac{\overline{n+1}}{2}sdd = \left(\Sigma - \frac{s^3}{nn} =\right)\frac{nn\Sigma - s^3}{nn}$,

And - - $dd = \frac{\overline{nn\Sigma - s^3}\times 4}{s.\overline{n-1}.\overline{n+1}.nn}$;

Th. - - $d = \frac{2}{n}\sqrt{\frac{\overline{nn\Sigma - s^3}}{s.\overline{n-1}.\overline{n+1}}}$;

In this example $d = \left(\frac{2}{10}\sqrt{\frac{\overline{10^2 \times 21960 - 120^3}}{120.9.11}} =\right) 2.$

Having found d, a may be found by quest. 18.

QUEST. XLII. There are two places 462 miles asunder, from which two persons A and B set out at the same time to meet each other; A goes 1 mile the first day and encreases each succeeding days journey by 1 mile; and B travels each day the cube of the miles that A travels: In what time will they meet?

Suppose $x =$ the time of their meeting;

Then x terms of $1+2+3+4$, &c. $= \frac{\overline{x+1}\times x}{2}$ } by cor. qu. 1.

And x terms of $1+8+27+64$, &c. $= \frac{\overline{x+1}^2 \times x^2}{2\times 2}$ } by cor. qu. 39.

Now by quest. $\frac{\overline{x+1}^2\times x^2}{2\times 2} + \frac{\overline{x+1}\times x}{2} = 462$;

And (putting $\frac{\overline{x+1}\times x}{2} = y$) $yy + y = 462$:

But $yy + y + \frac{1}{4} = \left(462 + \frac{1}{4} =\right) \frac{1849}{4}$;

Th. $y + \frac{1}{2} = \frac{43}{2}$; And $y = \left(\frac{43-1}{2} =\right) 21$:

But

But $\left(y=\frac{\overline{x+1}\times x}{2}=\right)\frac{xx+x}{2}=21$;

Th. - - $xx+x=42$:

But - - $x+xx+\frac{1}{4}=\left(42+\frac{1}{4}=\right)\frac{169}{4}$;

Th. $x+\frac{1}{2}=\frac{13}{2}$; And $x=\left(\frac{13-1}{2}=\right)6$.

QUEST. XLIII. It is required to find the sum C of n terms of the series 0, 1, 4, 10, 20, &c. the terms of which (0, 0+1, 0+1+3, 0+1+3+6, &c.) are the successive sums of the series 0, 1, 3, 6, 10, &c.?

Let $xn^4-yn^3+un^2-zn=C$; as in quest. 39.

Now by cor. quest. 38, the $\overline{n+1}$th term of this series is

$$\left(\frac{n\cdot\overline{n+1}\cdot\overline{n+2}}{1.\ 2.\ 3.}=\right)\frac{n^3}{6}+\frac{3n^2}{6}+\frac{2n}{6};$$

Therefore by process in quest. 39.

$$\left.\begin{array}{rrrr} 4xn^3 & +6xn^2 & +4xn & +x \\ & -3y & -3y & -y \\ & & +2u & +u \\ & & & -z \end{array}\right\}=\frac{n^3}{6}+\frac{3n^2}{6}+\frac{2n}{6},$$

$$\text{Or}\left\{\begin{array}{rrrr} 4xn^3 & +6xn^2 & +4xn & +x \\ & & +2u & +u \end{array}\right\}=\left\{\begin{array}{rrrr} \frac{1}{6}n^3 & +\frac{3}{6}n^2 & +\frac{2}{6}n & \\ & +3y & +3y & +y+z \end{array}\right\}:$$

$$\text{Now by corol. q. 250.}\left\{\begin{array}{l} 4x=\frac{1}{6}, \\ 6x=3y+\frac{3}{6}, \\ 4x+2u=3y+\frac{2}{6}, \\ x+u=y+z; \end{array}\right\}\text{Therefore}\left\{\begin{array}{r} x=\frac{1}{24}, \\ -\frac{1}{12}=y, \\ u=-\frac{1}{24}, \\ \frac{1}{12}=z: \end{array}\right.$$

And

And $\frac{n^4}{24}+\frac{n^3}{12}-\frac{n^2}{24}-\frac{n}{12}=\mathfrak{C}.=\frac{\overline{n-1}\,.\,n\,.\,\overline{n+1}\,.\,\overline{n+2}}{1\,.\,2\,.\,3\,.\,4}$.

Corol. 1. By writing $n+1$, for n, in the value of $\mathfrak{C}$; it will appear that n terms of 1, 4, 10, 20, &c. $=\frac{n\,.\,\overline{n+1}\,.\,\overline{n+2}\,.\,\overline{n+3}}{1\,.\,2\,.\,3\,.\,4}$.

Corol. 2. If

$\mathfrak{A}$	represent the sums of n terms of the series.	1, 2, 3, 4, 5, &c.	whose ter. are the successive sums of the series	1, 1, 1, 1, 1, &c.
$\mathfrak{B}$		1, 3, 6, 10, 15, &c.		1, 2, 3, 4, 5, &c.
$\mathfrak{C}$		1, 4, 10, 20, 35, &c.		1, 3, 6, 10, 15, &c.
$\mathfrak{D}$ *		1, 5, 15, 35, 70, &c.		1, 4, 10, 20, 35, &c.
&c.		&c.		&c.

Then from the several corollaries to questions 1, 3, 8, and 43;

$$\mathfrak{A}=\frac{n\,.\,\overline{n+1}}{1\,.\,2},$$

$$\mathfrak{B}=\frac{n\,.\,\overline{n+1}\,.\,\overline{n+2}}{1\,.\,2\,.\,3},$$

$$\mathfrak{C}=\frac{n\,.\,\overline{n+1}\,.\,\overline{n+2}\,.\,\overline{n+3}}{1\,.\,2\,.\,3\,.\,4},$$

$$\mathfrak{D}=\frac{n\,.\,\overline{n+1}\,.\,\overline{n+2}\,.\,\overline{n+3}\,.\,\overline{n+4}}{1\,.\,2\,.\,3\,.\,4\,.\,5}.$$

&c. &c.

---

* The numbers of which these series consist are, in general, called figurate numbers.

QUEST. XLIV. Three persons A, B, and C, agreed to play as many games at Piquet, as could be done without the same two persons, playing against each other, twice: The number of games is required?

Now $\left\{\begin{array}{l} A \text{ and } B \\ A \text{ and } C \\ B \text{ and } C \end{array}\right\}$ are all the variations that can be made, of two persons, out of three;

Th. they must play three games.

Corol. 1. If the number of persons were four, A, B, C, and D; then,

$\left.\begin{array}{lll} AB, & & \\ AC, & BC, & \\ AD, & BD, & CD, \end{array}\right\}$ are the required variations;

And the number of games will be 6:

Corol. 2. If there were five persons, A, B, C, D, E;

Then,

$\left.\begin{array}{llll} AB, & & & \\ AC, & BC, & & \\ AD, & BD, & CD, & \\ AE, & BE, & CE, & DE, \end{array}\right\}$ will be the different players;

Corol. 3. The number of different pairs or combinations of 2 in n things will be the nth term of the series

0, 1, 3, 6, 10, &c. $= \frac{n \cdot \overline{n-1}}{1 \cdot 2}$ by quest. 1.

QUEST.

QUEST. XLV. Four persons *A*, *B*, *C*, and *D*, engaged to play at Ombre, as often as they could make a different sett: How many times did they play?

Now $\left\{\begin{array}{l} AB \text{ and } C, \\ AB \text{ and } D, \\ AC \text{ and } D,\ BC \text{ and } D, \end{array}\right\}$ will be the players, at all the possible different setts of 3, in 4,

Th. they must play 4 times.

Corol. 1. If the number of persons were five, *A*, *B*, *C*, *D*, and *E*; Then,

$\left.\begin{array}{l} ABC, \\ ABD, \\ ABE, \\ ACD,\ BCD, \\ ACE,\ BCE, \\ ADE,\ BDE,\ CDE, \end{array}\right\}$ will be the different setts;

And they must play 10 times.

Corol. 2. The different combinations of 3, in n things will be the $\overline{n-1}$th term of the series 0, 1, 4, 10, 20, &c. $= \frac{n \cdot \overline{n-1} \cdot \overline{n-2}}{1 \cdot 2 \cdot 3}$ by quest. 38.

QUEST. XLVI. How often can a different set at Whist, be made by five persons, *A*, *B*, *C*, *D*, and *E*?

$$\text{Now}\left\{\begin{array}{l} ABCD, \\ ABCE, \\ ABDE, \\ ACDE,\ BCDE, \end{array}\right\}\text{will be the setts;}$$

Th. there will be five setts.

Corol. 1. If there were six persons *A*, *B*, *C*, *D*, *E*, and *F*; Then,

$$\left.\begin{array}{l} ABCD, \\ ABCE, \\ ABCF, \\ ABDE, \\ ABDF, \\ ABEF, \\ ACDE,\ BCDE, \\ ACDF,\ BCDF, \\ ACEF,\ BCEF, \\ ADEF,\ BDEF,\ CDEF, \end{array}\right\}\text{are the setts;}$$

And, the number of setts is 15.

Corol. 2. The number of combinations of 4 in n things will be the $\overline{n-2}$th term of the series 0, 1, 5, 15, 35, &c. $=\frac{n \cdot \overline{n-1} \cdot \overline{n-2} \cdot \overline{n-3}}{1 \cdot 2 \cdot 3 \cdot 4}$ by quest. 43.

QUEST. XLVII. The number of the combinations of m, in n things is required?

$$\text{By quest.}\left\{\begin{array}{c}44\\ \\45\\ \\46\end{array}\right\}\text{The Combinations of}\left\{\begin{array}{c}2\\ \\3\\ \\4\end{array}\right\}\text{in } n \text{ things} = \left\{\begin{array}{l}\dfrac{n\,.\,\overline{n-1}}{1\,.\,2};\\[2ex] \dfrac{n\,.\,\overline{n-1}\,.\,\overline{n-2}}{1\,.\,2\,.\,3};\\[2ex] \dfrac{n\,.\,\overline{n-1}\,.\,\overline{n-2}\,.\,\overline{n-3}}{1\,.\,2\,.\,3\,.\,4};\end{array}\right.$$

$$\left.\begin{array}{l}\text{Th. the com-}\\ \text{binations of } m,\\ \text{in } n \text{ things.}\end{array}\right\} = \frac{n\,.\,\overline{n-1}\,.\,\overline{n-2}\,.\,\overline{n-3}, \text{ \&c. to } \overline{n-m-1}}{1\,.\,2\,.\,3\,.\,4 \quad \text{\&c. to} \quad m}.$$

QUEST. XLVIII. A gentleman who had seven children, each a year older than the next younger, determined to new cloath them, and to bestow as many yards of lace on the trimming of the youngest's suit as he was years old; as many on the second, as the sum of his and the youngest's age; as many on the third as the sum of their three ages; &c. and agreed with the taylor to pay him (in pence) the product of each child's age, by the number of yards of lace on his cloaths, for the making each suit.

Now the work being done, the taylor's built amounted to 7*l.* 10*s.* 6*d.* Required the age of each child?

Let $7 = n$; ($7l.\ 10s.\ 6d. =$) $1806d. = m$; and $x =$ the youngest child's age:

Then x, $x+1$, $x+2$, &c. to $x+\overline{n-1}$ are the children's ages,

And x, $2x+1$, $3x+3$, &c. to $nx+\dfrac{n\cdot\overline{n-1}}{2}$ are the yards of lace on their cloaths;

$$\text{Whence}\left\{\begin{array}{l} xx+0\ +0 \\ 2xx+3x+1 \\ 3xx+9x+6 \\ \quad\text{\&c. to} \\ nxx+\dfrac{3n\cdot\overline{n-1}}{2}x+\dfrac{n\cdot\overline{n-1}^2}{2} \end{array}\right\}\text{are the prices of making each suit.}$$

$$\text{Th. the taylor's bill will be}\left\{\begin{array}{l} \overline{n \text{ terms of } 1,\ 2,\ 3, \text{ \&c. to } n\ .\ xx,} \\ +\ n \text{ terms of } 0,\ 3,\ 9, \text{ \&c. to } \dfrac{3n\cdot\overline{n-1}}{2}: \\ +\ n \text{ terms of } 0,\ 1,\ 6, \text{ \&c. to } \dfrac{n\cdot\overline{n-1}^2}{2}: \end{array}\right.$$

Now 1st. n terms of 1, 2, 3, &c. to $n = \dfrac{n\cdot\overline{n+1}}{2}$ by Corol. quest. 43.

2. Let $xn^3 - yn^2 + zn = n$ terms of 0, 3, 9, &c. to $\frac{3n \cdot \overline{n-1}}{2}$;

Then by writing $\overline{n+1}$ for n, the $\overline{n+1}$ term will be $\frac{3n \cdot \overline{n+1}}{2}$,

And from procefs in queft. 33.

$$\left.\begin{array}{r} 3xn^2 + 3xn + x \\ -2y \quad -y \\ +z \end{array}\right\} = \left(\frac{3n \cdot \overline{n+1}}{2} = \right) \frac{3n^2}{2} + \frac{3n}{2},$$

Or $3xn^2 + 3xn + x + z = \frac{3n^2}{2} + \overline{2y + \frac{3}{2}} \times n + y$;

By Cor. to qu. 250. $\left\{\begin{array}{r} 3x = \frac{3}{2}, \\ 3x = 2y + \frac{3}{2}, \\ x + z = y, \end{array}\right\}$ therefore $\left\{\begin{array}{l} x = \frac{1}{2}, \\ 0 = y, \\ z = -\frac{1}{2}, \end{array}\right\}$

And $\left(\frac{1}{2}n^3 - \frac{1}{2}n = \right) \frac{\overline{n+1} \cdot n \cdot \overline{n-1}}{2} = n$ terms of 0, 3, 9, &c. to $\frac{3n \cdot \overline{n-1}}{2}$.

3. Let $xn^4 - yn^3 + un^2 - zn = n$ terms of 0, 1, 6, &c. to $\frac{n \cdot \overline{n-1}^2}{2}$;

Then by writing $n+1$ for n, the $\overline{n+1}$th term will be $\left(\frac{\overline{n+1} \times nn}{2} =\right) \frac{n^3}{2} + \frac{nn}{2}$,

And from procefs in queft 39.

$$\left.\begin{array}{rrrr} 4xn^3 + 6xn^2 & + 4xn & + x \\ -3y & -3y & -y \\ & +2u & +u \\ & & -z \end{array}\right\} = \frac{n^3}{2} + \frac{nn}{2},$$

Or $\left\{\begin{array}{r} 4xn^3 + 6xn^2 + 4xn + x \\ +2u \quad +u \end{array}\right\} = \left\{\begin{array}{l} \frac{1}{2}n^3 + \frac{1}{2}nn, \\ \quad +3y + 3yn + y + z; \end{array}\right.$

By Corol. queft. 250. $\left\{\begin{array}{r} 4x = \frac{1}{2} \\ 6x = \frac{1}{2} + 3y, \\ 4x + 2u = 3y, \\ x + u = y + z; \end{array}\right\}$ therefore $\left\{\begin{array}{r} x = \frac{1}{8}, \\ \frac{1}{12} = y, \\ u = -\frac{1}{8}, \\ -\frac{1}{12} = z; \end{array}\right.$

And $\frac{1}{8}n^4 - \frac{1}{12}n^3 - \frac{1}{8}n^2 + \frac{1}{12}n$

Or $\frac{\overline{n+1} \cdot n \cdot \overline{n-1} \cdot \overline{3n-2}}{1 \cdot 2 \cdot 3 \cdot 4}$ $\Big\} = n$ terms of 0, 1, 6, &c. $\frac{n \cdot \overline{n-1}^2}{2}$.

Th. $\frac{n\cdot\overline{n+1}}{1\cdot 2}x^2+\frac{\overline{n+1}\cdot n\cdot\overline{n-1}}{1\cdot 1\cdot 2}x+\frac{\overline{n+1}\cdot n\cdot\overline{n-1}\cdot\overline{3n-2}}{1\cdot 2\cdot 3\cdot 4}=m,$

And $x^2+\overline{n-1}\times x+\frac{\overline{n-1}\cdot\overline{3n-2}}{3\cdot 4}=\frac{2m}{n\cdot\overline{n+1}},$

Or $x^2+\overline{n-1}\times x=\frac{2m}{n\cdot\overline{n+1}}-\frac{\overline{n-1}\cdot\overline{3n-2}}{3\cdot 4};$

But $\overline{x^2+\overline{n-1}\times x+\frac{n-1}{2}}\Big|^2=\frac{2m}{n\cdot\overline{n+1}}+\frac{\overline{n-1}\cdot\overline{n-1}\cdot\overline{n-1}\cdot\overline{3n-2}}{1\cdot 4\cdot 3\cdot 4};$

(Now $\frac{\overline{n-1}\cdot\overline{n-1}}{1\cdot 4}-\frac{\overline{n-1}\cdot\overline{3n-2}}{3\cdot 4}=\overline{\frac{n-1}{1}-\frac{3n-2}{3}}\times\frac{n-1}{4};$

And $\frac{n-1}{1}-\frac{3n-2}{3}=\left(\frac{3n-3}{3}-\frac{3n-2}{3}=\right)\frac{-1}{3};$

Whence $\frac{\overline{n-1}\cdot\overline{n-1}}{1\cdot 4}-\frac{\overline{n-1}\cdot\overline{3n-2}}{3\cdot 4}=\left(-\frac{1}{3}\times\frac{n-1}{4}=\right)-\frac{n-1}{12}\Big)$

Th. - $x^2+\overline{n-1}\times x+\overline{\frac{n-1}{2}}\Big|^2=\frac{2m}{n\cdot\overline{n+1}}-\frac{n-1}{12}$;

And - - - $x+\frac{n-1}{2}=\sqrt{\frac{2m}{n\cdot\overline{n+1}}-\frac{n-1}{12}}$;

And - - - - $x=\sqrt{\frac{2m}{n\cdot\overline{n+1}}-\frac{n-1}{12}}-\frac{n-1}{2}$.

In this Example - - $x=\left(\sqrt{\frac{2\cdot 1806}{7\cdot 8}-\frac{6}{12}}-\frac{6}{2}=\right)5$.

QUEST. XLIX. The ſum (Z) of n terms of the ſeries of mth powers (0^m, 1^m, 2^m, 3^m, 4^m, &c. to $\overline{n-1}^m$) is required?

Aſſume $\quad an^{m+1} - bn^m \quad + cn^{m-1} \quad - dn^{m-2} \quad + en^{m-3}$ &c. to $zn \quad = Z$;

Then $a \times \overline{n+1}^{m+1} - b \times \overline{n+1}^m + c \times \overline{n+1}^{m-1} - d \times \overline{n+1}^{m-2} + e \times \overline{n+1}^{m-3}$ &c. to $z \times \overline{n+1} = Z + n^m$:

And by proceeding as in queſtions 1, 33, 38, 39, and 43.)

$$\left.\begin{array}{llllll}
\frac{m+1}{1}an^m & +\frac{\overline{m+1}\cdot m}{1\cdot 2}an^{m-1} & +\frac{\overline{m+1}\cdot m\cdot\overline{m-1}}{1\cdot 2\cdot 3}an^{m-2} & +\frac{\overline{m+1}\cdot m\cdot\overline{m-1}\cdot\overline{m-2}}{1\cdot 2\cdot 3\cdot 4}an^{m-3} & + \text{\&c. to } +z \\
 & -\frac{m}{1}b & -\frac{m\cdot\overline{m-1}}{1\cdot 2}b & -\frac{m\cdot\overline{m-1}\cdot\overline{m-2}}{1\cdot 2\cdot 3}b & - \text{\&c. to } -b \\
 & & +\frac{m-1}{1}c & +\frac{\overline{m-1}\cdot\overline{m-2}}{1\cdot 2}c & + \text{\&c. to } +c \\
 & & & -\frac{m-2}{1}d & - \text{\&c. to } -d
\end{array}\right\} = n^m;$$

$$\text{Or}\left\{\begin{array}{l} \frac{m+1}{1}an^{m}+\frac{\overline{m+1}\cdot m}{1\cdot 2}an^{m-1}+\frac{\overline{m+1}\cdot m\cdot\overline{m-1}}{1\cdot 2\cdot 3}an^{m-2}+\frac{\overline{m+1}\cdot m\cdot\overline{m-1}\cdot\overline{m-2}}{1\cdot 2\cdot 3\cdot 4}an^{m-3}\ \&c.+a \\ \qquad\qquad\qquad +\frac{m-1}{1}c\qquad\qquad +\frac{\overline{m-1}\cdot\overline{m-2}}{1\cdot 2}c\qquad\qquad \&c.+c \end{array}\right\}=$$

$$\left\{\begin{array}{l} n^{m}+\frac{m}{1}bn^{m-1}+\frac{m\cdot\overline{m-1}}{1\cdot 2}bn^{m-2}+\frac{m\cdot\overline{m-1}\cdot\overline{m-2}}{1\cdot 2\cdot 3}bn^{m-3}\ \&c.+b \\ \qquad\qquad\qquad\qquad\qquad\qquad +\frac{m-2}{1}d\qquad\qquad \&c.+e \end{array}\right\}:$$

Now 1 From Cor. to quest. 250 - - - $\frac{m+1}{1}a=1$;

Th. - - - - - $a=\frac{1}{m+1}$.

2 Ditto (by writing $\frac{1}{m+1}$ for a) - - - $\frac{m}{2}=\frac{m}{b}b$;

Th. - - - - - $\frac{1}{2}=b$.

3 Ditto (and writing $\frac{1}{2}$ for b) - $\frac{m\cdot\overline{m-1}}{2\cdot 3}+\frac{m-1}{1}c=\frac{m\cdot\overline{m-1}}{1\cdot 2}\times\frac{1}{2}$,

Or by division - - $\frac{m}{2\cdot 3}+ \quad c=\frac{m}{2\cdot 2}$;

Th. - - - - - $c=\left(\frac{m}{4}-\frac{m}{6}=\frac{m}{12}=\right)\frac{m}{2\times 2.3}$.

4 Ditto - $\frac{m\cdot\overline{m-1}\cdot\overline{m-2}}{2\cdot 3\cdot 4}+\frac{\overline{m-1}\cdot\overline{m-2}}{1\cdot 2}\times\frac{m}{3\cdot 4}=\frac{m\cdot\overline{m-1}\cdot\overline{m-2}}{1\cdot 2\cdot 3}\times\frac{1}{2}+\frac{m-2}{1}d$,

Or - - - - - $\frac{m\,.\,\overline{m-1}}{2\,.\,3\,.\,4}+\frac{m\,.\,\overline{m-1}}{2\,.\,3\,.\,4}=\frac{m\,.\,\overline{m-1}}{2\,.\,3\,.\,4}+d,$

That is - - - - - - $\frac{m\,.\,\overline{m-1}}{3\,.\,4}=\frac{m\,.\,\overline{m-1}}{3\,.\,4}+d,$

Th. - - - - - $o=d.$

5th. $\frac{m\,.\,\overline{m-1}\,.\,\overline{m-2}\,.\,\overline{m-3}}{2\,.\,3\,.\,4\,.\,5}+\frac{\overline{m-1}\,.\,\overline{m-2}\,.\,\overline{m-3}}{1\,.\,2\,.\,3}\times\frac{m}{3\,.\,4}+\frac{m-3}{1}e=\frac{m\,.\,\overline{m-1}\,.\,\overline{m-2}\,.\,\overline{m-3}}{1\,.\,2\,.\,3\,.}\times\frac{1}{2}+o,$

Or - - $\frac{m\,.\,\overline{m-1}\,.\,\overline{m-2}}{2\,.\,3\,.\,4\,.\,5}+\frac{m\,.\,\overline{m-1}\,.\,\overline{m-2}}{2\,.\,3\,.\,3\,.\,4}+e=\frac{m\,.\,\overline{m-1}\,.\,\overline{m-2}}{2\,.\,2\,.\,3\,.\,4},$

That is - - $e=\frac{m\,.\,\overline{m-1}\,.\,\overline{m-2}}{2\,.\,3\,.\,4}\times\overline{\tfrac{1}{2}-\tfrac{1}{5}-\tfrac{1}{3}};$

Th. - - - - - $e=-\frac{m\,.\,\overline{m-1}\,.\,\overline{m-2}}{2\,.\,3\,.\,4\times5\,.\,6}.$

And by a similar process the rest may be discovered, as follows, *viz.*

Now if

$$a=\frac{1}{m+1},\qquad \alpha=\left(\frac{1}{6}\right.\qquad\qquad\left.=\right)\frac{1}{6},$$

$$b=\tfrac{1}{2},\qquad \beta=\left(\frac{4}{2}\alpha-\frac{1}{2}+\frac{1}{5}\right.\qquad\left.=\right)\frac{1}{30},$$

$$c=\frac{m}{2\times 6},$$

$$d=0,\qquad \gamma=\left(\frac{6.5.4}{2.3.4}\beta-\frac{6}{2}\alpha+\frac{1}{2}-\frac{1}{7}\right.\qquad\left.=\right)\frac{1}{42},$$

$$e=-\frac{m.\overline{m-1}.\overline{m-2}}{2.\ 3.\ 4\times 30},\qquad \delta=\left(\frac{8.7.6.5.4}{2.3.4.5.6}\gamma-\frac{8.7.6}{2.3.4}\beta+\frac{8}{2}\alpha-\frac{1}{2}+\frac{1}{9}=\right)\frac{1}{30};$$

$$f=0,$$

$$g=\frac{m.\overline{m-1}.\overline{m-2}.\overline{m-3}.\overline{m-4}}{2.\ 3\ .\ 4\ .\ 5\ .\ 6\times 42};\ \&c,\qquad \&c.$$

&c. &c.

Then

$$Z=\frac{1}{m+1}n^{m+1}-\frac{1}{2}n^{m}+\frac{m\alpha}{2}n^{m-1}-\frac{m.\overline{m-1}.\overline{m-2}.\beta}{2\,.\,3\,.\,4}n^{m-3}+\frac{m.\overline{m-1}.\overline{m-2}.\overline{m-3}.\overline{m-4}.\gamma}{2\,.\,3\,.\,4\,.\,5\,.\,6}n^{m-5},\ \&c.$$

Also if $\left.\begin{array}{r} A \\ B \\ C \\ D \\ \&c. \end{array}\right\}$ represent the sum of n terms of the series of $\left\{\begin{array}{l} \text{1st powers } 0,\ 1,\ 2,\ 3,\ \&c. \\ \text{2d powers } 0,\ 1,\ 4,\ 9,\ \&c. \\ \text{3d powers } 0,\ 1,\ 8,\ 27,\ \&c. \\ \text{4th powers } 0,\ 1,\ 16,\ 81,\ \&c. \\ \&c. \end{array}\right.$

Then $\left.\begin{array}{l} A=\frac{n^2}{2}-\frac{n}{2}, \\ B=\frac{n^3}{3}-\frac{n^2}{2}+\frac{n}{6}, \\ C=\frac{n^4}{4}-\frac{n^3}{2}+\frac{n^2}{4}, \\ D=\frac{n^5}{5}-\frac{n^4}{2}+\frac{n^3}{3}-\frac{n}{30} \end{array}\right\}$ Cor. 2. to Quest. XLIII.

$$E=\frac{n^6}{6}-\frac{n^5}{2}+\frac{5n^4}{12}-\frac{n^2}{12},$$

$$F=\frac{n^7}{7}-\frac{n^6}{2}+\frac{n^5}{2}-\frac{n^3}{6}+\frac{n}{42},$$

$$G=\frac{n^8}{8}-\frac{n^7}{2}+\frac{7n^6}{12}-\frac{7n^4}{24}+\frac{n^2}{12},$$

$$H=\frac{n^9}{9}-\frac{n^8}{2}+\frac{2n^7}{3}-\frac{7n^5}{15}+\frac{2n^3}{9}-\frac{n}{30},$$

$$I=\frac{n^{10}}{10}-\frac{n^9}{2}+\frac{3n^8}{4}-\frac{7n^6}{10}+\frac{n^4}{2}-\frac{3n^2}{20},$$

$$K=\frac{n^{11}}{11}-\frac{n^{10}}{2}+\frac{5n^9}{6}-\frac{n^7}{1}+\frac{n^5}{1}-\frac{n^3}{2}+\frac{5n}{66},$$

$$L=\frac{n^{12}}{12}-\frac{n^{11}}{2}+\frac{11n^{10}}{12}-\frac{11n^8}{8}+\frac{11n^6}{6}-\frac{11n^4}{8}+\frac{5n^2}{12}.$$

&c. &c.

QUEST. L. The ſum $\mathfrak{B}$ of n terms of the mth powers of an arithmetical progreſſion, whoſe leaſt term is a, and common difference d, is required?

Let A, B, C, &c. be as in queſt. laſt.

Then $a^m \quad = a^m$,

$$\overline{a+d}^m = a^m + ma^{m-1} \times d + \frac{m \cdot \overline{m-1}}{1 \cdot 2} a^{m-2} \times dd + \&c.$$

$$\overline{a+2d}^m = a^m + ma^{m-1} \times 2d + \frac{m \cdot \overline{m-1}}{1 \cdot 2} a^{m-2} \times 4d^2 + \&c.$$

$$\overline{a+3d}^m = a^m + ma^{m-1} \times 3d + \frac{m \cdot \overline{m-1}}{1 \cdot 2} a^{m-2} \times 9d^2 + \&c.$$

Th. $\mathfrak{B} = na^m + Ama^{m-1}d + B\,\frac{m \cdot \overline{m-1}}{1 \cdot 2} a^{m-2} d +^2$

QUEST. LI. and LII. The nth term (x) and the sum of n Terms (s) of a rank or series of numbers, the difference of whose differences (or whose second differences) are equal, are required?

Put a for the least, and z for the greatest term; D for the difference of the two first terms (or for the first of the first differences;) and d for the difference of the first differences (or the second difference:)

Then if it be an ascending series,

$$\left.\begin{array}{l} a, \\ a+D, \\ a+2D+d, \\ a+3D+3d, \\ a+4D+6d, \\ a+5D+10d, \end{array}\right\} \text{will be the terms of the series; for} \left\{\begin{array}{l} D, \\ D+d, \\ D+2d, \\ D+3d, \\ D+4d, \end{array}\right\} \text{the differences of those terms, dif. by} \left\{\begin{array}{l} d, \\ d, \\ d, \\ d, \end{array}\right\} \text{the above substituted second diff.}$$

Or, if it be a descending series,

$$\left.\begin{array}{l} z, \\ z-D, \\ z-2D+d, \\ z-3D+3d, \\ z-4D+6d, \\ z-5D+10d, \end{array}\right\} \text{will be the terms of the series; for} \left\{\begin{array}{l} D, \\ D-d, \\ D-2d, \\ D-3d, \\ D-4d, \end{array}\right\} \text{the differences of those ter. differ by} \left\{\begin{array}{l} d, \\ d, \\ d, \\ d, \end{array}\right\} \text{the above substituted second diff.}$$

Th

Th. $x = \left\{\begin{matrix} a+\overline{n-1}\times D \\ z-\overline{n-1}\times D \end{matrix}\right\} + \times d\overline{n-2}$ th term of 1, 3, 6, 10, &c.

But the $\overline{n-2}$ th. term of 1, 3, 6, 10, &c. is $\dfrac{\overline{n-2}\,.\,\overline{n-1}}{2\,.\,2}$, by Corol. quest. 1.

Th. $x = \left\{\begin{matrix} a+\overline{n-1}\times D \\ z-\overline{n-1}\times D \end{matrix}\right\} + \dfrac{\overline{n-1}\,.\,\overline{n-2}}{1\,.\,2}d.$

Also $s = \left\{\begin{matrix} na+\overline{n \text{ terms of } 0, 1, 2, \&c.}\times D \\ nz-\overline{n \text{ terms of } 0, 1, 2, \&c.}\times D \end{matrix}\right\}$
$+\overline{n-1}$ terms of 0, 1, 3, &c. $\times d$.

But, n terms of 0, 1, 2, 3, &c. $= \dfrac{n\,.\,\overline{n-1}}{1\,.\,2}$ by quest. 1.

And $\overline{n-1}$ terms of 0, 1, 3, 6, &c. $= \dfrac{n\,.\,\overline{n-1}\,.\,\overline{n-2}}{1\,.\,2\,.\,3}$ by q. 38.

Th. $s = \left\{\begin{matrix} na+\dfrac{n\,.\,\overline{n-1}}{1\,.\,2}D \\ nz-\dfrac{n\,.\,\overline{n-1}}{1\,.\,2}D \end{matrix}\right\} + \dfrac{n\,.\,\overline{n-1}\,.\,\overline{n-2}}{1\,.\,2\,.\,3}d.$

Exam-

Example. What is the ſum of n terms, of the ſeries of ſquare numbers, 0, 1, 4, 9, 16, &c.?

Square numbers.	Firſt differences.	Second differences.
0		
1	1	2
4	3	2
9	5	2
16	7	

It appears above, that in this example, $a=0$; $D=1$; and $d=2$:

$$\left.\begin{array}{ll}\text{Th.} & 0\times n+\frac{n\,.\,\overline{n-1}}{2}1+\frac{n\,.\,\overline{n-1}\,.\,\overline{n-2}}{1\,.\,2\,.\,3}2,\\ \text{Or} & \frac{n\,.\,\overline{n-1}}{2}+\frac{n\,.\,\overline{n-1}}{2}\times\frac{2n-4}{3},\\ \text{Or} & \frac{n\,.\,\overline{n-1}}{2}\times\overline{1+\frac{2n-4}{3}},\\ \text{Or} & \frac{n\,.\,\overline{n-1}}{2}\times\frac{3+2n-4}{3},\\ \text{Th.} & \frac{n\,.\,\overline{n-1}\,.\,\overline{2n-1}}{1\,.\,2\,.\,3},\end{array}\right\}=\text{ſum req.}$$

See queſt. 33.

QUEST. LIII. and LIV. The nth. term (x) and the sum of n terms (s) of a series of numbers whose third differences are equal, are required?

Putting $a=$ first term; $D=$ first of the first differences; $\Delta=$ first of the second differences; and $d=$ third dffiorences: the terms of the series with their first, second, and third differences may be expressed as below.

Terms of the series.	First differences.	second diff.	third diff.
$a.$			
$a\pm D.$	$D.$		
$a\pm 2D+\Delta.$	$D\pm\Delta.$	$\Delta.$	
$a\pm 3D+3\Delta\pm d.$	$D\pm 2\Delta+d.$	$\Delta\pm d.$	$d.$
$a\pm 4D+6\Delta\pm 4d.$	$D\pm 3\Delta+3d.$	$\Delta\pm 2d.$	$d.$
$a\pm 5D+10\Delta\pm 10d.$	$D\pm 4\Delta+6d.$	$\Delta\pm 3d.$	$d.$
$a\pm 6D+15\Delta\pm 20d.$	$D\pm 5\Delta+10d.$	$\Delta\pm 4d.$	$d.$

Th.

$$\text{Th. } x = \begin{cases} a \pm \overline{n-1} \times D + \Delta \times \overline{\overline{n-2}\text{th term of } 1,\ 3,\ 6,\ \&c.} \\ \pm d \times \overline{\overline{n-3}\text{th term of } 1,\ 4,\ 10,\ \&c.} \end{cases}$$

$$\text{And } s = \begin{cases} na \pm \overline{\overline{n \quad \text{terms of } 0,\ 1,\ 2,\ 3,\ \&c.}} \times D, \\ \pm \overline{\overline{n-1} \text{ terms of } 0,\ 1,\ 3,\ 6,\ \&c.} \times \Delta, \\ \pm \overline{n-2} \text{ terms of } 0,\ 1,\ 4,\ 10,\ \&c. \times d: \end{cases}$$

$$\text{Or } x = a \pm \overline{n-1} \times D + \frac{\overline{n-1} \cdot \overline{n-2}}{1 \cdot 2} \Delta + \frac{\overline{n-1} \cdot \overline{n-2} \cdot \overline{n-3}}{1 \cdot 2 \cdot 3} d;$$

$$\text{And } s = na \pm \frac{n \cdot \overline{n-1}}{1 \cdot 2} D + \frac{n \cdot \overline{n-1} \cdot \overline{n-2}}{1 \cdot 2 \cdot 3} \Delta \pm \frac{n \cdot \overline{n-1} \cdot \overline{n-2} \cdot \overline{n-3}}{1 \cdot 2 \cdot 3 \cdot 4} d.$$

See queſtions 1, 38, and 43.

Example. What is the ſum of n terms of the ſeries of cube numbers 0, 1, 8, 27, 64, &c.

Numbers.	First Differences.	Second differences.	third differences.
0			
1	1		
8	7	6	
27	19	12	6
64	37	18	6
125	61	24	6

It appears from above, that in this example $a=0$; $D=1$; $\Delta=6$; and $d=6$;

Th.

$$\left.\begin{aligned}
&\text{Th. } 0\times n+\frac{n\cdot\overline{n-1}}{2}+\frac{n\cdot\overline{n-1}\cdot\overline{n-2}}{1\cdot 2\cdot 3}6+\frac{n\cdot\overline{n-1}\cdot\overline{n-2}\cdot\overline{n-3}}{1\cdot 2\cdot 3\cdot 4}6,\\
&\text{Or } \frac{n\cdot\overline{n-1}}{2}+\frac{n\cdot\overline{n-1}}{2}\times\frac{6\times\overline{n-2}}{3}+\frac{n\cdot\overline{n-1}}{2}\times\frac{6\times\overline{n-2}\cdot\overline{n-3}}{3\cdot 4},\\
&\text{Or } \frac{n\cdot\overline{n-1}}{2}\times\overline{1+\frac{6\cdot\overline{n-2}}{3}+\frac{6\times\overline{n-2}\cdot\overline{n-3}}{3\cdot 4}},\\
&\text{Or } \frac{n\cdot\overline{n-1}}{2}\times\overline{1+\frac{6\cdot\overline{n-2}}{3}\times\overline{1+\frac{n-3}{4}}},\\
&\text{Or } \frac{n\cdot\overline{n-1}}{2}\times\overline{1+\frac{6\cdot\overline{n-2}}{3}\times\frac{n+1}{4}},\\
&\text{Or } \frac{n\cdot\overline{n-1}}{2}\times\overline{1+\frac{nn-n-2}{2}},\\
&\text{Or } \frac{n\cdot\overline{n-1}}{2}\times\frac{2+nn-n-2}{2};\\
&\text{Th. } \left(\frac{n\cdot\overline{n-1}}{2}\times\frac{nn-n}{2}=\right)\frac{n^2\cdot\overline{n-1}^2}{2\cdot 2}.
\end{aligned}\right\}=\text{sum required.}$$

See quest. 39.

QUEST.

QUEST. LV. and LVI. The nth term (z), and the ſum of the terms (s), of a ſeries of numbers whoſe mth differences are equal, are required?

Then (by queſtions 2, 3, 50, 51, 52, and 53.)

$$\text{In a ſeries whoſe}\left\{\begin{array}{c}\text{1ſt}\\ \text{2d}\\ \text{3d}\end{array}\right\}\text{diff. are equal.}\left\{\begin{array}{l} a\pm\overline{n-1}\,.\,d \\ a\pm\overline{n-1}\,.\,D+\dfrac{\overline{n-1}\,.\,\overline{n-2}}{1\,.\,2}d \\ a\pm\overline{n-1}\,.\,D+\dfrac{\overline{n-1}\,.\,\overline{n-2}}{1\,.\,2}\Delta\pm\dfrac{\overline{n-1}\,.\,\overline{n-2}\,.\,\overline{n-3}}{1\,.\,2\,.\,3}d. \end{array}\right\}=n\text{th terms.}$$

$$\text{And}\left\{\begin{array}{l} na\pm\dfrac{n\,.\,\overline{n-1}}{1\,.\,2} \\ na\pm\dfrac{n\,.\,\overline{n-1}}{1\,.\,2}D+\dfrac{n\,.\,\overline{n-1}\,.\,\overline{n-2}}{1\,.\,2\,.\,3}d \\ na\pm\dfrac{n\,.\,\overline{n-1}}{1\,.\,2}D+\dfrac{n\,.\,\overline{n-1}\,.\,\overline{n-2}}{1\,.\,2\,.\,3}\Delta\pm\dfrac{n\,.\,\overline{n-1}\,.\,\overline{n-2}\,.\,\overline{n-3}}{1\,.\,2\,.\,3\,.\,4}d. \end{array}\right\}=\text{the ſum.}$$

Therefore putting $d^{i}=$ the first of the first differences; $d^{ii}=$ the first of the second differences; $d^{iii}=$ the first of the third differences, &c.

$$z = a \pm \overline{n-1}\, d^{i} + \frac{\overline{n-1}\,.\,\overline{n-2}}{1\,.\,2} d^{ii} \pm \frac{\overline{n-1}\,.\,\overline{n-2}\,.\,\overline{n-3}}{1\,.\,2\,.\,3} d^{iii} + \frac{\overline{n-1}\,.\,\overline{n-2}\,.\,\overline{n-3}\,.\,\overline{n-4}}{1\,.\,2\,.\,3\,.\,4} d^{iv}$$

&c. to $\overline{m+1}$ terms;

$$s = na \pm \frac{n\,.\,\overline{n-1}}{1\,.\,2} d^{i} + \frac{n\,.\,\overline{n-1}\,.\,\overline{n-2}}{1\,.\,2\,.\,3} d^{ii} \pm \frac{n\,.\,\overline{n-1}\,.\,\overline{n-2}\,.\,\overline{n-3}}{1\,.\,2\,.\,3\,.\,4} d^{iii} + \frac{n\,.\,\overline{n-1}\,.\,\overline{n-2}\,.\,\overline{n-3}\,.\,\overline{n-4}}{1\,.\,2\,.\,3\,.\,4\,.\,5} d^{iv}$$

&c. to $\overline{m+1}$ terms.

To find how many terms of a series (a, b, c, d, &c.) whose mth differences are equal, have been added together to compose a number s.

1. Let $p=$ the continual product of m terms of the series 2, 3, 4, 5, &c.

2. And let $ps=A$; $A-pa=B$; $B-pb=C$; $C-pc=D$, &c.

3. Find the divisors of A, B, C, &c. ranging them against their respective numbers.

4. Among the divisors of A find n; so that $\overline{n-1}$ may be a divisor of B; $\overline{n-2}$, of C; $\overline{n-3}$, of D, &c.

5. Then will n, be the number of terms required.

The reason of this will appear, if the equation expressing the value of s in quest. 56, be multiplied by (2. 3. 4. &c. $=$) p: For then n (being a factor in every term on the right side of the equation) must be a divisor of ($ps=$) A; and when the number of terms is lessened by one, then $\overline{n-1}$ must be a divisor of B, &c.

QUEST. LVII. What is the nth power of $\overline{a+b}$?

The following powers may be easily obtained by multiplication; viz.

$$\overline{a+b}^1=a+b,$$
$$\overline{a+b}^2=a^2+2ab+b^2,$$
$$\overline{a+b}^3=a^3+3a^2b+3ab^2+b^3,$$
$$\overline{a+b}^4=a^4+4a^3b+6a^2b^2+4ab^3+b^4;$$

And by questions 2d, 50, 52, 54, it appears
that the 2d 3d 4th 5th term of a series,
whose 1st 2d 3d 4th differences
are equal, is $a+D$;
$a+2D+\Delta$;
$a+3D+3\Delta\ \delta$;
$a+4D+6\Delta+4\delta+d$.

In which, the numeral coeficients of the
2d 3d 4th 5th term, are the same with those of
the 1st 2d 3d 4th Power.

But the nth term of such a series is expressed

$$\text{by } a+\overline{n-1}\times D+\frac{\overline{n-1}\,.\,\overline{n-2}}{1\,.\,2}\Delta$$
$$+\frac{\overline{n-1}\,.\,\overline{n-2}\,.\,\overline{n-3}}{1\,.\,2\,.\,3}\delta, \text{ \&c. (q. 54.)}$$

Th. the numeral coefficients (or unciæ) which arise in Powers, may be expressed in a like manner, by writing n for $\overline{n-1}$;

$$\text{Th. } \overline{a+b}^n=a^n+na^{n-1}b+\frac{n\,.\,\overline{n-1}}{1\,.\,2}a^{n-2}b^2$$
$$+\frac{n\,.\,\overline{n-1}\,.\,\overline{n-2}}{1\,.\,2\,.\,3}a^{n-3}b^3,$$
&c. to $\overline{n+1}$ terms.

COROL.

COROL. 1. If A, B, C, &c. represent the 1st 2d 3d, &c. term of the above series; Then, $\overline{a+b}|^n$

$$\overset{A}{=a^n}+\overset{B}{\frac{n}{1}.\frac{b}{a}A}+\overset{C}{\frac{n-1}{2}.\frac{b}{a}B}+\overset{D}{\frac{n-2}{3}.\frac{b}{a}C}+\overset{E}{\frac{n-3}{4}.\frac{b}{a}.D},\text{\&c.}$$

COROL. 2. If the nth Power of $\overline{a-b}$ be required; Then, $\overline{a-b}|^n$

$$=a^n-\frac{n}{1}.\frac{b}{a}A+\frac{n-1}{2}.\frac{b}{a}B-\frac{n-2}{3}.\frac{b}{a}C+\frac{n-3}{4}.\frac{b}{a}.D,\text{\&c.}$$

COROL. 3. $\overline{a+b}|^{\frac{m}{p}}$

$$=a^{\frac{m}{p}}+\frac{m}{p}.\frac{b}{a}.A+\frac{m-p}{2p}.\frac{b}{a}B+\frac{m-2p}{3p}.\frac{b}{a}.C+\text{\&c.}$$

COROL. 4. $\overline{a+b}|^{\frac{1}{p}}$

$$=a^{\frac{1}{p}}+\frac{1}{p}.\frac{b}{a}.A-\frac{p-1}{2p}.\frac{b}{a}B+\frac{2p-1}{3p}.\frac{b}{a}.C-\text{\&c.}$$

COROL. 5. $\overline{a+b}|^{-n}=\frac{1}{\overline{a+b}|^n}$

$$=\frac{1}{a^n}-\frac{nb}{a^{n+1}}+\frac{n.\overline{n+1}.b^2}{1.2\,a^{n+2}}-\frac{n.\overline{n+1}.\overline{n+2}.b^3}{1.2.3\,a^{n+3}}+\text{\&c}$$

$$=\frac{1}{a^n}-\frac{n}{1}.\frac{b}{a}.A+\frac{n+1}{2}.\frac{b}{a}.B-\frac{n+2}{3}.\frac{b}{a}.C+\text{\&c.}$$

COROL. 6. $\overline{a-b}|^{-n}$

$$=\frac{1}{a^n}+\frac{n}{1}.\frac{b}{a}.A+\frac{n+1}{2}.\frac{b}{a}.B+\frac{n+2}{3}.\frac{b}{a}.C+\text{\&c.}$$

COROL. 7. $\overline{a+b}|^{-\frac{m}{p}}$

$$=\frac{1}{a^{\frac{m}{p}}}-\frac{n}{1}.\frac{b}{a}.A+\frac{m+p}{2p}.\frac{b}{a}.B-\frac{m+2p}{3p}.\frac{b}{a}.C+\text{\&c.}$$

QUEST. LVIII. In a series of numbers whose first differences are equal, two terms are given; to find the third?

Let a, b, and c, represent the three terms, of an ascending series:

Then $b-a=c-b$;

Or $c-2b+a=0$.

Th. Case
1. If a, and b; be given, $2b-a=c$;
2. If b, and c; be given, $2b-c=a$;
3. If a, and c; be given, $\frac{a+c}{2}=b$.

QUEST. LIX. In a series of numbers, whose second differences are equal, three terms are given; to find the fourth?

Let a, b, c, and d, represent four terms of an ascending series;

Then

Terms.	First difference.	Second difference.
d		
c	$d-c$	$d-2c+b$
b	$c-b$	$c-2b+a$
a	$b-a$	

Th. $d-2c+b=c-2b+a$,

Or $d-3c+3b-a=0$.

The Case
1. If a, b, and c; be given, $\overline{c-b}\times 3+a=d$;
2. If a, b, and d; be given, $\frac{\overline{d-a}+3b}{3}=c$;
3. If a, c, and d; be given, $\frac{3c-\overline{d-a}}{3}=b$;
4. If b, c, and d; be given, $d-\overline{c-b}\times 3=a$.

QUEST.

QUEST. LX. In a series of numbers, whose third differences are equal; four terms are given to find the fifth?

If a, b, c, d, and e, represent five numbers of an ascending series;

Then

Terms.	First difference.	Second difference.	Third difference.
e			
d	$e-d$	$e-2d+c$	$e-3d+3c-b$
c	$d-c$	$d-2c+b$	$d-3c+3b-a$
b	$c-b$	$c-2b+a$	
a	$c-a$		

Th. $e-3d+3c-b=d-3c+3b-a$;
Or $e-4d+6c-4b+a=0$.

Th. Case:

1. If a, b, c, and d; be given, $e=4d-6c+4b-a$;
2. If a, b, c, and e; be given, $d=\dfrac{e+6c-4b+a}{4}$;
3. If a, b, d, and e; be given, $c=\dfrac{\overline{d+b}\times 4-e-a}{6}$;
4. If a, c, d, and e; be given, $b=\dfrac{e+6c-4d+a}{4}$;
5. If b, c, d, and e; be given, $a=4d-6c-4b-e$.

Corol. If a, b, c, d, e, f, &c. represent a series whose m th differences are equal; then if $\overline{m+1}$ terms be given, another term may be found, by the following equation; *viz.*

$$a-\overline{m+1}\,.\,b+\frac{\overline{m+1}\,.\,m}{1\,.\,2}c-\frac{\overline{m+1}\,.\,m\,.\,\overline{m-1}}{1\,.\,2\,.\,3}d, \text{ \&c. to } \overline{m+2}$$

terms $=0$.

See process in quest. 56.

QUEST. LXI. All the divisors of the number six, are required?

The number six, is composed of the prime numbers two, and three;

Or generally, ab is composed of a and b;

But ab may be divided by 1, a, b, and ab;

Therefore (ab or) 6 has 4 divisors, 1, 2, 3, and 6.

QUEST. LXII. All the divisors of the number 30 (or abc) which is composed of the 3 prime numbers 2, 3, and 5; are required?

abc may be divided by $\begin{cases} 1, \\ a, b, c, \\ ab, ac, bc; \\ abc. \end{cases}$

Th. 30, has 8 divisors, viz. $\begin{cases} 1; \\ 2, 3, 5; \\ 6, 10, 15; \\ 30. \end{cases}$

QUEST. LXIII. It is required to find all the divisors of 210 (or $abcd$) which is composed of the four prime numbers 2, 3, 5, and 7?

$$abcd \text{, may be divided by } \begin{cases} 1; \\ a, b, c, d; \\ ab, ac, ad, bc, bd, cd; \\ abc, abd, acd, bcd; \\ abcd. \end{cases}$$

$$\text{Th. 210 has 16 divisors, viz. } \begin{cases} 1; \\ 2, 3, 5, 7; \\ 6, 10, 14, 15, 21, 35; \\ 30, 42, 70, 105; \\ 210. \end{cases}$$

Corol. Hence a number, or quantity that is composed of n prime numbers, or letters, has 2^n divisors.

Schol. The three last questions relate to elections;

For out of two things proposed, a, and b; neither, either, or both, may be chosen;

Out of three things proposed, a, b, and c; neither any one; any two; or all three, may be chosen: &c.

Therefore the number of elections in n things is 2^n.

QUEST. LXIV. All the divisors of $729=3^6$ are required?

aa has 3 divisors $1 . a, aa$;
aaa has 4 divisors $1 . a, aa, a^3$;
$aaaa$ has 5 divisors $1 . a, aa, a^3, a^4$;
&c. &c.

Th. a^n has $\overline{n+1}$ divisors $1 . a, a^2, a^3$, &c. to a^n,

And $729=3^6$, has 7 divisors, *viz.* 1, 3, 9, 27, 81, 243, 729.

QUEST. LXV. It is required to find the number of the divisors of the quantity $a^n b$?

By last a^n has $n+1$ divisors $1, a, a^2, a^3$, &c.

Beside which $a^n b$ has $b, ab . a^2b . a^3b$, &c.

Th. $a^n b$, has $\overline{n+1} \times 2$ divisors.

QUEST.

QUEST. LXVI. How many divisors has the quantity $a^n bc$?

The divisors of $a^n bc$ are $\begin{cases} 1, & a, & a^2, & a^3, & \&c. \\ b, & ba, & ba^2, & ba^3, & \&c. \\ c, & ca, & ca^2, & ca^3, & \&c. \\ bc, & bca, & bca^2, & bca^3, & \&c. \end{cases}$

Th. $a^n bc$, has $\overline{n+1} \times 4$ divisors.

COROL.

$$\left.\begin{array}{lll} \text{Since } a^n b, & \text{has} & \overline{n+1} \times 2, \\ \text{And } a^n bc, & \text{has} & \overline{n+1} \times 4, \\ \text{Th. } a^n bcd, & \text{has} & \overline{n+1} \times 8, \\ \quad a^n bcde, & \text{has} & \overline{n+1} \times 16, \end{array}\right\} \text{divisors;}$$

And putting the number of single letters, b, c, d, e, and $e = m$.

Then $a^n bcde$, &c. has $\overline{n+1} \times 2^m$, divisors;

And $a^n b^r c^p def$, &c. has $\overline{n+1} \times \overline{r+1} \times \overline{p+1} \times 2^m$, divisors.

QUEST. LXVII. How many ways may the order of n things be varied?

If the given things, 2 in number, be represented by a, and b;

Then they may be placed in two different orders; *viz.* $\left.\right\}$ *ab*, *ba*;

If the number of things be three, *viz.* *a*, *b*, and *c*;

Then they may be placed in six different orders; *viz.* $\left.\right\}$ *abc*, *acb*; *bac*, *bca*; *cab*, *cba*;

If the number of things be four, *viz.* *a*, *b*, *c*, and *d*;

Then they may be placed in twenty-four different orders; *viz.* $\left.\right\}$
abcd, *abdc*, *acbd*, *acdb*, *adcb*, *adbc*;
bacd, *badc*, *bcad*, *bcda*, *bdac*, *bdca*;
cabd, *cadb*, *cbad*, *cbda*, *cdab*, *cdba*;
dabc, *dacb*, *dbac*, *dbca*, *dcab*, *dcba*;

But $2=1\times2$; $6=1\times2\times3$; and $24=1\times2\times3\times4$;

Th. the permutation of n things $=1\times2\times3\times4$, &c. to n.

In

In computing the interest of money;

$$\text{If}\left\{\begin{array}{l} p \\ r \\ n \\ m \end{array}\right\}\text{represent}\left\{\begin{array}{l}\text{the principal,}\\ \text{the interest of 1}l.\text{ for 1 time,}\\ \text{the number of times,}\\ \text{the amount of principal and int.}\end{array}\right.$$

Then

QUEST. LXVII. Given p, r, and n to find m?

Now $pr=$ the interest of p for 1 time,
$2pr=$ Ditto - - - 2,
$3pr=$ Ditto - - - 3;
&c. &c.
Th. $npr=$ the interest of p for n times:
And $p+npr=m$.

QUEST. LXIX. Given p, r, and m; to find n?

(Since $p+npr=m$;) $npr=m-p$;
Th. - - $n=\dfrac{m-p}{pr}$.

QUEST. LXX. Given p, n, and m, to find r?

(Since $npr=m-p$;) Th. $r=\dfrac{m-p}{np}$.

QUEST.

QUEST. LXXI. Given m, r, and n; to find p.

(Since $p+npr=m$.) Th. $p=\frac{m}{1+nr}$.

QUEST. LXXII. An usurer lent 186*l.* for x months and gained thereby 31 pounds; and at the same rate of interest lending 360*l.* for y months, he gained 90 pounds; the values of x and y are required; when $x+y=20$?

If $r=$ the interest of 1 *l.* for 1 year;
Then $186r=$ the interest of 186 for 1 year,
And $360r=$ Ditto - 360;

$$\text{Also}\left\{\begin{array}{l}\frac{186rx}{12}=\text{Ditto - 186 for } x\\[1ex] \frac{360ry}{12}=\text{Ditto - 360 for } y\end{array}\right\}\text{Months.}$$

Th. $\frac{186rx}{12}=31$; Or $\frac{r}{12}=\left(\frac{31}{186x}=\right)\frac{1}{6x}$.

Ad $\frac{360ry}{12}=90$; Or $\frac{r}{12}=\left(\frac{90}{360y}=\right)\frac{1}{4y}$:

Th. - $\frac{1}{6x}=\frac{1}{4y}$; Or $4y=6x$:

But $x+y=20$; Th. $x=20-y$;

Whence $4y=(6\times\overline{20-y})\ 120-6y$,

Or - $10y=120$; Th. $y=12$.

QUEST.

QUEST. LXXIII. The duties of certain goods amounted to 2460*l.* out of which a discount of $2\frac{1}{2}$ per cent. was allowed, in consideration of prompt payment, on the sum actually paid: What did the discount amount to?

Let $2460=m$; $\left(\frac{2,5}{100}=\right)\frac{1}{40}=r$; $1=n$; and $d=$ discount required.

(Where m, r, and n; are of the same kind as in the last questions; that is $m-d=p$:)

Then by quest. 71. $m-d=\frac{m}{1+nr}$;

Th. $\left(m-\frac{m}{1+nr}=\right)\frac{mnr}{1+nr}=d$.

In this example - $d=\left(\frac{2460\times\frac{1}{40}}{1+\frac{1}{40}}=\frac{2460}{41}=\right)60$.

QUEST. LXXIV. A person has now due to him 320*l.* and at the end of 5 years 96*l.* more, will be due from the same debtor; they agree that the whole shall be discharged at one payment, at that time when the simple interest of the 320*l.* shall be equal to the discount of the 96*l.* both being calculated at 5*l.* per cent. per annum: The time of payment is required?

Let

Let $320=p$; $5=t$; $96=m$; $\left(\frac{5}{100}=\right)\frac{1}{20}=r$;
And $t-x=$ time required;

Then by quest. 68. $\overline{t-x}\times pr=$ interest of p for $t-x$;

And by quest. 73. $\frac{mxr}{1+xr}=$ discount of m for x;

$$\text{Th.}\quad (\overline{t-x}\times pr=)\ tpr-xpr=\frac{mxr}{1+xr},$$

$$\text{Or}\quad tp-xp=\frac{mx}{1+xr},$$

$$\text{Or}\quad tp-xp+tpxr-x^2pr=mx,$$

$$\text{Or}\quad \left.\begin{array}{r} prx^2+m \\ +p \\ -tpr \end{array}\right\} x=tp;$$

$$\text{Th.}\quad x^2+\frac{m+p-tpr}{pr}x=\left(\frac{tp}{pr}=\right)\frac{t}{r}:$$

$$\text{Substitute}\quad \frac{m+p-tpr}{pr}=2b,$$

$$\text{Then}\quad xx+2bx=\frac{t}{r}:$$

$$\text{But}\quad xx+2bx+bb=\frac{t}{r}+bb;$$

$$\text{Th.}\quad x+b=\sqrt{\frac{t}{r}+bb};\ \text{and}\ x=\sqrt{\frac{t}{r}+bb}-b.$$

In this example $\left(\frac{96+320-5\,.\,320\times\frac{1}{20}}{320\times\frac{1}{20}}=\right)21=2b.$

$$\text{And}\ x=\sqrt{5\times 20+\frac{21\times 21}{4}}-\frac{21}{2}=4;$$

Th. $(5-4=)$ 1 year $=$ time required.

QUEST.

QUEST. LXXV. The diſcounting of a bill came to 5*l.* 12*s.* 0*d.* Had the rate per cent been 1*l.* more, it would have coſt 6*l.* 6*s.* 0*d.* and if the rate per cent. had been 1*l.* leſs, only 4*l.* 16*s.* 0*d.*

The value of the bill, time when due, and rate of intereſt are required?

Let $p=$ principal, $t=$ time, and $r=$ int. of 100*l.* in 1 year, $a=5$*l.* 12*s.* 0*d.* $b=6$*l.* 6*s.* 0*d.* and $c=4$*l.* 16*s.* 0*d.*

Then by queſt. 73. $\left\{\begin{array}{l} \dfrac{rtp}{100+rt}=a \\ \dfrac{\overline{r+1}\times tp}{100+\overline{r+1}\times t}=b \\ \dfrac{\overline{r-1}\times tp}{100+\overline{r-1}\times t}=c \end{array}\right\}$ therefore $\left\{\begin{array}{l} p=\dfrac{100a+art}{rt}, \\ p=\dfrac{100b+bt\times\overline{r+1}}{\overline{r+1}\times t}, \\ p=\dfrac{100c+ct\times\overline{r-1}}{\overline{r-1}\times t}. \end{array}\right.$

Th. $\dfrac{100a+art}{r}=\dfrac{100b+bt\times\overline{r+1}}{r+1}$,

Or $\overline{r+1}\times 100a+\overline{r+1}\times art=100br+btr\times\overline{r+1}$;

Th. $\dfrac{\overline{r+1}\times 100a-100br}{\overline{b-a}\times r\times\overline{r+1}}=t$;

Alſo

Let $320=p$; $5=t$; $96=m$; $\left(\frac{5}{100}=\right)\frac{1}{20}=r$;

And $t-x=$ time required;

Then by queſt. 68. $\overline{t-x}\times pr=$ intereſt of p for $t-x$;

And by queſt. 73. $\frac{mxr}{1+xr}=$ diſcount of m for x;

Th. $\left(\overline{t-x}\times pr=\right)tpr-xpr=\frac{mxr}{1+xr}$,

Or - - $tp-xp=\frac{mx}{1+xr}$,

Or $tp-xp+tpxr-x^2pr=mx$,

Or - $\left.\begin{array}{r} prx^2+m \\ +p \\ -tpr \end{array}\right\} x=tp$;

Th. $x^2+\frac{m+p-tpr}{pr}x=\left(\frac{tp}{pr}=\right)\frac{t}{r}$:

Subſtitute - $\frac{m+p-tpr}{pr}=2b$,

Then - - $xx+2bx=\frac{t}{r}$: -

But - $xx+2bx+bb=\frac{t}{r}+bb$;

Th. $x+b=\sqrt{\frac{t}{r}+bb}$; and $x=\sqrt{\frac{t}{r}+bb}-b$.

In this example $\left(\frac{96+320-5\cdot 320\times\frac{1}{20}}{320\times\frac{1}{20}}=\right)21=2b$.

And $x=\sqrt{5\times 20+\frac{21\times 21}{4}}-\frac{21}{2}=4$;

Th. $(5-4=)$ 1 year = time required.

QUEST.

QUEST. LXXV. The diſcounting of a bill came to 5 *l.* 12 *s.* 0 *d.* Had the rate per cent been 1 *l.* more, it would have coſt 6 *l.* 6 *s.* 0 *d.* and if the rate per cent. had been 1 *l.* leſs, only 4 *l.* 16 *s.* 0 *d.*

The value of the bill, time when due, and rate of intereſt are required?

Let $p=$ principal, $t=$ time, and $r=$ int. of 100 *l.* in 1 year, $a=5$ *l.* 12 *s.* 0 *d.* $b=6$ *l.* 6 *s.* 0 *d.* and $c=$ 4 *l.* 16 *s.* 0 *d.*

$$\text{Then by queſt. 73.}\left\{\begin{array}{l} \dfrac{rtp}{100+rt}=a \\ \dfrac{\overline{r+1}\times tp}{100+\overline{r+1}\times t}=b \\ \dfrac{\overline{r-1}\times tp}{100+\overline{r-1}\times t}=c \end{array}\right\}\text{therefore}\left\{\begin{array}{l} p=\dfrac{100a+art}{rt}, \\ p=\dfrac{100b+bt\times\overline{r+1}}{\overline{r+1}\times t}, \\ p=\dfrac{100c+ct\times\overline{r-1}}{\overline{r-1}\times t}. \end{array}\right.$$

Th. $\dfrac{100a+art}{r}=\dfrac{100b+bt\times\overline{r+1}}{r+1}$,

Or $\overline{r+1}\times 100a+\overline{r+1}\times art=100br+btr\times\overline{r+1}$;

Th. $\dfrac{\overline{r+1}\times 100a-100br}{\overline{b-a}\times r\times\overline{r+1}}=t$:

Alſo

QUEST. LXXVII. To compute the amount of an annuity in arrear by simple interest?

$$\text{Let}\left\{\begin{array}{c} a \\ r \\ n \\ m \end{array}\right\} \text{represent} \left\{\begin{array}{l} \text{the annuity;} \\ \text{the interest of 1}l.\text{ for 1 time,} \\ \text{the number of times,} \\ \text{the sum due at the end of } n \text{ times;} \end{array}\right.$$

$$\text{Then}\left\{\begin{array}{l} a \\ a+ar \\ a+2ar \\ a+3ar \\ \quad\&c. \\ a+\overline{n-1}\times ar \end{array}\right\} \begin{array}{c}\text{becomes due} \\ \text{in the}\end{array} \left\{\begin{array}{c} \text{first} \\ \text{second} \\ \text{third} \\ \text{fourth} \\ \&c. \\ n\text{th} \end{array}\right\} \text{time;}$$

Th. their Sum $na+\frac{n\,.\,\overline{n-1}}{1\,.\,2}ar=m.$ See quest. 3.

QUEST. LXXVIII. Given a, n, and m; to find r?

From above $\frac{n\,.\,\overline{n-1}}{1\,.\,2}ar=m-na$;

Th. - - $r=\frac{\overline{m-na}\times 2}{n\,.\,\overline{n-1}\,.\,a}$.

QUEST. LXXIX. Given m, n, and r; to find a?

From above $2na+n\,.\,\overline{n-1}\,.\,ar=2m$;

Th. - - $\frac{2m}{2n+n\,.\,\overline{n-1}\,.\,r}=a$,

Or - - $\frac{2m}{\overline{2+\overline{n-1}\,.\,r}\times n}=a$.

QUEST.

QUEST. LXXX. Given a, r, and m; to find n?

From above $2na+n\,.\,\overline{n-1}\,.\,ar=2m$,

That is $2na+nnar-nar=2m$,

Or $arnn+\overline{2-r}\times an=2m$,

Or - $nn+\frac{2-r}{r}n=\frac{2m}{ar}$:

But - $nn+\frac{2-r}{r}n+\overline{\frac{2-r}{2r}}\Big|^2=\frac{2m}{ar}+\overline{\frac{2-r}{2r}}\Big|^2$:

Th. - - $n+\frac{2-r}{2r}=\sqrt{\frac{2m}{ar}+\overline{\frac{2-r}{2r}}\Big|^2}$,

And $\sqrt{\frac{2m}{ar}+\overline{\frac{2-r}{2r}}\Big|^2}-\frac{2-r}{2r}=n$.

QUEST. LXXXI. The amount of an annuity in arrear computed at simple interest was 215*l.* now if the annuity had continued unpaid one year longer, it would have amounted to 275*l.* but if one year less, to no more than 157*l.* 10*s.* the annuity, time, and rate are required?

If $a=$ annuity; $n=$ time; and $r=$ interest of 1*l.*
Also $215=b$; $275=c$; and $157,5=d$;

$$\text{Then}\left\{\begin{array}{l} d=\overline{n-1}\times a+\frac{\overline{n-1}\,.\,\overline{n-2}}{1\,.\,2}ar, \\ b=na+\frac{n\,.\,\overline{n-1}}{1\,.\,2}ar, \\ c=\overline{n+1}\times a+\frac{\overline{n+1}\,.\,n}{1\,.}ar, \end{array}\right\}\text{by quest. 77.}$$

Or

QUEST. LXXXIV. and LXXXV. In a geometrical progression are given a, r, and z; to find n and s?

By quest. 82; $A+\overline{n-1}\times R=Z$;

Th. - - - - $n=\frac{Z-A}{R}+1$.

From Equat. in qu. 83. $s-a=sr-zr$,

Or - - $zr-z+z-a=sr-s$,

Or - - $\overline{r-1}\times z+z-a=\overline{r-1}\times s$;

Th. - $z+\frac{z-a}{r-1}=s$.

QUEST. LXXXVI. and LXXXVII. In a geometrical progression are given a, r, and s; to find z, and n?

By Equat. 2; in quest. 85. $zr-a=sr-s$;

Th. - - - - - $z=\frac{sr-s+a}{r}$,

Or - - - - - $z=s-\frac{s-a}{r}$.

By quest. 82. - - $z=ar^{n-1}$;

By quest. 87. - - $z=\frac{sr-s+a}{r}$;

Th. - - - - $ar^{n-1}=\frac{sr-s+a}{r}$;

Or - - - - $ar^{n}=sr-s+a$;

Th. - - - - $r^{n}=\frac{\overline{r-1}\times s}{a}+1$;

In

In Logarithms; Let $\frac{\overline{r-1}\times s}{a}+1=b$;

Then - - - $n=\frac{B}{R}$.

QUEST. LXXXVIII. and LXXXIX. In a geometrical progression are given, a, z, and s; to find r and n?

From equation first in quest. 83. $s-a=sr-zr$;

Th. - - - - - $\frac{s-a}{s-z}=r$.

By last; $L.\overline{s-a}-L.\overline{s-z}=R$;

By quest. 84. - $n=\frac{Z-A}{R}+1$;

Th. - - $n=\frac{Z-A}{L.\overline{s-a}-L.\overline{s-z}}+1$;

QUEST. XC. and XCI. In a geometrical progression are given, a, n, and z; to find r and s?

By quest. 82. $A+\overline{n-1}\times R=Z$;

Th. - - - $R=\frac{Z-A}{n-1}$;

By last; - - $R=\frac{Z-A}{n-1}$;

By quest. 85; - $s=z+\frac{z-a}{r-1}$.

QUEST. XCII. and XCIII. In a geometrical progression are given, a, n, and s; to find r; and z?

By equation 4th quest. 87. $ar^n = sr - s + a$;

Th. - $ar^n - sr + \overline{s-a} = 0$:

But $\dfrac{ar^n - sr + \overline{s-a}}{r-1} = ar^{n-1} + ar^{n-2}$, &c. to $ar - \overline{s-a}$;

Th. $ar^{n-1} + ar^{n-2} + ar^{n-3}$, &c. to $ar - \overline{s-a} = 0$.

If $r = -1$; and n be an $\left\{\begin{matrix}\text{even}\\ \text{odd}\end{matrix}\right\}$ N° $\left\{\begin{matrix}s\\ s-a\end{matrix}\right\} = 0$.

If $r = 0$; Then - - - $s - a = 0$,

* If $r = 1$; Then - - $s - na = 0$;

Th. (r being either a whole number or rational fraction) when n is an $\left\{\begin{matrix}\text{even}\\ \text{odd}\end{matrix}\right\}$ number, if among the respective divisors of $\left\{\begin{matrix}s\\ s-a\end{matrix}\right\}$, $\overline{s-a}$, and $\overline{s-na}$; there be found three, that differ by d (some divisor of a): (That is n being $\left\{\begin{matrix}\text{even}\\ \text{odd}\end{matrix}\right\}$ if $\overline{m+d}$, m and $\overline{m-d}$, where $m =$ any whole number, are severally divisors of $\left\{\begin{matrix}s\\ s-a\end{matrix}\right\}$, $\overline{s-a}$, and $\overline{s-na}$): Then $\dfrac{m}{d} = r$.

From above r (if rational) $= \dfrac{m}{d}$;

By quest. 86 - $z = s - \dfrac{s-a}{r}$.

* *This is an application of Sir* Isaac Newton's *method of finding divisors (before cited in part 1st) to the general solution of this question.*

Exam-

Example; Required the Ratio of that geometrical progression, whose least term is 5; number of terms 10; and sum 5115?

$$\left.\begin{array}{l} s \quad\;\; =5115, \\ s-\;a=5110, \\ s-na=5065, \end{array}\right\} \text{Its divisors} \left\{\begin{array}{l} 1,\ 3,\ 5,\ 11,\ 15,\ 31, \\ 1,\ 2,\ 5,\ \ 7,\ 10,\ 35; \\ 1,\ 5,\ 1013; \end{array}\right.$$

Where $\left\{\begin{array}{l} 3,\ \ 2,\ 1 \\ 15,\ 10,\ 5 \end{array}\right\}$ differ by $\left\{\begin{array}{l} 1 \\ 5 \end{array}\right\}$ a divisor of 5;

And $\frac{2}{1}=\frac{10}{5}=2=r$.

QUEST. XCIV. and XCV. In a geometrical progression are given, z, r, and n; to find a, and s?

By quest. 82. $A+\overline{n-1}\times R=Z$;

Th. - - - $A=Z-\overline{n-1}\times R$.

By equation 1st in quest. 83. $s-a=sr-zr$;

And (by quest. 82.) - - $ar^{n-1}=z$,

Or - - - - - $a=\frac{z}{r^{n-1}}$;

Th. - - - - $s-\frac{z}{r^{n-1}}=sr-zr$,

Th. $\left(\frac{zr^{n}-z}{r^{n}-r^{n-1}}=\right)\frac{\overline{r^{n}-1}\times z}{\overline{r-1}\times r^{n-1}}=s$.

In Logarithms (if $nR=B$)

Then $L.\overline{b-1}+Z-L.\overline{r-1}-\overline{n-1}\times R=S$.

QUEST. XCVI. and XCVII. In a geometrical progression are given, z, r, and s; to find a, and n?

By equation in quest. 83. $s-a=sr-zr$;

Th. - - $s-\overline{s-z}\times r=a$.

By equ. 5. quest. 87; $r^n=\frac{\overline{r-1}\times s}{a}+1$;

Th. (by above) $r^n=\left(\frac{\overline{r-1}\times s}{s-\overline{s-z}\times r}+1=\right)\frac{zr}{s-\overline{s-z}\times r}$:

In Logarithms: let $b=\frac{zr}{s-\overline{s-z}\times r}$;

Then - - $r=\frac{B}{R}$.

QUEST. XCVIII. and XCIX. In a geometrical progression are given, n, r, and s; to find a, and z?

By equation 4. quest. 87. $ar^n=sr-s+a$,

Th. - - - $a=\frac{s\times\overline{r-1}}{r^n-1}$:

In logarithms (if $nR=B$) $A=S+L.\overline{r-1}-L.\overline{b-1}$.

By quest. 95. $s=\frac{\overline{r^n-1}\times z}{\overline{r-1}\times r^{n-1}}$;

Th. $\frac{r^{n-1}\times\overline{r-1}\times s}{r^n-1}=z$:

In logarithms; if $nR=B$;

Then - - $Z=\overline{n-1}\times R+L.\overline{r-1}+S-L.\overline{b-1}$.

QUEST.

QUEST. C. and CI. In a geometrical progression are given, z, n, and s; to find r and a?

By quest. 95. $zr^n - z = sr^n - sr^{n-1}$;

Th. - - - $0 = \overline{s-z} \times r^n - sr^{n-1} + z$;

But $\frac{\overline{s-z} \times r^n - sr^{n-1} + 1}{r-1} = \overline{s-z} \times r^{n-1} - zr^{n-2} - zr^{n-3}$, &c. to z;

Th. $\overline{s-z} \times r^{n-1} - zr^{n-2} - zr^{n-3} - zr^{n-4}$, &c. to $-zr - z = 0$.

If $r = -1$; and n an $\left\{\begin{matrix}\text{even}\\\text{odd}\end{matrix}\right\}$ N° $\left\{\begin{matrix}s\\s-z\end{matrix}\right\} = 0$,

If $r = 0$; then - - - $z = 0$,

If $r = 1$; then - - $nz - s = 0$;

Th. (r being either an integer or rational fraction) when n is an $\left\{\begin{matrix}\text{even}\\\text{odd}\end{matrix}\right\}$ number, if among the respective divisors of $\left\{\begin{matrix}s\\\overline{s-z}\end{matrix}\right\}$, z, and $\overline{nz-s}$; there be found three that differ by d (some divisor of $\overline{s-z}$): that is (n being $\left\{\begin{matrix}\text{even}\\\text{odd}\end{matrix}\right\}$ if $\overline{m+d}$, m, $\overline{m-d}$, where $m =$ any whole number, are severally divisors of $\left\{\begin{matrix}s\\s-z\end{matrix}\right\}$ z; and $\overline{nz-s}$); then $\frac{m}{d} = r$.

By last; r, (if rational) $= \frac{m}{d}$;

By quest. 96. - $a = s - \overline{s-z} \times r$.

Example; Required the ratio of that geometrical progression, whose greatest term is 2560; number of terms 10; and sum 5115?

$$\left.\begin{matrix} s = 5115, \\ z = 2560, \\ nz - s = 20485. \end{matrix}\right\} \text{its diviosrs} \left\{\begin{matrix} 1, 3, 5, 11, 15, 31, \text{\&c.} \\ 1, 2, 4, 5, 8, 10, 16, \text{\&c.} \\ 1, 5, 17, 85, 241, \text{\&c.} \end{matrix}\right.$$

And $\frac{2}{1} = \frac{10}{5} = 2 = r$.

M 3 QUEST.

QUEST. CII. In a geometrical progreſſion are given, the ratio, number of terms, and product of the firſt and laſt term, to find the terms?

Here r, n, and az, are given; to find the reſt;
By queſt. 82. $z=ar^{n-1}$;
Th. $az=aar^{n-1}$; Hence $\sqrt{\frac{az}{r^{n-1}}}=a$.

QUEST. CIII. Three numbers in geometrical progreſſion are required, ſo that the difference of the firſt and ſecond may be 6; and of the ſecond and third 15?

If x, y, and z, be the numbers required;
Then $y-x=6$; Or $y-6=x$;
And $z-y=15$; $z=15+y$:
But $x:y::y:z$,
That is $y-6:y::y:15+y$;
Th $yy+9y-90=yy$, Th. $y=10$.

QUEST. CIV. It is required to find three numbers in geometrical progreſſion, the ſum of the firſt and ſecond of which may be 14; and of the ſecond and third 35?

If x, y, and z, be the numbers required;
Then $x+y=14$; Or $x=14-y$;
And $y+z=35$; Or $z=35-y$:
But $x:y::y:z$,
That is $14-y:y::y:35-y$;
Th. $490-49y+yy=yy$, Th. $10=y$.

QUEST.

QUEST. CV. There are three numbers in geometrical progression whose product is 512; and the sum of the first and last 34: What are the numbers?

If x, y, and z, be the numbers required;
Then $x : y :: y : z$; Th. $xz = yy$;
And $(xyz = xz \times y =)$ $y^3 = 512$;
Th. - - $y = 8$:
And - $(yy =)$ $xz = 64$.
But by quest. - $z + x = 34$;

Th. by qu. 159. $\left\{\begin{array}{l} z = \overline{34 + \sqrt{34 \times 34 - 4 \times 64}} \times \frac{1}{2} = 32: \\ x = 34 - \sqrt{34 \times 34 - 4 \times 64} \times \frac{1}{2} = 2. \end{array}\right.$

QUEST. CVI. What three numbers in geometrical progression are those, whose sum is 95; and the sum of their squares 3225?

If x, y, and z, be the numbers required;
Then $x : y :: y : z$; Th. $xz = yy$;
And $x + y + z = 95$; Th. $x + z = 95 - y$;
Also $xx + yy + zz = 3325$; Th. $xx + xz + z^2 = 3325$.
Now (by 2d) $xx + 2xz + zz = 9025 - 190y + yy$,
And (by first) - - $xz =$ - yy;
Th. $xx + xz + zz = 9025 - 190y$ by subtraction;
Whence $3325 = 9025 - 190y$;
Th. - $y = \left(\frac{9025 - 3325}{190} =\right) 30$:
Th. $x + z = (95 - 30 =)\ 65$; and $xz = (30 \times 30 =)\ 900$:

And by qu. 159. $\left\{\begin{array}{l} x = \overline{(65 + \sqrt{65 \times 65 - 4 \times 900}} \times \frac{1}{2} =)\ 45; \\ z = (65 - \sqrt{65 \times 65 - 4 \times 900} \times \frac{1}{2} =)\ 20. \end{array}\right.$

QUEST. CVII. There are three numbers in geometrical progression; if the second of them be taken from the sum of the first and third, and that difference be severally multiplied by the sum of the first and third, and by the sum of the three numbers; the products will be 1120, and 1456: What are those numbers?

If x, y, and z, represent the numbers required;
And $x+z-y$ (the said difference) $=u$;

Then $\overline{x+z}\times u=1120$; Or $x+z=\frac{1120}{u}$;

And $\overline{x+z}\times u+uy=1456$; Or $uy=336$;

But $(\overline{x+z}-\overline{x+z-y}=)\,y=\frac{1120}{u}-u$;

Th. $\frac{1120}{u}-u=(y=)\,\frac{336}{u}$, (by second)

Th. - $28=(u=)\,x+z-y$, and $xz=yy=144$,
And - $y=12$,
Also $x+z=40$:

Th. by quest. 159. $\left\{\begin{array}{l} z=(40+\sqrt{40^2-4\times144}\times\frac{1}{2}=)\ 36, \\ x=(40-\sqrt{40^2-4\times144}\times\frac{1}{2}=)\ 4. \end{array}\right.$

QUEST. CVIII. There are three numbers in geometrical progression, the third exceeds the first by 15; and the sum of their square is 525: What are those numbers?

If x, y, and z, represent the numbers required?
Then $x+15=z$; And $\overline{x+15}^2=zz$:
But $x:y::y:x+15$; Th. $xx+15x=yy$;
By quest. $xx+\overline{xx+15x}+\overline{xx+30x+225}=525$,

But $\overline{xx+15x+\frac{15}{2}}^2=\left(\frac{225}{4}+100=\right)\frac{625}{4}$;

Th. $x+\frac{15}{2}=\frac{25}{2}$; And $x=\left(\frac{25-15}{2}=\right)5$.

QUEST.

QUEST. CIX. Of three numbers in geometrical progression, there are given the sum of the first and second 14; and the difference of the third and second 15; To find the numbers?

If x, y, and z, be the numbers required;
Then $x+y=14$; Or $x=14-y$;
And $z-y=15$; Or $z=15+y$:
But $x:y::y:z$; per qu. That is $14-y:y::y:15+y$;
Th. $210-y-yy=yy$,
Or $2yy+y=210$, Or $yy+\frac{1}{2}y=105$:
But $yy+\frac{1}{2}y+\frac{1}{16}=(105+\frac{1}{16}=)\frac{1681}{10}$;
Th. $y+\frac{1}{4}=\frac{41}{4}$; And $y=\left(\frac{41-1}{4}=\right)10$.

QUEST. CX. There are three numbers in geometrical progression, the greatest of which exceeds the least by 15; also the difference of the squares of the greatest and least numbers, is to the sum of the squares of all the three numbers; as 5, to 7?

If x, y, and z, represent the three numbers;
Then $x+15=z$; by quest.
And $x:y::y:x+15$; Th. $xx+15x=yy$;
Now $(\overline{x+15}^2-xx=)\ 30x+225=zz-xx$,
And $(x^2+x^2+15x+\overline{x+15}^2=)\ 3x^2+45x+225=xx+yy$ [$+zz$;
Th. $30x+225:3x^2+45x+225::5:7$ per quest.
Th. $210x+1575=15xx+225x+1125$,
Th. $30=xx+x$; And $5=x$.

QUEST. CXI. The sum (35) of three numbers in geometrical progression, and the proportion of the mean term to the difference of the extremes (as 2 to 3) being given; to find the numbers?

If x, y, and z, be the numbers required;
Then $x+y+z=35$; Or $y=35-x-z$;
And $y : z-x :: 2 : 3$; Or $3y=2z-2x$;

Th. $2z-2x=(3y=)\ 105-3x-3z$, And $z=\frac{105-x}{5}$:

Now $3y=\left(\frac{210-2x}{5}-2x=\right)\frac{210-12x}{5}$; And $y=\frac{70-4x}{5}$.

Th. $y=\frac{70-4x}{5}$.

But by qu. $x : \frac{70-4x}{5} :: \frac{70-4x}{5} : \frac{105-x}{5}$;

Th. $\frac{105x-xx}{5}=\frac{4900-560x+16xx}{25}$,

Th. $xx-\frac{155}{3}x=-\frac{700}{3}$: And $5=x$.

QUEST. CXII. There are three numbers in geometrical progression, whose sum is 28 and the sum of their cubes 4672; What are those numbers?

If x, y, and z, denote the numbers required;

Then by quest. $\begin{cases} x+y+z=28, \\ x^3+y^3+z^3=4672; \end{cases}$

Or by Transposition $\begin{cases} x+z=28-y, \\ x^3+z^3=4672-y^3; \end{cases}$

Th. - - $\overline{x+z}^3=\overline{28-y}^3$:

Now (the diff. of two last is) $3x^2z+3xz^2=17280-2352y+84y^2$,

But $x+z=28-y$; And $xz=yy$;

Th. - $\overline{28-y}\times 3yy=17280-2352y+84y^2$,

Th. $y^3-784y+5760=0$: Where $y=8$,

Now $(yy=)\ xz=64$; and $28-y=)\ x+z=20$;

Th. by qu. $\begin{cases} z=\overline{20+\sqrt{20\times 20-4\times 64}\times\frac{1}{2}}=16, \\ x=\overline{20-\sqrt{20\times 20-4\times 64}\times\frac{1}{2}}=4. \end{cases}$

159

QUEST.

QUEST. CXIII. There are three numbers in geometrical progression, whose sum is 14; and the difference of the squares of the greatest and least is 60: What are those numbers?

If $x=$ the least number; and y the ratio;
Then x, xy, and xy^2, will be the numbers required;

Now by quest. $\begin{cases} x+xy+xy^2=14, \\ x^2y^4-xx=60; \end{cases}$

Th. - - $\begin{cases} x=\dfrac{14}{1+y+yy} \\ xx=\dfrac{60}{y^4-1}: \end{cases}$

But (by 3d) $\dfrac{196}{1+2y+3y^2+2y^3+y^4}=xx$;

Th. - $\dfrac{196}{1+2y+3y^2+2y^3+y^4}=\dfrac{60}{y^4-1}$;

Whence $34y^4-30y^3-45y^2-30y-64=0$, by reduction,

But $\dfrac{34y^4-30y^3-45y^2-30y-64}{y-2}=34y^3+38y^2+31y+32$

Th. $y=2$. And $x=\left(\dfrac{14}{1+2+4}=\right)2$.

So the the numbers are 2. 4. 8.

QUEST. CXIV. It is required to find four mean proportionals, between 5 and 160?

Let $r=$ the ratio of the progression,
Then 5, $5r$, $5rr$, $5r^3$, $5r^4$, $(160=)\ 5r^5$, are six terms in geometrical progression;

Th. $\left(\dfrac{160}{5}=\right)32=r^5$,

Th. $(\overline{32}|^{\frac{1}{5}}=)\ 2=r$;

Whence $5r=10$; $5r^2=20$; $5r^3=40$; and $5r^4=80$: are the numbers required.

QUEST. CXV. There are four numbers in geometrical progression, the sum of the first and third is 25; and the sum of the second and fourth is 50: What are those numbers?

If x, y, u, and z, represent the numbers required;
Then $x+u=25$; and $y+z=50$ per question;
Also $x:y::u:z$,
Or $x:u::y:z$,
Or $x:x+u::y:y+z$;
Th. $x:\ y\ ::x+u:y+z$;
That is $x:\ y\ ::\ 25\ :\ 50$;
Th. - $50x=25y$; and $2x=y$:
Also ($x:2x::2x:$) $4x=u$:
But ($x+4x=$) $5x=25$; Th. $x=5$.

QUEST. CXVI. What four numbers in geometrical progression, the sum of the two least of which is 15; and the sum of the two greater 60?

If x, y, u, and z, represent those numbers;
Then $x+y=15$; and $u+z=60$ per quest.
But $x:y::y:u$,
Th. $x+y:y::y+u:u$;
Also $y\ :u::\ u\ :z$;
Th. $y+u:u::u+z:z$;
Whence $x+y:y+u::y+u:u+z$,
That is $15:y+u::y+u:60$;
Th. ($15\times60=$) $900=\overline{y+u}^2$,
And - $30=y+u$:
Now by second $15:y::30:u$,
That is - $15u=30y$, And $u=2y$,
Th. ($y+2y=$) $3y=30$, And $y=10$.

QUEST.

QUEST. CXVII. There are four numbers in geometrical progression, the product of the two least terms is 50; and of the two greatest 800: What are those numbers?

If x, y, u, and z, represent those numbers;

$$\left.\begin{array}{ll} \text{Then } xy=50; & \text{Or } x=\frac{50}{y}, \\ \quad uz=800; & \text{Or } z=\frac{800}{u}, \\ \text{And } xz=uy; & \text{Or } \frac{50}{y}\times\frac{800}{u}=uy; \end{array}\right\}\text{per quest.}$$

Th. $50\times800=u^2y^2$,

Th. $50\times\ \ 4=uy$, And $\frac{200}{y}=u$;

But $x:y::y:u$,

That is $\frac{50}{y}:y::y:\frac{200}{y}$;

Th. $\frac{10000}{yy}=yy$, Th. $10=y$.

QUEST. CXVIII. It is required to find four numbers in geometrical progression, so that the sum of the means may be 30; and the sum of the extremes 45?

If x, y, u, and z, represent the numbers required;

$$\text{Then}\left\{\begin{array}{ll} y+u=30; & \text{Or } u=30-y, \\ x:y::y:u; & \text{Or } x=\frac{yy}{30-y}; \\ y:u::u:z; & \text{Or } z=\frac{\overline{30-y}^2}{y}; \end{array}\right.$$

But $(x+z=)\ \frac{yy}{30-y}+\frac{\overline{30-y}^2}{y}=45$ by quest.

And $y=10$; $u=20$; $x=5$; $z=40$.

QUEST.

QUEST. CXXII. There are four numbers in geometrical progression whose sum is 75; and the sum of the squares of the extremes is 1625?

Let x, y, u, and z, represent the numbers required;
And let $y+u=\alpha$; Then $x+z=75-\alpha$:

Now - $(\overline{x+z}^2=)\ 1625+2xz=5625-150\alpha+\alpha^2$,
Or $2uy(=2xz)\ =4000-150\alpha+\alpha^2$:

Now (per last) $y^3+y^2u+yu^2+u^3=75uy$,
And - $y^3+y^2u+yu^2+u^3=\overline{yy+uu}\times\alpha$,
Also - $yy+2uy+uu=\alpha^2$;
The diff. of third and last $yy+uu=150\alpha-4000$;
Th. - $y^3+y^2u+yu^2+u^3=150\alpha^2-4000\alpha$,
Whence - - $75uy=150\alpha^2-4000\alpha$,
Or - - - - $3uy=\ 6\alpha^2-160\alpha$;

Th. - $\dfrac{6\alpha^2-160\alpha}{3}=(uy=)\dfrac{4000-150\alpha+\alpha^2}{2}$,

Th. - $\alpha=\left(\dfrac{335-65}{9}=\right)\ 30=y+u$;

And - - $uy=\dfrac{6\times30\times30-160\times30}{3}=200.$

Th. - $u=20\ y=10.$ as in the last.

QUEST.

QUEST CXXIII. There are four numbers in geometrical progression, the sum of the squares of the two means of which is 500; and the sum of the squares of the extremes 1625: What are those numbers?

If x, y, u, and z, represent the numbers required;

Now becouse x, y, u, and z, } are in geometrical
Therefore x^2, y^2, u^2, and z^2, } progression.

Then by qu. 116. $x^2+y^2 : y^2+u^2 :: y^2+u^2 : u^2+z^2$:

But - $\left.\begin{array}{l} x^2+z^2=1625, \\ y^2+u^2=\ \ 500, \end{array}\right\}$ by quest.

Th. - - $x^2+y^2+u^2+z^2=2125$:

Now (if $x^2+y^2=a$; Then) $u^2+z^2=2125-a$;

And - $a : 500 :: 500 : 2125-a$;

Th. - $2125a-a^2=2500$:

Whence $(a=)\ x^2+y^2=125$:

And - $u^2+z^2=(2125-125=)\ 2000$:

But by qu. 116. $x^2+y^2 : y^2 :: y^2+u^2 : u^2$,

That is - $125 : y^2 :: 500 : u^2$,

Th. $125u^2=500y^2$; Or $u^2=4y^2$:

But $(u^2+y^2=4y^2+y^2=)\ 5y^2=500$;

Th. - - $y^2=100$:

And - - $y=10$.

QUEST.

QUEST. CXXIV. The ſum 75, of four numbers in geometrical progreſſion; and the ſum of their ſquares, 2125, being given; to find the numbers?

Let x, y, u, and z, repreſent thoſe numbers;

Then $x+y+u+z=75$, And $x^2+y^2+u^2+z^2=2125$, by queſt.

Now $\overline{x+y+u+z}^2-\overline{x^2+y^2+u^2+z^2}=3500$,

Or $2xy+2xu+2xz+2yu+2yz+2uz=3500$,

Or $xy+\left\{\begin{matrix}xu\\yy\end{matrix}\right\}+\left\{\begin{matrix}xz\\yu\end{matrix}\right\}+yu+\left\{\begin{matrix}yz\\uu\end{matrix}\right\}+uz=1750$,

Or $xy+yy+2yu+uu+uz=1750$; and if $a=y+u$

Then - - $xy+uz=1750-aa$;

But $\left.\begin{matrix}x^2y+y^3+u^2y+z^2y\\+x^2u+y^2u+u^3+z^2u\end{matrix}\right\}=\overline{x^2+y^2+u^2+z^2}\times\overline{y+u}$,

And $\left.\begin{matrix}x^2y+xy^2+xyu+xyz\\+xuz+yuz+u^2z+uz^2\end{matrix}\right\}=\overline{x+y+u+z}\times\overline{xy+uz}$,

Now by the property of ($\div\!\div$) continued proportionals. $\left\{\begin{matrix}yy=xu; & \text{Th. } xy^2=x^2u;\\ xu=yy; & \text{Th. } xyu=y^3;\\ xz=yu; & \text{Th. } xyz=y^2u;\\ xz=yu; & \text{Th. } xuz=yu^2;\\ yz=uu; & \text{Th. } yuz=u^3;\\ uu=yz; & \text{Th. } u^2z=yz^2;\end{matrix}\right.$

Th. $\overline{x^2+y^2+u^2+z^2}\times\overline{y+u}=\overline{x+y+u+z}\times\overline{xy+uz}$,

That is $2125a=75\times\overline{1750-aa}$;

Whence $a^2+\frac{85}{3}a=1750$, And $(a=)\ y+u=30$.

Now $x+z=(75-30=)\ 45$; and $\overline{x+z}\times\overline{y+u}=1350$,

That is $xy+yz+ux+uz=1350$;

The difference of the 5th and laſt is $2yu=(1750-1350=)\ 400$;

Th. $yu=200$; And $u=20$; $y=10$. See qu. 121.

QUEST. CXXV. The ſum 32; the ſum of the ſquares 340; and the ſum of the cubes 4256; of four numbers in disjunct geometrical proportion, being given, to find the numbers?

If x, y, u, and z, repreſent the numbers required;

$$\text{Then}\left\{\begin{array}{l} uy=xz, \\ 32=x+y+u+z, \\ 340=x^2+y^2+u^2+z^2, \\ 4256=x^3+y^3+u^3+z^3, \end{array}\right\}\text{by queſt.}$$

$$\text{Now }\frac{\overline{32}|^2-340}{2}=342=xy+xu+xz+yu+yz+uz,$$
$$=xy+xu+2yu+yz+uz;$$
$$=\overline{x+z}\times\overline{y+u}+2yu;$$

$$\text{And}\left(\frac{\overline{32}|^3-4256}{3}=\right)\ 9504=\left\{\begin{array}{l} xxy+xxu+xxz+xy^2, \\ +xu^2+xz^2+y^2u+y^2z, \\ +yu^2+yz^2+u^2z+uz^2; \end{array}\right.$$

$$\text{Th. }\frac{9504}{342}=x+y+u+z-\frac{xyu+xyz+xuz+yuz}{\overline{x+z}\times\overline{y+u}+2yu};$$
$$=x+y+u+z-\frac{\overline{x+y+u+z}\times yu}{\overline{z+x}\times\overline{y+u}+2yu};$$
$$=32-\frac{32yu}{342},$$

$$\text{Th. }yu=\left(\frac{10944-9504}{32}=\right)45=xz:$$

From above, $342=\overline{x+z}\times\overline{y+u}+90$,

By ſecond $32=\overline{x+z}+\overline{y+u}$;

$$\left.\begin{array}{l}\text{Th. by} \\ \text{qu. 159}\end{array}\right\{\begin{array}{l} \overline{x+z}=\overline{32+\sqrt{32\times32-4\times252}}\times\frac{1}{2}=18; \\ \overline{y+u}=\overline{32-\sqrt{32\times32-4\times252}}\times\frac{1}{2}=14; \end{array}$$

QUEST.

Th. by qu. 159. $\begin{cases} u=\overline{14+\sqrt{14\times14-4\times45}}\times\frac{1}{2}=9; \\ y=\overline{14-\sqrt{14\times14-4\times45}}\times\frac{1}{2}=5: \end{cases}$

By Dit. $\begin{cases} z=\overline{18+\sqrt{18\times18-4\times45}}\times\frac{1}{2}=15; \\ x=\overline{18-\sqrt{18\times18-4\times45}}\times\frac{1}{2}=3. \end{cases}$

QUEST. CXXVI. There are four numbers in geometrical progreſſion; the ſum of the ſecond and fourth is 50; and the difference between the third and firſt is 15: What are thoſe numbers?

If x, y, u, and z, repreſent thoſe numbers;

Then $y+z=50$; Or $z=50-y$;

And $u-x=15$; Or $x=u-15$;

Alſo (becauſe $yy=ux$), Th. $yy=u\times\overline{u-15}$;

Th. $\sqrt{u\times\overline{u-15}}=y$; And $z=50-\sqrt{u\times\overline{u-15}}$;

Now $u-15:\sqrt{u\times\overline{u-15}}::u:50-\sqrt{u\times\overline{u-15}}$,

Th. $\overline{u-15}\times50-\overline{u-15}\times\sqrt{u\times\overline{u-15}}=u\times\sqrt{u\times\overline{u-15}}$,

Or - $\overline{2u-15}\times\sqrt{u\times\overline{u-15}}=\overline{u-15}\times50$;

Th. - - - - $\frac{\overline{u-15}\times50}{2u-15}=\sqrt{u\times\overline{u-15}}$,

And - - $\frac{\overline{u-15}^2\times2500}{\overline{2u-15}^2}=u\times\overline{u-15}$,

Or - $4u^3-60u^2-2275u+37500=0$: Th. $u=20$.

QUEST.

QUEST. CXXVII. There are four numbers in geometrical progression, the sum of the two least is 15; and the difference of the two greatest 20: What are those numbers?

If x, y, u, and z, be the numbers required;

Then $x+y=15$; Th. $x=15-y$;

And $z-u=20$; Th. $z=20+u$;

Also (because $u=\frac{yy}{x}$;) Th. $z=20+\frac{yy}{15-y}$:

But $15-y : y :: \frac{yy}{15-y} : 20+\frac{yy}{15-y}$,

That is $\overline{15-y}\times 20+yy=\frac{y^3}{15-y}$, And $y=10$.

QUEST. CXXVIII. The sum of the means, 30; and the sum of the squares of the extremes, 1625; of 4 numbers in geometrical progression being given, to find the numbers?

If x, xy, xyy, and xy^3, represent the numbers required;

Then $xy+xyy=30$; Or $x=\frac{30}{y+yy}$;

And $x^2+x^2y^6=1625$; Or $x^2=\frac{1625}{1+y^6}$;

Th. $\left(\frac{30}{y+yy}\right)^2=\right)\frac{900}{yy+2y^3+y^4}=\frac{1625}{1+y^6}$,

Th. $36y^6-65y^4-130y^3-65y^2+36=0$: And $y=2$.

QUEST.

QUEST. CXXIX. The sum 75; and the sum of the cubes, 73125; of four numbers in geometrical progression, being given; to find the numbers?

Let x, y, u, and z, represent the numbers required;
And let - $y+u=a$; Then $x+z=75-a$:
Now - $\overline{y+u}^3=a^3$,
And - $\overline{x+z}^3=421875-16875a+225a^2-a^3$;
Th. $\overline{y+u}^3+\overline{x+z}^3=421875-16875a+225a^2$:
But $\overline{y+u}|^3+\overline{x+z}|^3=\begin{cases} x^3+y^3+u^3+z^3+3yu\times\overline{y+u} \\ +3xz\times\overline{x+z}, \end{cases}$

$=73125+3yu\times\overline{x+y+u+z}$,
$=73125+3yu\times 75$;

Th. $73125+225yu=421875-16875a+225a^2$;
Th. - $yu=1550-75a+a^2$:
But - $\left\{\begin{matrix} y^3=x^2z \\ u^3=xz^2 \end{matrix}\right\}$ see quest. 119;
Th. - $73125=x^3+x^2z+xz^2+z^3$,

$=\overline{x+z}^3-2xz\times\overline{x+z}$,
$=\overline{x+z}^3-2uy\times\overline{75-a}$,
$=\begin{cases} 421875-16875a+225a^2-a^3 \\ -uy\times\overline{150-2a}; \end{cases}$

Or $\overline{150-2a}\times uy=348750-16875a+225a^2-a^3$;
Th. - $uy=\dfrac{348750-16875a+225a^2-a^3}{150-2a}$:
Th. $1550-75a+a^2=\dfrac{348750-16875a+225a^2-a^3}{150-2a}$:
By Reduction $0=a^3-75a^2-2525a+116250$:
Th. - $a=30=y+u$;
And - $yu=(1550-75\times 30+30\times 30=)\ 200$:
Th. - $\left\{\begin{matrix} u=20 \\ y=10 \end{matrix}\right\}$ as in quest. 121.

QUEST.

QUEST. CXXX. There are four numbers in geometrical progression, whose sum is 9620; and if the product of the two extremes be added to the product of their cube roots, the sum will be 3581730: What are those numbers?

If, x^3, y^3, u^3, and z^3, represent the numbers required;
Then $x^3z^3+xz=3581730$ by quest.
But $\frac{x^3z^3+xz-3581730}{xz-153}=x^2z^2+153xz+23410$:
Th. - - $xz=153$:
Now $x^2z=y^3$; And $xz^2=u^3$ (by quest. 119)
Th. $x^3+x^2z+xz^2+z^3=9620$ by quest.
Substitute - $x+z=a$;
Th. $x^3+3x^2z+3xz^2+z^3=a^3$;
And $(2x^2z+2xz^2=)\,306a=a^3-9620$, by substr.
Th. $a^3-306a-9620=0$: And $(a=)\;x+z=26$.

And by qu. 159. $\begin{cases} z=(26+\sqrt{26\times26-4\times153}\times\frac{1}{2}=)\,17; \\ x=(26-\sqrt{26\times26-4\times153}\times\frac{1}{2}=)\;9. \end{cases}$

QUEST. CXXXI. Of five numbers in geometrical progression, there are given, the sum of the first and third 25; and the sum of the third and fifth 100; to find the numbers?

If x, y, u, e, and z, represent those numbers;
Then $x+u=25$; And $u+z=100$ per quest.
Now $x:y::u:e$,
Th. $x:x+u::y:y+e$,
Again $y:u::e:z$,
Th. $y:y+e::u:u+z$,
Whence $x+u:y+e::y+e,\ u+z$,
That is $25:y+e::y+e:100$;
Th. $(100\times25=)\ 2500=\overline{y+e}^2$, and $50=y+e$
But $x:y::x+u:y+e$
That is $x:y::25:50$; Th. $2x=y$:
Now $x:2x::2x:u$; Th. $4x=u$:
And $(x+4x=)\ 5x=25$; Th. $x=5$.

QUEST.

QUEST. CXXXII. Of 5 numbers in geometrical progression, there are given, the sum of the first and second, 15; and the sum of the fourth and fifth, 120; to find those numbers?

If x, y, u, e, and z, represent those numbers;

Then $x+y=15$; and $e+z=120$ per quest.

Now by quest. 116; $\begin{cases} x+y : y+u :: y+u : u+e, \\ y+u : u+e :: u+e : e+z; \end{cases}$

That is - $\begin{cases} 15 : y+u :: y+u : u+e, \\ y+u : u+e :: u+e : 120; \end{cases}$

Th. - - $\begin{cases} 15 \times \overline{u+e} = \overline{y+u}|^2, \\ \dfrac{\overline{u+e}|^2}{120} = y+u; \end{cases}$

Whence - - $\dfrac{\overline{u+e}|^4}{120 \times 120} = 15 \times \overline{u+e}$,

Th. - $(\sqrt[3]{\overline{120}|^2 \times 15} =)\ 60 = u+e$,

And - $\left(\dfrac{60 \times 60}{120} =\right) 30 = y+u$:

But because $x : y :: y : u$; $x : x+y :: y : y+u$,

That is - - $x : 15 :: y : 30$;

Th. $30x = 15y$; And $2x = y$;

And $(x+y = x+2x =)\ 3x = 15$; Th. $x = 5$.

QUEST. CXXXIII. Of five numbers in geometrical progression, there is given the sum of the three least 35; and the sum of the three greatest, 140; to find those numbers?

If x, y, u, e, and z, represent those numbers;

Then $x+y+u=35$; And $u+e+z=140$:

But $x : y :: u : e$;

Th. $x+y : y :: u+e : e$:

But $y : e :: u : z$;

Th. $x+y : u :: u+e : z$;

Whence $x+y+u : u :: u+e+z : z$:

That is $35 : u :: 140 : z$;

Th. - - $z = 4u$:

But $zu = ee$: Th. $4u \times u = ee$, and $2u = e$:

Now $(u+2u+4u=)\ 7u = 140$; Th. $u = 20$.

QUEST.

QUEST. CXXXIV. In five numbers in geometrical progression, there are given the sum of the two extremes 85; and the sum of the three means, 70: To find those numbers?

If x, y, u, e, and z, be the numbers required;
Then $x+z=85$; and $y+u+e=70$ per quest.
Now $ye=uu$; Th. $2ye=2uu$,
Also - - - $yy=xu$, $ee=zu$:
Th. - $\overline{y+e}^2$ $(=yy+2ye+ee)$ $=\overline{2u+x+z}\times u$:
But $y+e=70-u$; Th. $\overline{y+e}^2=4900-140u+uu$,
And - - $\overline{2u+x+z}\times u=\overline{85+2u}\times u$;
Th. - - - - $85u+2uu=4900-140u+uu$,
Or - - $uu+225u=4900$: and $u=20$:
Now $ye=(uu=)$ 400; And $y+e=(70-20=)$ 50:

Th. by qu. 159. $\left\{\begin{array}{l} e=\overline{50+\sqrt{50\times50-4\times400}}\times\frac{1}{2}=40; \\ y=\overline{50-\sqrt{50\times50-4\times400}}\times\frac{1}{2}=10. \end{array}\right.$

QUEST. CXXXV. In five numbers in geometrical progression, there are given the sum of the first, third, and fifth, 105; and the sum of the second, and fourth 50: To find those numbers?

If x, y, u, e, and z, represent the numbers required;
Then $x+u+z=105$; And $y+e=50$.
Now (by last) $\overline{x+z+2u}\times u=\overline{y+e}^2$,
That is - $\overline{105+u}\times u=(50\times50=)$ 2500:
Th. - - - $u=20$:
Whence the rest may be found as in the last.

QUEST. CXXXVI. In five numbers in geometrical progression, there is given the mean term 20; and the sum of the other four, 135; to find the numbers?

If x, y, 20, e, and z, represent the numbers required;
Then $x+y+e+z=135$ per quest.
Substitute, $a=y+e$; Then $x+z=135-a$:

Now (by qu. 134.) $\overline{x+z+40}\times 20=\overline{y+e}^2$;

That is $\left\{\begin{array}{c}\overline{135-a+40}\times 20\\ \hline \overline{175-a}\times 20\end{array}\right\}=aa$;

Th. - - - $aa+20a=3500$.
And - - $(a=)y+e=50$:
But - - $(20\times 20=)\ 400=ye$;
Whence - - - $\left\{\begin{array}{l}e=40\\ y=10\end{array}\right\}$ as in qu. 134.

QUEST. CXXXVII. In five numbers in geometrical progression; the sum of the extremes, 85; and the sum of the squares of the second and fourth, 1700, are given; to find the numbers?

If x, y, u, e, and z, represent the numbers required;
Then $x+z=85$; And $yy+ee=1700$ per quest.
But $\overline{x+z+2u}\times u=\overline{y+e}^2$ (by quest. 134.)
That is $\overline{85+2u}\times u=yy+2ye+ee$,
Th. - $85u=(yy+ee=)\ 1700$,
And - $u=20$; Also $(uu=)\ xz=400$;

Th. by qu. 159. $\left\{\begin{array}{l}x=\overline{85+\sqrt{85\times 85-4\times 400}}\times\frac{1}{2}=80;\\ z=\overline{85-\sqrt{85\times 85-4\times 400}}\times\frac{1}{2}=5.\end{array}\right.$

QUEST.

QUEST. CXXXVIII. The sum of five numbers in geometrical progression is, 155; and the sum of their squares, 8525: What are those numbers?

If x, y, u, e, and z, represent those numbers;

And - $y+e=a$; Then $x+z=155-a-u$:

Now - $a^2=(yy+2ye+ee=)\ yy+2uu+ee$;

Th. $a^2-uu=yy+uu+ee$.

And $\overline{x+z}^2=24025-310a-310u+a^2+2au+uu$:

But - $2xz=2uu$;

Th. $x^2+z^2=24025-310a-310u+a^2+2au-uu$;

Now $8525=x^2+y^2+u^2+e^2+z^2$ by quest.

Th. - $8525=24025-310a-310u+2a^2+2au-2uu$;

But - $\overline{y+e}^2=\overline{x+z+2u}\times u$ (by quest. 134.)

That is $a^2=(\overline{155-a+u}\times u=)\ 155u-au+uu$,

Or - $155u=a^2+au-uu$;

Th. - $310u=2a^2+2au-2uu$:

Now $8525=24025-310a-310u+310u$,

Or $310a=(24025-8525=)\ 15500$;

Th. - $a=50=y+e$;

But (above) $a^2=155u-au+uu$,

That is $2500=(155u-50u+uu=)\ uu+105u$;

Th. - $20=u$:

And the rest will be found as in quest. 134.

QUEST. CXXXIX. What is the sum M, of n terms of the series 1, 3, 7, 15, 31, &c. the terms of which are the successive sums of the geometrical progression 1, 2, 4, 8, 16, &c.

If $a=1$; and $r=2$; then the sum of

$$\left.\begin{array}{c}1\\2\\3\\4\\\&c.\\n\end{array}\right\}\text{Terms of the geometrical progression, } a,\ ar,\ ar^2,\ ar^3,\ \&c.\text{ will be}\left\{\begin{array}{l}\frac{ar-a}{r-1}=\overline{r-1}\times\frac{a}{r-1}\\ \frac{ar^2-a}{r-1}=\overline{r^2-1}\times\frac{a}{r-1}\\ \frac{ar^3-a}{r-1}=\overline{r^3-1}\times\frac{a}{r-1}\\ \frac{ar^4-a}{r-1}=\overline{r^4-1}\times\frac{a}{r-1}\\ \&c.\qquad \&c.\qquad \&c.\\ \frac{ar^n-a}{r-1}=\overline{r^n-1}\times\frac{a}{r-1}\end{array}\right\}\text{By quest. 83.}$$

$$\text{Th. } M=\frac{a}{r-1}\times\left\{\begin{array}{l}n\text{ terms of } r,\ r^2,\ r^3,\ \&c.\\ -n\text{ terms of } 1+1+1,\ \&c.\end{array}\right.$$

But n terms of r, r^2, r^3, &c. $=\overline{r^n-1}\times\frac{r}{r-1}$ by qu. 83.

$$\text{Th. } M=\overline{\frac{r\times\overline{r^{n}-1}}{r-1}-n}\times\frac{a}{r-1}:$$

In this example if $n=5$; Then,

$$M=\left(\overline{\frac{2\times\overline{2^{5}-1}}{2-1}-5}\times\frac{1}{2-1}=2\times31-5=\right)57.$$

QUEST.

QUEST. CXL. What is the sum ($\mathfrak{S}$) of n terms of the series $\frac{1}{1}+\frac{3}{2}+\frac{7}{4}+\frac{15}{8}+\frac{31}{16}$, &c. The terms of which are the successive sums of the geometrical progression $\frac{1}{1}+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}$, &c.

If $z=1$, and $r=2$; then the sum of,

$$\left.\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ \&c. \\ n \end{array}\right\} \text{terms of the geometrical progression, } z, \frac{z}{r}, \frac{z}{r^2}+\frac{z}{r^3}, \text{ \&c. will by qu. 95. be } \left\{\begin{array}{l} \frac{\overline{r-1}\times z}{\overline{r-1}\times 1} = \overline{r-1}\times\frac{z}{r-1} \\ \frac{\overline{r^2-1}\times z}{\overline{r-1}\times r} = \overline{r-\frac{1}{r}}\times\frac{z}{r-1} \\ \frac{\overline{r^3-1}\times z}{\overline{r-1}\times r^2} = \overline{r-\frac{1}{r^2}}\times\frac{z}{r-1} \\ \frac{\overline{r^4-1}\times z}{\overline{r-1}\times r^3} = \overline{r-\frac{1}{r^3}}\times\frac{z}{r-1} \\ \&c. \\ \frac{\overline{r^n-1}\times z}{\overline{r-1}\times r^{n-1}} = \overline{r-\frac{1}{r^{n-1}}}\times\frac{z}{r-1} \end{array}\right.$$

$$\text{Th. } \mathfrak{S}=\frac{z}{r-1}\times\left\{\begin{array}{l} n \text{ terms of } r+r+r+ \text{ \&c.}- \\ n \text{ terms of } \frac{1}{1}+\frac{1}{r}+\frac{1}{r^2} \text{ \&c.} \end{array}\right.$$

But n terms of $\frac{1}{1}+\frac{1}{r}+\frac{1}{r^2}$ &c. $=\frac{r^n-1}{\overline{r-1}\times r^{n-1}}$ by q. 95.

$$\text{Th. } \mathfrak{S}=\frac{z}{r-1}\times\overline{nr-\frac{r^n-1}{\overline{r-1}\times r^{n-1}}}:$$

In this Exam. $\left\{ \mathfrak{S}=\left(\frac{1}{2-1}\times 5\times 2-\frac{2^n-1}{\overline{2-1}\times 2^4}=10-\frac{31}{16}=\right)\right.$ $\frac{129}{16}$.

In computations relating to compound interest and annuities.

$$\text{Let}\left\{\begin{array}{c} p\\ a\\ r\\ n\\ m\end{array}\right\}\text{represent}\left\{\begin{array}{l}\text{the principal or present worth,}\\ \text{the annuity,}\\ \text{the interest of 1}l.\text{ in 1 time,}\\ \text{the number of times,}\\ \text{the amount;}\end{array}\right.$$

Then, QUEST. CXLI. p, r, and n, are given; to find m?

$$\text{Now because 1}l.\text{ will in 1 time amount to }\overline{1+r}\left\{\begin{array}{l} p\\ p\times\overline{1+r}\\ p\times\overline{1+r}^2\\ \quad\&c.\\ p\times\overline{1+r}^{n-1}\end{array}\right\}\text{will amount to}\left\{\begin{array}{l} p\times\overline{1+r}\\ p\times\overline{1+r}^2\\ p\times\overline{1+r}^3\\ \quad\&c.\\ p\times\overline{1+r}^n\end{array}\right\}=\text{the amount of }p\text{ at the end of}\left\{\begin{array}{c} 1\\ 2\\ 3\\ \&c.\\ n\end{array}\right\}\text{times;}$$

Th. $p\times\overline{1+r}^n=m$;

In Logarithms $P+n\times L,\overline{1+r}=M$.

QUEST. CXLII. When p, r, and m, are given; to find n?

Then - $n=\dfrac{M-P}{L.\overline{1+r}}$.

QUEST. CXLIII. When p, n, and m, are given; to find r?

Then $L.\overline{1+r}=\dfrac{M-P}{n}$.

QUEST. CXLIV. When r, n, and m, are given to find p?

Then $P=M-n\times L.\overline{1+r}$.

QUEST.

QUEST. CXLV. In annuities computed by compound interest a, r, and n, are given; to find m?

$$\text{Then}\left\{\begin{array}{l} a \\ a\times\overline{1+r} \\ a\times\overline{1+r}^2 \\ a\times\overline{1+r}^3 \\ \&c. \\ a\times\overline{1+r}^{n-1} \end{array}\right\}\text{becomes due in the}\left\{\begin{array}{l} \text{1st} \\ \text{2d} \\ \text{3d} \\ \text{4th} \\ \&c. \\ n\text{th.} \end{array}\right\}\text{Time.}$$

Th. their sum $\frac{a\times\overline{1+r}^n - a}{r} = m$ by quest. 83.

In logar. let $n\times L.\overline{1+r}=B$; then $A+L.\overline{b-1}-R=M$.

QUEST. CXLVI. When m, r, and n, are given; to find a?

Then $a=\frac{rm}{\overline{1+r}^n-1}$;

Or (if $n\times L.\overline{1+r}=B$) $A=M+R-L.\overline{b-1}$.

QUEST. CXLVII. When a, m, and r, are given; to find n?

Then $a\times\overline{1+r}^n=mr+a$;

Th. $\overline{1+r}^n=\frac{mr+a}{a}$;

And $n=\frac{L.\overline{mr+a}-A}{L.\overline{1+r}}$.

QUEST. CXLVIII. When a, n, and m, are given; to find r?

Then $\frac{a\times\overline{1+r}|^{n}-a}{\overline{1+r}-1}=m$,

Or $a\times\overline{1+r}|^{n}-a=m\times\overline{1+r}-m$;

Th. $a\times\overline{1+r}|^{n}-m\times\overline{1+r}+\overline{m-a}=0$:

If $\overline{1+r}$ could reasonably be expected to be a whole number, or rational fraction, it might be found in the same manner as r in quest. 92; but as that will seldom or never happen; use the following approximation *.

Let $D=\frac{M-A-N}{\frac{1}{2}\times\overline{n-1}}$; and $e=\frac{6}{n+1}$;

Also $F=\frac{L.2\times\overline{\overline{d-1}+e}+E}{2}$; Then $r=f-e$.

For example; if an annuity of 50 *l.* forborn 18 years amounts to 1342 *l.* 15 *s.* What rate of interest was allowed?

Here $50=a$, $M=3,12800$,
$18=n$, $A=1,69897$,
$1342,75=m$, $N=1,25527$;
$8,5=\frac{1}{2}\times\overline{n-1}$, $8,5)0,17375(0,02044=D$
$0,31579=\frac{6}{n+1}=e$; And $1,04819=d$.
Then $2\times0,04819=0,09638$, $E=\bar{1},49940$,
$e=0,31579$, $L,2\times\overline{\overline{d-1}+e}=\bar{1},61508$,
Th. $2\times\overline{d-1}+e=0,41217$; $2)\bar{1},11448$,
$\bar{1},55724,=F$

Then $(f=)\ 0,36078-0,31579=(0,04499=)\ 0,045=r$.
Answer $4\frac{1}{2}$ *l.* per cent.

* *See Gardiner's Edit. of Vlaq's Logarithms, page 8. Dodson's Antilogarithmic Canon page 53. Philosophical Transactions for 1770, page 508.*

QUEST. CXLIX. In annuities computed by compound interest a, r, and n, are given; to find p?

$$\text{Then}\left\{\begin{array}{c}\frac{a}{1+r}\\ \frac{a}{\overline{1+r}^2}\\ \frac{a}{\overline{1+r}^3}\\ \text{\&c.}\\ \frac{a}{\overline{1+r}^n}\end{array}\right\}\begin{array}{l}\text{is the present value}\\ \text{of the sum payable}\\ \text{at the end of the}\end{array}\left\{\begin{array}{c}\text{1st}\\ \text{2d}\\ \text{3d}\\ \text{\&c.}\\ n\text{th}\end{array}\right\}\text{time;}$$

Th. their sum $\frac{\overline{\overline{1+r}|^n-1}\times a}{r\times\overline{1+r}|^n}=p$, by quest. 95.

In Log. Let $n\times L.\overline{1+r}=B$;

Then $A+L.\overline{b-1}-R-B=P$.

QUEST. CL. When p, r, and n, are given; to find a?

Then - - $a=\frac{pr\times\overline{1+r}^n}{\overline{1+r}^n-1}$:

Or (if $n\times L.\overline{1+r}=B$; $A=\frac{P+R+B}{L.\overline{b-1}}$:

QUEST. CLI. When a, p, and r, are given; to find n?

Then $\overline{1+r}^n\times a=pr\times\overline{1+r}^n+a$,

Th. - $\overline{1+r}^n=\frac{a}{a-pr}$:

And - $n=\frac{A-L.\overline{a-pr}}{L.\overline{1+r}}$.

QUEST. CLII. When a, n, and p, are given; to find r?

Then $\dfrac{a\times\overline{1+r}^{n}-a}{\overline{1+r}-1\times\overline{1+r}^{n}}=p$,

Th. $p\times\overline{1+r}^{n-1}-\overline{a+p}\times\overline{1+r}^{n}+a=0$:

And here, as in queſt. 148. (ſince the value of $\overline{1+r}$ is not to be expected in a whole number or rational fraction) the following approximation (ſee the places quoted in page 272) may be uſed:

Let $G=\dfrac{A+N-P}{\frac{1}{2}\times\overline{n+1}}$; $b=\dfrac{6}{n-1}$;

$K=\dfrac{L.\overline{b-2\times\overline{g-1}}+H}{2}$; Then $r=b-k$:

For example; ſuppoſe a bookſeller purchaſes a work for 40 *l.* and pays for printing a thouſand copies thereof 15 *l.* for paper 20 *l.* and for advertiſing and other incident charges 10 *l.* Now if he ſells the edition at 3 *s.* each copy in 10 years: (*i. e.* 100 copies every year) What does he gain per cent.?

Here the bookſeller lays out 85 *l.* to purchaſe an annuity of 15 *l.* per year, to continue 10 years;

Th. $15=a$, $A=1,17609$,
$10=n$, $N=1,00000$,
$2,17609$,
$85=p$, $P=1,92942$,
$\frac{2}{3}=\dfrac{6}{n-1}=b$,
$5,5)0,24667(=0,04485=G$;
$5,5=\frac{1}{2}\times\overline{n+1}$; Th. $1,10879=g$.
$b=0,66667$, $H=1,82391$,
$2\times 0,10879=0,21758$, $L\,\overline{b-2\times\overline{g-1}}$ $=\bar{1},65233$,
$b-2\times\overline{g-1}=0,44909$: $2)\bar{1},47624$,
$K=\bar{1},73812$,

Then $0,66667-(k=)\ 0,5417=0,11950=r$.

Anſwer 11 *l.* 19 *s.* per cent.

QUEST.

QUEST. CLIII. In annuities computed by compound intereſt: a, p, and r, are given; to find m?

By queſt. 151. $\overline{1+r}^{n}=\frac{a}{a-pr}$:

By queſt. 141. $\overline{1+r}^{n}=\frac{m}{p}$;

Th. - - $\frac{m}{p}=\frac{a}{a-pr}$, And $m=\frac{pa}{a-pr}$.

QUEST. CLIV. When a, r, and m, are given; to find p?

Then - $\frac{ma}{a+mr}=p$.

QUEST. CLV. When a, p, and m, are given; to find r?

Then - $\frac{\overline{m-p}\times a}{mp}=r$.

QUEST. CLVI. When p, r, and m, are given; to find a?

Then - $a=\frac{mrp}{m-p}$.

QUEST. CLVII. In computing annuities by compound intereſt, p, m, and n, are given; to find a?

By queſt. 141. $1+r=\left.\frac{m}{p}\right|^{\frac{1}{n}}$

QUEST. CLXV. When p, a, x, and t, are given; to find r?

The following approximation (see the places quoted in quest. 148.) may be used.

Let $\dfrac{2x+n+1}{2}=b$; $u=\dfrac{12b}{tt-1}$;

And $\dfrac{A+T-P}{b}=V$; $z=\overline{u-v-1}\times 2$;

Then $\dfrac{Z+U}{2}=E$; And $u-e=r$.

QUEST. CLXVI. A gentleman who had 10 different annuities of 100 $l.$ each, the longest being to continue 60 years, the second 59, the third 58 years, &c. sold them all at 5 $l.$ per cent. compound interest: How much money did he receive?

If $a=100$, and $r=0{,}05$; then the present worth of 100 $l.$ a year to continue,

$$\left.\begin{array}{l}60\\[2em]59\\[2em]58\end{array}\right\}\text{years will be}\left\{\begin{array}{l}\dfrac{\overline{1+r}^{60}-1\times a}{r\times\overline{1+r}^{60}}=\overline{1-\dfrac{1}{\overline{1+r}^{60}}}\times\dfrac{a}{r}\\[1em]\dfrac{\overline{1+r}^{59}-1\times a}{r\times\overline{1+r}^{59}}=\overline{1-\dfrac{1}{\overline{1+r}^{59}}}\times\dfrac{a}{r}\\[1em]\dfrac{\overline{1+r}^{58}-1\times a}{r\times\overline{1+r}^{58}}=\overline{1-\dfrac{1}{\overline{1+r}^{58}}}\times\dfrac{a}{r}\end{array}\right\}\text{by qu. 149.}$$

&c. &c. &c.

Therefore the value of 10 such annuities will be

$$\overline{10-\dfrac{\overline{1+r}^{10}-1}{r\times\overline{1+r}^{60}}}\times\dfrac{a}{r}\text{ by quest. 83.}$$

Now $(\overline{1+0{,}05}^{10}=)\ \overline{1{,}05}^{10}=1{,}628895$;

Th. - - $\overline{1{,}05}^{10}-1=0{,}628895$;

Also - - $\overline{1{,}05}^{60}=18{,}679186$;

Th. - $\dfrac{0{,}628895}{0{,}05\times 18{,}679186}=0{,}673368$:

And - $\overline{10-0{,}673368}\times\frac{100}{0{,}05}=18653{,}26\ l.$

QUEST.

QUEST. CLXVII. If to enjoy the benefit of an eſtate for 23 years, after the expiration of 8 years, be worth 400 *l.* preſent money, what will the ſame eſtate be worth for 21 years, after the expiration of 10 years; allowing compound intereſt at 5 per cent?

Suppoſe the eſtate was *a L.* per year, and $x=$ the number required;

$$\left.\begin{array}{ll}\text{Then} & \dfrac{\overline{\overline{1,05}^{23}-1}\times a}{,05\times\overline{1,05}^{31}}=400 \\ \text{And} & \dfrac{\overline{\overline{1,05}^{21}-1}\times a}{,05\times\overline{1,05}^{31}}=x.\end{array}\right\}\text{ per queſtion.}$$

That is - - $a=\dfrac{400\times,05\times\overline{1,05}^{31}}{\overline{1,05}^{23}-1}$,

And - - $a=\dfrac{x\times,05\times\overline{1,05}^{31}}{\overline{1,05}^{21}-1}$;

Th. $\dfrac{x\times,05\times\overline{1,05}^{31}}{\overline{1,05}^{21}-1}=\dfrac{400\times,05\times\overline{1,05}^{31}}{\overline{1,05}^{23}-1}$,

Th. - - $x=\dfrac{400\times\overline{\overline{1,05}^{21}-1}}{\overline{1,05}^{23}-1}$,

That is - - $x=\left(\dfrac{400\times1,785963}{2,071524}=\right)344,8597.$

QUEST.

QUEST. CLXVIII. If 150*l.* be lent, on condition that 12*l.* per annum be paid, until the principal and its interest at 5 per cent. be satisfied; and that at every payment, the interest then due, shall be discharged, and the remainder, by which such payment exceeds that interest, applied to reduce the principal: How many years must the said payment continue?

If $p=150$; $a=12$; $r=,05$; and $n=$ the time required;

Then
$$\begin{cases} p+pr \\ = \text{sum due before 1st payment.} \\ p-a+pr \\ = \text{sum due after 1st payment.} \end{cases}$$

And
$$\begin{cases} \overline{p-a}\times r+pr^2 \\ = \text{interest due at 2d payment.} \\ \overline{2p-a}\times r+pr^2 \\ = \text{sum due after 2d payment.} \end{cases}$$

Also
$$\begin{cases} \overline{p-2a}\times r+\overline{2p-a}\times r^2+pr^3 \\ = \text{interest due at 3d payment.} \\ p-3a+\overline{3p-3a}\times r+\overline{3p-a}\times r^2+pr^3 \\ = \text{sum due after 3d payment.} \end{cases}$$

Again
$$\begin{cases} \overline{p-3a}\times r+\overline{3p-3a}\times r^2+\overline{3p-a}\times r^3+pr^4 \\ = \text{interest due at 4th payment.} \\ p-4a+\overline{4p-6a}\times r+\overline{6p-4a}\times r^2+\overline{4p-a}\times r^3+pr^4 \\ = \text{sum due after 4th payment.} \end{cases}$$

Th.
$$\begin{cases} p+npr+\frac{n.\overline{n-1}}{1\,.\,2}pr^2 \\ \quad +\frac{n.\overline{n-1}.\overline{n-2}}{1\,.\,2\,.\,3}pr^3,\ \&c. \\ -na-\frac{n.\overline{n-1}}{2}ar-\frac{n.\overline{n-1}.\overline{n-2}}{1\,.\,2\,.\,3}ar^2 \\ \quad -\frac{n.\overline{n-1}.\overline{n-2}.\overline{n-3}}{1\,.\,2\,.\,3\,.\,4}ar^3\ \&c. \end{cases}$$

will be the sum due after the nth payment; which by the question is nothing.

Th. $p\times\overline{1+nr+\frac{n.\overline{n-1}}{1\,.\,2}r^2,\ \&c.}=a\times\overline{n+\frac{n.\overline{n-1}}{1\,.\,2}r}$
$\overline{+\frac{n.\overline{n-1}.\overline{n-2}}{1\,.\,2\,.\,3}r^2,\ \&c.}$

Or

Or $p \times \overline{1+r}|^n = a \times \frac{\overline{1+r}|^n - 1}{r}$;

Or $\frac{pr \times \overline{1+r}|^n}{a} = \overline{1+r}|^n - 1$;

Or - $1 = \overline{1 - \frac{pr}{a}} \times \overline{1+r^n}$;

Or - $1 = \frac{a-pr}{a} \times \overline{1+r^n}$;

Th. $\frac{a}{a-pr} = \overline{1+r^n}$.

And $\frac{A - L.\overline{a-pr}}{L.\overline{1+r}} = n$.

Scholium. By comparing this result with quest. 151: it appears; that, where a debt is discharged by many equal payments, and the interest due at the time of each payment is cleared before any part of the principal, compound interest is allowed to the lender.

But it is both legal and customary, when money is paid in part of a debt, to deduct the interest then due, out of such payment; and to apply, only, the remaining part to the discharge of the principal.

Therefore, in computing the present values of annuities, &c. the rules found on the principles of compound interest, will give their legal value.

QUEST.

QUEST. CLXIX. In a geometrical progression infinitely decreasing are given, the greatest term z, and the ratio, r; to find the sum s?

By quest. 85. $z+\frac{z-a}{r-1}=s$.

Where a, in a finite progression, signifies the least term; but in an infinite progression the least term is inconsiderable;

Th. - $z+\frac{z}{r-1}=s$, Or $\frac{rz}{r-1}=s$.

QUEST. CLXX. What is the present value p of an estate of a per year, in fee simple, allowing compound interest at r per pound per annum?

Now $p=\frac{a}{1+r}+\frac{a}{\overline{1+r}^2}+\frac{a}{\overline{1+r}^3}$ &c. ad infinitum;

Th. $p=\left(\frac{\frac{a}{1+r}\times\overline{1+r}}{\overline{1+r}-1}=\right)\frac{a}{r}$ by quest. last.

QUEST. CLXXI. What is the present value p, of the reversion of an estate of a per annum, in fee simple; to commence at the end of x years; allowing compound interest at r per pound per annum?

By quest. 149. an annuity of a to continue x years $\Big\} = \dfrac{a\times\overline{1+r}|^x - a}{r\times\overline{1+r}|^x}$,

By quest. 170. a perpetuity a per annum $\Big\} = \dfrac{a}{r}$;

Th. $\dfrac{a}{r} - \dfrac{a\times\overline{1+r}|^x - a}{r\times\overline{1+r}|^x} = p$.

Or $\dfrac{a\times\overline{1+r}|^x - a\times\overline{1+r}|^x + a}{r\times\overline{1+r}|^x} = p$;

Th. $\dfrac{a}{r\times\overline{1+r}|^x} = p$.

Or $A - R - x\times L.\overline{1+r} = P$.

Corol. 1. $A = P + R + x\times L.\overline{1+r}$;

Corol. 2. $x = \dfrac{A-P-R}{L.\overline{1+r}}$.

QUEST.

QUEST. CLXXII. It is required to find the sum of the infinite series of the reciprocals of the triangular numbers, $\frac{1}{1}+\frac{1}{3}+\frac{1}{6}+\frac{1}{10}+\frac{1}{15}$, &c.

Let - - $s=\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}$, &c. ad infinitum;

Then - $s-1=\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}$, &c. by transposition;

And the diff. of these two equat. is $1=\frac{2-1}{1\times2}+\frac{3-2}{2\times3}+\frac{4-3}{3\times4}+\frac{5-4}{4\times5}$, &c.

That is - $1=\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\frac{1}{4.5}$, &c.

Twice which is $2=\frac{1}{1}+\frac{1}{3}+\frac{1}{3.2}+\frac{1}{2.5}$, &c.

Th. - - - $2=\frac{1}{1}+\frac{1}{3}+\frac{1}{6}+\frac{1}{10}+\frac{1}{15}$, &c.

QUEST. CLXXIII. The sum of n terms of the series, $\frac{1}{1}+\frac{1}{3}+\frac{1}{6}+\frac{1}{10}$, &c. is required?

If - - - - $z=\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$, &c. to $\frac{1}{n}$;

Then - $z-\frac{1}{1}+\frac{1}{n+1}=\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}$, &c. to $\frac{1}{n+1}$;

Th. - - $\frac{1}{1}-\frac{1}{n+1}=\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}$, &c. to $\frac{1}{n.\overline{n+1}}$,

Or - - - $\frac{n}{n+1}=\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}$, &c. to $\frac{1}{n.\overline{n+1}}$;

Th. - - - $\frac{2n}{n+1}=\frac{1}{1}+\frac{1}{3}+\frac{1}{6}+\frac{1}{10}$, &c. to $\frac{2}{n.\overline{n+1}}$.

QUEST.

QUEST. CLXXIV. It is required to find the ſum of the infinite ſeries of the reciprocals of the pyramidal numbers, $\frac{1}{1}+\frac{1}{4}+\frac{1}{10}+\frac{1}{20}+\frac{1}{35}$, &c.

By qu. 172. $2=\frac{1}{1}+\frac{1}{3}+\frac{1}{6}+\frac{1}{10}+\frac{1}{15}$, &c.

Then $2-1=\frac{1}{3}+\frac{1}{6}+\frac{1}{10}+\frac{1}{15}+\frac{1}{20}$, &c.

And by ſubt. $1=\frac{3-1}{1\times 3}+\frac{6-3}{3\times 6}+\frac{10-6}{6\times 10}+\frac{15-10}{10\times 15}$, &c.

That is - $1=\frac{2}{1.3}+\frac{3}{3.6}+\frac{4}{6.10}+\frac{5}{10.15}$, &c.

Or - - $1=\frac{2}{3}+\frac{1}{6}+\frac{1}{15}+\frac{1}{30}$, &c.

Th. - $\frac{3}{2}=\frac{1}{1}+\frac{1}{4}+\frac{1}{10}+\frac{1}{20}$, &c.

QUEST. CLXXV. It is required to find the ſum of the infinite ſeries, $\frac{1}{1}+\frac{1}{5}+\frac{1}{15}+\frac{1}{35}+\frac{1}{70}$, &c.

By queſt. 174. $\frac{3}{2}=\frac{1}{1}+\frac{1}{4}+\frac{1}{10}+\frac{1}{20}+\frac{1}{35}$, &c.

Then - $\frac{3}{2}-1=\frac{1}{4}+\frac{1}{10}+\frac{1}{20}+\frac{1}{35}+\frac{1}{56}$, &c.

And by ſubtr. $1=\frac{4-1}{1.4}+\frac{10-4}{4.10}+\frac{20-10}{10.20}+\frac{35-20}{20.35}$, &c.

Or - - $1=\frac{3}{4}+\frac{6}{40}+\frac{10}{200}+\frac{15}{700}$, &c.

Or - - - $1=\frac{3}{4}+\frac{3}{20}+\frac{1}{20}+\frac{3}{140}$, &c.

Or - - $1=\frac{3}{4}+\frac{3}{20}+\frac{3}{60}+\frac{3}{140}$, &c.

Then (by div.) $\frac{1}{3}=\frac{1}{4}+\frac{1}{20}+\frac{1}{60}+\frac{1}{140}$, &c.

And (by mult.) $\frac{4}{3}=\frac{1}{1}+\frac{1}{5}+\frac{1}{15}+\frac{1}{35}$, &c.

QUEST.

QUEST. CLXXVI. The sum of n terms of the series, $\frac{1}{1}+\frac{1}{4}+\frac{1}{10}+\frac{1}{20}$, &c. is required?

By quest. 173. - - - - $\frac{2n}{n+1}=\frac{1}{1}+\frac{1}{3}+\frac{1}{6}+\frac{1}{10}$,

&c. to $\frac{2}{n.\overline{n+1}}$;

Th. - - $\frac{2n}{n+1}-1+\frac{2}{\overline{n+1}.\overline{n+2}}=\frac{1}{3}+\frac{1}{6}+\frac{1}{10}+\frac{6}{30}$,

&c. to $\frac{2}{\overline{n+1}.\overline{n+2}}$;

Th. - - $1-\frac{2}{\overline{n+1}.\overline{n+1}}=\frac{2}{3}+\frac{3}{18}+\frac{4}{60}+\frac{5}{100}$,

&c. to $\frac{4}{n.\overline{n+1}.\overline{n+2}}$;

Or - - - $\frac{\overline{n+3}\times n}{\overline{n+1}\times\overline{n+2}}=\frac{2}{3}+\frac{2}{12}+\frac{2}{30}+\frac{2}{60}$,

&c. to $\frac{4}{n.\overline{n+1}.\overline{n+2}}$;

Th. - - - $\frac{\overline{n+3}\times 3n}{2.\overline{n+1}.\overline{n+2}}=\frac{1}{1}+\frac{1}{4}+\frac{1}{10}+\frac{1}{20}$,

&c. to $\frac{2.3}{n.\overline{n+1}.\overline{n+2}}$.

QUEST.

QUEST. CLXXVII. Required the sum of n terms of the series, $\frac{1}{1}+\frac{1}{5}+\frac{1}{15}+\frac{1}{35}$, &c.

By quest. 175 - - - - $\frac{3n.\overline{n+3}}{2.\overline{n+1}.\overline{n+2}} =$

$\frac{1}{1}+\frac{1}{4}+\frac{1}{10}+\frac{1}{20}$, &c. to n terms;

Th. - - - $\frac{3n.\overline{n+3}}{2.\overline{n+1}.\overline{n+2}}-\frac{1}{1}+\frac{2.3}{\overline{n+1}.\overline{n+2}.\overline{n+3}} =$

$\frac{1}{4}+\frac{1}{10}+\frac{1}{20}+\frac{1}{35}$, &c. to n terms;

Th. - - - - $1-\frac{2.3}{\overline{n+1}.\overline{n+2}.\overline{n+3}} =$

$\frac{3}{4}+\frac{3}{20}+\frac{3}{60}+\frac{3}{140}$, &c. to n terms;

And - - - - $\frac{4}{3}-\frac{4}{3}\times\frac{2.3}{\overline{n+1}.\overline{n+2}.\overline{n+3}} =$

$\frac{1}{1}+\frac{1}{5}+\frac{1}{15}+\frac{1}{35}$, &c. to n terms.

COROL. 1.

$$\text{Since}\ \left\{\begin{array}{l} \frac{1}{1}+\frac{1}{3}+\frac{1}{6}+\frac{1}{10},\ \&c. =\frac{2}{1} \\ \frac{1}{1}+\frac{1}{4}+\frac{1}{10}+\frac{1}{20},\ \&c. =\frac{3}{2} \\ \frac{1}{1}+\frac{1}{5}+\frac{1}{15}+\frac{1}{35},\ \&c. =\frac{4}{3} \end{array}\right\}\ \text{as before.}$$

$$\text{Th.}\ \left\{\begin{array}{l} \frac{1}{1}+\frac{1}{6}+\frac{1}{21}+\frac{1}{56},\ \&c. =\frac{5}{4}; \\ \frac{1}{1}+\frac{1}{7}+\frac{1}{28}+\frac{1}{84},\ \&c. =\frac{6}{5}; \\ \frac{1}{1}+\frac{1}{8}+\frac{1}{36}+\frac{1}{120},\ \&c. =\frac{7}{6}; \\ \qquad\qquad \&c. \qquad\qquad \&c. \end{array}\right.$$

COROL. 2.

$$\text{Since } n \text{ terms of}\ \left\{\begin{array}{l} \frac{1}{1}+\frac{1}{3}+\frac{1}{6},\ \&c. =\frac{2}{1}-\frac{2}{1}\times\frac{1}{n+1}; \\ \frac{1}{1}+\frac{1}{4}+\frac{1}{10},\ \&c. =\frac{3}{2}-\frac{3}{2}\times\frac{1.2}{\overline{n+1}.\overline{n+2}}; \\ \frac{1}{1}+\frac{1}{5}+\frac{1}{15},\ \&c. =\frac{4}{3}-\frac{4}{3}\times\frac{1.2.3}{\overline{n+1}.\overline{n+2}.\overline{n+3}}. \end{array}\right.$$

Therefore the manner of continuing these rules is also evident.

QUEST.

QUEST. CLXXVI. The sum of n terms of the series, $\frac{1}{1}+\frac{1}{4}+\frac{1}{10}+\frac{1}{20}$, &c. is required?

By quest. 173. - - - - $\frac{2n}{n+1}=\frac{1}{1}+\frac{1}{3}+\frac{1}{6}+\frac{1}{10}$,

&c. to $\frac{2}{n.\overline{n+1}}$;

Th. - - $\frac{2n}{n+1}-1+\frac{2}{\overline{n+1}.\overline{n+2}}=\frac{1}{3}+\frac{1}{6}+\frac{1}{10}+\frac{2}{30}$,

&c. to $\frac{2}{\overline{n+1}.\overline{n+2}}$;

Th. - - $1-\frac{2}{\overline{n+1}.\overline{n+1}}=\frac{2}{3}+\frac{3}{18}+\frac{4}{60}+\frac{5}{150}$,

&c. to $\frac{4}{n.\overline{n+1}.\overline{n+2}}$;

Or - - - $\frac{\overline{n+3}\times n}{\overline{n+1}\times\overline{n+2}}=\frac{2}{3}+\frac{2}{12}+\frac{2}{30}+\frac{2}{60}$,

&c. to $\frac{4}{n.\overline{n+1}.\overline{n+2}}$;

Th. - - - $\frac{\overline{n+3}\times 3n}{2.\overline{n+1}.\overline{n+2}}=\frac{1}{1}+\frac{1}{4}+\frac{1}{10}+\frac{1}{20}$,

&c. to $\frac{2.3}{n.\overline{n+1}.\overline{n+2}}$.

QUEST.

QUEST. CLXXX. The sum of the infinite ser[ies] $\frac{1}{1.2.3.4}+\frac{1}{2.3.4.5}+\frac{1}{3.4.5.6}$, &c. is required?

$$\text{By qu. 178. } \tfrac{1}{4}=\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}\text{, \&c.}$$

$$\text{Then } \tfrac{1}{4}-\tfrac{1}{6}=\frac{1}{2.3.4}+\frac{1}{3.4.5}+\frac{1}{4.5.6}\text{, \&c.}$$

$$\text{Th. } \tfrac{1}{6}=\frac{24-6}{1.4.9.4}+\frac{60-24}{2.9.16.5}+\frac{120-60}{3.16.25.6}\text{, \&c.}$$

$$\text{That is } \tfrac{1}{6}=\frac{18}{1.4.9.4}+\frac{36}{2.9.16.5}+\frac{60}{3.16.25.6}\text{, \&c.}$$

$$\text{Th. } \left(\frac{1}{1.2.3.3}=\right)\frac{1}{18}=\frac{1}{1.2.3.4}+\frac{1}{2.3.4.5}+\frac{1}{3.4.5.6}\text{, \&c.}$$

QUEST. CLXXXI. The sum of n terms of the series $\frac{1}{1.2.3.4}+\frac{1}{2.3.4.5}+\frac{1}{3.4.5.6}$, &c. is required?

By reasoning as in the preceding questions;

$$\left.\begin{aligned}&\tfrac{1}{6}-\frac{1}{\overline{n+1}.\overline{n+2}.\overline{n+3}}=\frac{3}{1.2.3.4}\\ \text{Th. } &\tfrac{1}{18}-\frac{1}{3.\overline{n+1}.\overline{n+2}.\overline{n+3}}=\frac{1}{1.2.3.4}\end{aligned}\right\}\text{\&c. to } n \text{ terms.}$$

COROL.

QUEST. CLXXVIII. Required the ſum of the infinite ſeries, $\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+\frac{1}{4.5.6}$, &c.

By equation 4, queſt. 172. $\Big\}$ $1=\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\frac{1}{4.5}$, &c.

Then - $1-\frac{1}{2}=\frac{1}{2.3}+\frac{1}{3.4}+\frac{1}{4.5}+\frac{5}{5.6}$, &c.

Th. - - - $\frac{1}{2}=\frac{6-2}{1.4.3}+\frac{12-6}{2.9.4}+\frac{20-12}{3.16.5}$, &c.

Or - - - $\frac{1}{2}=\frac{4}{1.4.3}+\frac{6}{2.9.4}+\frac{8}{3.16.5}$, &c.

Or - - - $\frac{1}{2}=\frac{2}{1.2.3}+\frac{2}{2.3.4}+\frac{2}{3.4.5}$, &c.

Th. $\left(\frac{1}{2.2}=\right)$ $\frac{1}{4}=\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}$, &c.

QUEST. CLXXIX. The ſum of n terms of the ſeries, $\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}$, &c. is required?

If - - $z=\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}$, &c. to $\frac{1}{\overline{n.n+1}}$;

Then $z-\frac{1}{2}+\frac{1}{\overline{n+1.n+2}}=\frac{1}{2.3}+\frac{1}{3.4}+\frac{1}{4.5}$, &c. to $\frac{1}{\overline{n+1.n+2}}$;

Th. $\frac{1}{2}-\frac{1}{\overline{n+1.n+2}}=\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}$, &c. to n terms;

Th. $\frac{1}{4}-\frac{1}{\overline{2.n+1.n+2}}=\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}$, &c. to n terms.

QUEST.

QUEST. CLXXX. The ſum of the infinite ſeries $\frac{1}{1.2.3.4}+\frac{1}{2.3.4.5}+\frac{1}{3.4.5.6}$, &c. is required?

By qu. 178. $\frac{1}{4}=\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}$, &c.

Then $\frac{1}{4}-\frac{1}{6}=\frac{1}{2.3.4}+\frac{1}{3.4.5}+\frac{1}{4.5.6}$, &c.

Th. $\frac{1}{6}=\frac{24-6}{1.4.9.4}+\frac{60-24}{2.9.16.5}+\frac{120-60}{3.16.25.6}$, &c.

That is $\frac{1}{6}=\frac{18}{1.4.9.4}+\frac{36}{2.9.16.5}+\frac{60}{3.16.25.6}$, &c.

Th. $\left(\frac{1}{1.2.3.3}=\right)\frac{1}{18}=\frac{1}{1.2.3.4}+\frac{1}{2.3.4.5}+\frac{1}{3.4.5.6}$, &c.

QUEST. CLXXXI. The ſum of n terms of the ſeries $\frac{1}{1.2.3.4}+\frac{1}{2.3.4.5}+\frac{1}{3.4.5.6}$, &c. is required?

By reaſoning as in the preceding queſtions;

$$\left.\begin{array}{l} \frac{1}{6}-\frac{1}{\overline{n+1}.\overline{n+2}.\overline{n+3}}=\frac{3}{1.2.3.4} \\ \text{Th. } \frac{1}{18}-\frac{1}{3.\overline{n+1}.\overline{n+2}.\overline{n+3}}=\frac{1}{1.2.3.4} \end{array}\right\}\text{ \&c. to } n \text{ terms.}$$

Corol. I.

$$\text{Since}\left\{\begin{array}{ll}\dfrac{1}{1\cdot2}+\dfrac{1}{2\cdot3}+\dfrac{1}{3\cdot4}+\dfrac{1}{4\cdot5}, \text{ \&c.} & =\dfrac{1}{1\cdot1}\\[2ex] \dfrac{1}{1\cdot2\cdot3}+\dfrac{1}{2\cdot3\cdot4}+\dfrac{1}{3\cdot4\cdot5}+\dfrac{1}{4\cdot5\cdot6}, \text{ \&c} & =\dfrac{1}{1\cdot2\cdot2}\\[2ex] \dfrac{1}{1\cdot2\cdot3\cdot4}+\dfrac{1}{2\cdot3\cdot4\cdot5}+\dfrac{1}{3\cdot4\cdot5\cdot6}, \text{ \&c.} & =\dfrac{1}{1\cdot2\cdot3\cdot3}\end{array}\right\}\text{as before;}$$

$$\text{Therefore}\left\{\begin{array}{ll}\dfrac{1}{1\cdot2\cdot3\cdot4\cdot5}+\dfrac{1}{2\cdot3\cdot4\cdot5\cdot6}, \text{ \&c.} & =\dfrac{1}{1\cdot2\cdot3\cdot4\cdot4}\\[2ex] \dfrac{1}{1\cdot2\cdot3\cdot4\cdot5\cdot6}+\dfrac{1}{2\cdot3\cdot4\cdot5\cdot6\cdot7}, \text{ \&c.} & =\dfrac{1}{1\cdot2\cdot3\cdot4\cdot5\cdot5}.\end{array}\right.$$

Corol. II.

Since n terms of

$$\frac{1}{1\cdot2}+\frac{1}{2\cdot3}, \text{ \&c.} = \frac{1}{1\cdot1}-\frac{1}{n+1}$$

$$\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}, \text{ \&c.} = \frac{1}{1\cdot2\cdot2}-\frac{1}{2\cdot\overline{n+1}\cdot\overline{n+2}}$$

$$\frac{1}{1\cdot2\cdot3\cdot4}+\frac{1}{2\cdot3\cdot4\cdot5}, \text{ \&c.} = \frac{1}{1\cdot2\cdot3\cdot3}-\frac{1}{3\cdot\overline{n+1}\cdot\overline{n+2}\cdot\overline{n+3}}$$

Therefore $\dfrac{1}{1\cdot2\cdot3\cdot4\cdot5}+\dfrac{1}{2\cdot3\cdot4\cdot5\cdot6}$,

$$=\frac{1}{1\cdot2\cdot3\cdot4\cdot4}-\frac{1}{4\cdot\overline{n+1}\cdot\overline{n+2}\cdot\overline{n+3}\cdot\overline{n+4}}, \text{ \&c.}$$

QUEST. CLXXXII. In the series of the reciprocals of the natural numbers, $\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}$, &c. infinitely continued; it is required to find what proportion the sum of the odd terms, has to the sum of the even terms?

$$\text{Now}\left\{\begin{array}{l}\frac{1}{1}+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\\ \frac{1}{3}+\frac{1}{6}+\frac{1}{12}+\frac{1}{24}\\ \frac{1}{5}+\frac{1}{10}+\frac{1}{20}+\frac{1}{40}\\ \frac{1}{7}+\frac{1}{14}+\frac{1}{28}+\frac{1}{56}\end{array}\right\}\begin{array}{l}\text{\&c. ad}\\ \text{infinit.}\end{array}\left\{\begin{array}{l}=(\frac{1}{1}\times 2=)\ \frac{2}{1},\\ =(\frac{1}{3}\times 2=)\ \frac{2}{3},\\ =(\frac{1}{5}\times 2=)\ \frac{2}{5},\\ =(\frac{1}{7}\times 2=)\ \frac{2}{7},\end{array}\right\}\text{by qu. 169.}$$

&c.

$$\text{And (by transposit.)}\left\{\begin{array}{l}\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\\ \frac{1}{6}+\frac{1}{12}+\frac{1}{24}\\ \frac{1}{10}+\frac{1}{20}+\frac{1}{40}\\ \frac{1}{14}+\frac{1}{28}+\frac{1}{56}\end{array}\right\}\begin{array}{l}\text{\&c. ad}\\ \text{infinit.}\end{array}\left\{\begin{array}{l}=(\frac{2}{1}-\frac{1}{1}=)\ \frac{1}{1},\\ =(\frac{2}{3}-\frac{1}{3}=)\ \frac{1}{3},\\ =(\frac{2}{5}-\frac{1}{5}=)\ \frac{1}{5},\\ =(\frac{2}{7}-\frac{1}{7}=)\ \frac{1}{7},\end{array}\right.$$

Th. by addition $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}$, &c.
$=\frac{1}{1}+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}$, &c.

QUEST. CLXXXIII. In the infinite series of the reciprocals of the square numbers, $\frac{1}{1}+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}$, &c. the proportion which the sum of the odd terms, has to the sum of the even terms, is required?

$$\text{Now}\left\{\begin{array}{l} \frac{1}{1}+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}, \text{ \&c.} = \frac{4}{3}, \\ \frac{1}{9}+\frac{1}{36}+\frac{1}{144}+\frac{1}{576}, \text{ \&c.} = \frac{4}{3\times 9}, \\ \frac{1}{25}+\frac{1}{100}+\frac{1}{400}+\frac{1}{1600}, \text{ \&c.} = \frac{4}{3\times 25}, \end{array}\right\}\text{per quest. 169.}$$

$$\left.\begin{array}{r}\text{And by}\\ \text{Addition.}\end{array}\right\}\frac{1}{1}+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}, \text{ \&c.} = \frac{4}{3}+\frac{4}{3\times 9}+\frac{4}{3\times 25}, \text{ \&c.}$$

$$\text{Or } \frac{3}{1}+\frac{3}{4}+\frac{3}{9}+\frac{3}{16}, \text{ \&c.} = \frac{4}{1}+\frac{4}{9}+\frac{4}{25}+\frac{4}{49}, \text{ \&c.}$$

$$\text{Or } \frac{3}{4}+\frac{3}{16}+\frac{3}{36}+\frac{3}{64}, \text{ \&c.} = \frac{1}{1}+\frac{1}{9}+\frac{1}{25}+\frac{1}{49}, \text{ \&c.}$$

$$\text{Th. } \frac{1}{1}+\frac{1}{9}+\frac{1}{25}+\frac{1}{49}, \text{ \&c.} : \frac{1}{4}+\frac{1}{16}+\frac{1}{36}+\frac{1}{64}, \text{ \&c.} :: 3 : 1.$$

QUEST.

QUEST. CLXXXIV. In the infinite series of the reciprocals of the cube numbers $\frac{1}{1}+\frac{1}{8}+\frac{1}{27}+\frac{1}{64}+\frac{1}{125}$, &c. the proportion which the sum of the odd terms, has to the sum of the even terms, is required?

$$\text{Now}\left\{\begin{array}{l}\frac{1}{1}+\frac{1}{8}+\frac{1}{64}, \text{ \&c.} \quad - \quad =\frac{8}{7}, \\ \frac{1}{27}+\frac{1}{216}+\frac{1}{1728}, \text{ \&c.} \quad =\frac{8}{7\times 27}, \\ \frac{1}{125}+\frac{1}{1000}+\frac{1}{8000}, \text{ \&c.} =\frac{8}{7\times 125},\end{array}\right\}\text{By quest. 169.}$$

And by Addition. $\left\}\frac{1}{1}+\frac{1}{8}+\frac{1}{27}, \text{ \&c.} =\frac{8}{7}+\frac{8}{7\times 27}+\frac{8}{7\times 125}, \text{ \&c.}\right.$

Or $\frac{7}{1}+\frac{7}{8}+\frac{7}{27}$, &c. $=\frac{8}{1}+\frac{8}{27}+\frac{8}{125}$, &c.

Or $\frac{7}{8}+\frac{7}{64}+\frac{7}{216}$, &c. $=\frac{1}{1}+\frac{1}{27}+\frac{1}{125}$, &c.

Th. $\frac{1}{1}+\frac{1}{27}+\frac{1}{125}$, &c. : $\frac{1}{8}+\frac{1}{64}+\frac{1}{216}$, &c. :: 7 : 1.

COROL. Since by the preceding questions

$$\frac{1}{1}+\frac{1}{3}+\frac{1}{5}, \text{ \&c.} : \frac{1}{2}+\frac{1}{4}+\frac{1}{6}, \text{ \&c.} :: 1 : 1$$

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}, \text{ \&c.} : \frac{1}{2^2}+\frac{1}{4^2}+\frac{1}{6^2}, \text{ \&c.} :: 3 : 1$$

$$\frac{1}{1^3}+\frac{1}{3^3}+\frac{1}{5^3}, \text{ \&c.} : \frac{1}{2^3}+\frac{1}{4^3}+\frac{1}{6^3}, \text{ \&c.} :: 7 : 1$$

Therefore

$$\frac{1}{1^m}+\frac{1}{3^m}+\frac{1}{5^m}, \text{ \&c.} : \frac{1}{2^m}+\frac{1}{4^m}+\frac{1}{6^m}, \text{ \&c.} :: 2^m-1 : 1$$

QUEST. CLXXXV. The sum (a) of the infinite series $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$, &c. is required?

This is solved by quest. 169. but as it is an introductory question to the following class, the sum may be investigated as follow:

Let $\frac{1}{2}=x$;

Then $a=x+x^2+x^3+x^4+x^5$, &c.

Put $\frac{z}{1-x}=(a=)\ x+x^2+x^3+x^4$, &c.

Then $z=\overline{1-x}\times\overline{x+x^2+x^3+x^4, \&c.}$

But $x=\overline{1-x}\times\overline{x+x^2+x^3+x^4, \&c.}$

See the work.

$$\begin{array}{l} x+x^2+x^3+x^4+x^5, \&c. \\ 1-x \\ \hline x+x^2+x^3+x^4+x^5, \&c. \\ \quad -x^2-x^3-x^4-x^5, \&c. \\ \hline x+0\ +0\ +0\ +0\ ;\ \&c. \end{array}$$

Th. $z=x$,

And $\frac{x}{1-x}=(a=)\ x+x^2+x^3+x^4$, &c.

QUEST.

QUEST. CLXXXVI. The sum (a) of the infinite series $\frac{1}{2}-\frac{1}{4}+\frac{1}{8}-\frac{1}{16}$, &c. is required?

Put $\frac{1}{2}=x$; And let $\frac{x}{1+x}=a$;

Then $\frac{x}{1+x}=x-x^2+x^3-x^4+x^5$, &c.

And - $x=\overline{1+x}\times\overline{x-x^2+x^3-x^4}$, &c.

But - $x=\overline{1+x}\times\overline{x-x^2+x^3-x^4}$, &c.

See the work.

$$\begin{array}{l} x-x^2+x^3-x^4+x^5-x^6, \text{ \&c.} \\ 1+x \\ \hline x-x^2+x^3-x^4+x^5-x^6, \text{ \&c.} \\ \quad +x^2-x^3+x^4-x^5+x^6, \text{ \&c.} \\ \hline x+0+0+0+0+0, \text{ \&c.} \end{array}$$

Th. - - - $x=x$,

And - - $\frac{x}{1+x}=(a=)\ x-x^2+x^3-x^4+x^5$, &c.

In this Example. $\left.\right\}$ $\frac{\frac{1}{2}}{1+\frac{1}{2}}=\frac{1}{2}-\frac{1}{4}+\frac{1}{8}-\frac{1}{16}+\frac{1}{32}$, &c.

That is $\frac{3}{2})\frac{1}{2}(\frac{2}{6}=\frac{1}{3}=\frac{1}{2}-\frac{1}{4}+\frac{1}{8}-\frac{1}{16}+\frac{1}{32}$, &c.

QUEST. CLXXXVII. The sum (b) of the infinite series $\frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\frac{4}{16}$, &c. is required?

Put $\frac{1}{2}=x$; And let $\frac{z}{\overline{1-x}^2}=b$;

Then $\frac{z}{\overline{1-x}^2}=x+2x^2+3x^3+4x^4+5x^5$, &c.

And - - $z=\overline{1-x}^2\times\overline{x+2x^2+3x^3+4x^4, \text{ \&c.}}$

But - - $x=\overline{1-x}^2\times\overline{x+2x^2+3x^3+4x^4, \text{ \&c.}}$ as will appear by actually multiplying.

Th. - - $z=x$,

And - $\frac{x}{\overline{1-x}^2}=(b=)\ x+2x^2+3x^3+4x^4$, &c.

In this Exam. $\frac{\frac{1}{2}}{\overline{1-\frac{1}{2}}^2}=\frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\frac{4}{16}+\frac{5}{32}$, &c.

That is $\frac{1}{4})\ \frac{1}{2}\ (\frac{4}{2}=2=\frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\frac{4}{16}+\frac{5}{32}$, &c.

COROL.

$\frac{x}{\overline{1+x}^2}=(b=)\ x-2x^2+3x^3-4x^4$, &c.

That is $\frac{9}{4})\ \frac{1}{2}\ (\frac{4}{18}=\frac{2}{9}=\frac{1}{2}-\frac{2}{4}+\frac{3}{8}-\frac{4}{16}+\frac{5}{32}$, &c.

This will appear by a similar process.

QUEST. CLXXXVIII. The sum (p) of n terms of the series $\frac{a}{m}+\frac{a+d}{mr}+\frac{a+2d}{mr^2}+\frac{a+3d}{mr^3}$, &c. is required?

The $\overline{n+1}$th terms of this series is $\frac{a+nd}{mr^n}$;

And the series after the nth term will be $\Big\}\ \frac{a+nd}{mr^n}+\frac{a+\overline{n+1}\times d}{mr^{n+1}}$, &c.

Th. putting $a+nd$ for a; and mr^n for m in the result of the last question.

The sum of all the terms after the nth will be $\Big\}\ \frac{\overline{a+nd}\times\overline{1-x}+dx}{mr^n\times\overline{1-x}^2}$;

But the sum of n terms

$$p=\frac{a\times\overline{1-x}+dx}{m\times\overline{1-x}^2}-\frac{\overline{a+nd}\times\overline{1-x}+dx}{mr^n\times\overline{1-x}^2};$$

Or $$p=\frac{\overline{\overline{r^n-1}\times a-nd}\times\overline{r-1}+\overline{r^n-1}\times d}{mr^n\times\overline{r-1}^2}r.$$

See the following Question.

QUEST. CLXXXIX. It is required to find the sum (c) of the infinite series $\frac{a}{m}+\frac{a+d}{mr}+\frac{a+2d}{mr^2}+\frac{a+3d}{mr^3}+\frac{a+4d}{mr^4}$, &c. arising by dividing the terms of an arithmetical, by those of a geometrical progression?

Let $\frac{1}{r}=x$; And $\frac{z}{m\times\overline{1-x}^2}=c$;

Then $\frac{z}{m\times\overline{1-x}^2}=\frac{a}{m}+\frac{a+d}{mr}+\frac{a+2d}{mr^2}$, &c.

Or $\frac{z}{\overline{1-x}^2}=a+\frac{a+d}{r}+\frac{a+2d}{r^2}$, &c.

That is $\frac{z}{\overline{1-x}^2}=a+\overline{a+d}\times x+\overline{a+2d}\times x^2$, &c.

Th. $z=\overline{1-x}^2\times\left\{\begin{matrix}a+\overline{a+d}\times x\\+\overline{a+2d}\times x^2\end{matrix}\right\}$ &c.

But $\overline{1-x}\times a+dx$

$=\overline{1-x}^2\times\overline{a+\overline{a+d}\times x+\overline{a+2d}\times x^2\text{, \&c.}}$

Which will appear by performing the Operation.

Th.

Th[illegible] $z = a \times \overline{1-x+dx}$,

And $\dfrac{a \times \overline{1-x+dx}}{m \times \overline{1-x}^2} = (t=)\ \dfrac{a}{m} + \dfrac{a+d}{mr} + \dfrac{a+2d}{mr^2}$, &c.

Corol.

$$\frac{a \times \overline{1+x-dx}}{m \times \overline{1+x}^2} = (t=)\ \frac{a}{m} - \frac{a+d}{mr} + \frac{a+2d}{mr^2}, \text{ \&c.}$$

This will be obtained by a similar process.

Now if $\dfrac{1}{r}$ be wrote for x;

Then $t = \dfrac{a \times \overline{1 - \frac{1}{r} + \frac{d}{r}}}{m \times \overline{1 - \frac{2}{r} + \frac{1}{rr}}} = \dfrac{a \times \overline{r-1+d}}{m \times \overline{r-1}^2} \times r$:

$$t = \frac{a \times \overline{1 + \frac{1}{r} - \frac{d}{r}}}{m \times \overline{1 + \frac{2}{r} - \frac{1}{rr}}} = \frac{a \times \overline{r+1-d}}{m \times \overline{r+1}^2} \times r.$$

QUEST. CXC. The sum (d) of the infinite series $\frac{1}{3}+\frac{4}{9}+\frac{9}{27}+\frac{16}{81}$, &c. is required?

Put $\frac{1}{3}=x$; And let $\frac{z}{\overline{1-x}^3}=d$;

Then $\frac{z}{\overline{1-x}^3}=x+4x^2+9x^3+16x^4+25x^5$, &c.

And $z=\overline{1-x}^3\times\overline{x+4x^2+9x^3+16x^4\text{, \&c.}}$

But $x+x^2=\overline{1-x}^3\times\overline{x+4x^2+9x^3+16x^4\text{, \&c.}}$

See the operation.

$$
\begin{array}{l}
x+4x^2+9x^3+16x^4+25x^5+36x^6\text{, \&c.}\\
1-3x+3x^2-x^3\\
\hline
x+4x^2+9x^3+16x^4+25x^5+36x^6\text{, \&c.}\\
\quad -3x^2-12x^3-27x^4-48x^5-75x^6\text{, \&c.}\\
\qquad\qquad +3x^3+12x^4-27x^5+48x^6\text{, \&c.}\\
\qquad\qquad\qquad -x^4-4x^5-9x^6\text{, \&c.}\\
\hline
x+xx+0+0+0+0;\\
\hline
\end{array}
$$

Th. $x+x^2=z$,

Or $\frac{x\times\overline{1+x}}{\overline{1-x}^3}=(d=)\ x+4x^2+9x^3+16x^4$, &c.

QUEST.

QUEST. CXCI. The sum (q) of n terms of the series $\frac{a}{m}+\frac{a+D}{mr}+\frac{a+2D+d}{mrr}+\frac{a+3D+3d}{mr^3}$, is required?

The $\overline{n+1}$th term of this series is $\frac{a+nD+\frac{1}{2}.n.\overline{n-1}.d}{mr^n}$,

The $\overline{n+2}$th - - - - - - $\frac{a+\overline{n+1}.D+\frac{1}{2}\overline{n+1}.n.d}{mr^{n+1}}$.

Th. (putting $\left\{\begin{array}{ll} a+nD+\frac{1}{2}.n.\overline{n-1}.d, & \text{for } a, \\ D+ \quad nd, & \text{for } D, \\ mr^n, & \text{for } m, \end{array}\right\}$ in the result of the next qu.)

The sum of all the series after the nth term

$$=\frac{\overline{a+nD+\frac{1}{2}.n.\overline{n-1}.d}\times\overline{1-x}^2+\overline{D+nd}\times\overline{1-x}\times x+dx^2}{mr^n\times\overline{1-x}^3},$$

And q

$$=\frac{a\times\overline{1-x}^2+Dx\times\overline{1-x}+dx^2}{m\times\overline{1-x}^3}$$

$$-\frac{\overline{a+nD+\frac{1}{2}.n.\overline{n-1}.d}\times\overline{1-x}^2+\overline{D+nd}\times x\times\overline{1-x}+dx^2}{mr^n\times\overline{1-x}^3}.$$

Or $q=$

$$\frac{1}{mr^n\times\overline{1+x}^3}\times\left\{\frac{\overline{\overline{r^n-1}\times a-nD-\frac{1}{2}n.\overline{n-d}}.\times\overline{1-x}^2}{+\overline{r^n-1}\times.\overline{D-nd}\times x\times\overline{1-x}\times\overline{r^n-1}\times dx^2}\right.:$$

If $\frac{1}{r}$ be wrote for x;

Then $q=$

$$\frac{1}{mr^n\times\overline{r-1}^3}\times\left\{\frac{\overline{\overline{r^n-1}\times a-nD-\frac{1}{2}n.\overline{n-1}.d}\times\overline{r-1}^2}{+\overline{r^n-1}\times\overline{D-nd}\times\overline{r-1}+\overline{r^n-1}\times d\times r.}\right.$$

QEST.

QUEST. CXCII It is required to find the sum (s) of the infinite series $\frac{a}{m}+\frac{a+D}{mr}+\frac{a+2D+d}{mr^2}+\frac{a+3D+3d}{mr^3}$, &c. arising by dividing the terms of a series whose second differences are equal, by those of a geometrical progression?

Let $\frac{1}{r}=x$; And $\frac{z}{m\times\overline{1-x}^3}=s$;

Then $\frac{z}{m\times\overline{1-x}^3}=\frac{a}{m}+\frac{a+D}{mr}+\frac{a+2D+d}{mr^2}$, &c.

Or $\frac{z}{\overline{1-x}^3}=a+\overline{a+D}\times x+\overline{a+2D+d}\times xx$, &c.

Th. $z=\overline{1-x}^3\times\left\{\begin{array}{l}a+\overline{a+D}\times x+\\ \overline{a+2D+d}\times x^2, \text{ \&c.}\end{array}\right.$

See the operation.

$$a+\overline{a+D}\times x+\overline{a+2D+d}\times xx, \text{ \&c.}$$
$$1-\quad 3x+\quad 3x^2\quad -x^3=\overline{1-x}^3$$

$$a+\overline{a+D}\times x+\overline{a+2D+d}\times x^2, \text{ \&c.}$$
$$-3a\times x-\overline{3a+3D}\times x^2, \text{ \&c.}$$
$$+3a\times x^2, \text{ \&c.}$$

$$a-2ax+Dx+ax^2-Dx^2+dx^2, \qquad +0$$

Th. $z=a\times\overline{1-2x+x^2}+D\times\overline{x-x^2}+dx^2$,

Or $z=a\times\overline{1-x}^2+D\times\overline{1-x}\times x+dx^2$;

Th.

$$\text{Th. } \frac{a\times\overline{1-x}^2+Dx\times\overline{1-x}+dx^2}{m\times\overline{1-x}^3}$$

$$=(\varrho=)\frac{a}{m}+\frac{a+D}{mr}+\frac{a+2D+d}{mrr}, \text{ \&c.}$$

$$\text{Corol. } \frac{a\times\overline{1+x}^2-Dx\times\overline{1+x}+dx^2}{m\times\overline{1+x}^3}$$

$$=(\varrho=)\frac{a}{m}-\frac{a+D}{mr}+\frac{a+2D+d}{mrr}, \text{ \&c.}$$

If $\frac{1}{r}$ be wrote for x;

$$\text{Then } \varrho=\frac{a\times\overline{1-\frac{2}{r}+\frac{1}{rr}}+\frac{D}{r}\times\overline{1-\frac{1}{r}}+\frac{d}{rr}}{m\times\overline{1-\frac{3}{r}+\frac{3}{rr}-\frac{1}{r^3}}}$$

$$=\frac{a\times\overline{r-1}^2+D\times\overline{r-1}+d}{m\times\overline{r-1}^3}r;$$

$$\text{And } \varrho=\frac{a\times\overline{r+1}^2-D\times\overline{r+1}+d}{m\times\overline{r+1}^3}r.$$

Quest.

QUEST. CXCIII. The sum (f) of the infinite series $\frac{a}{m}+\frac{a+D}{mr}+\frac{a+2D+\Delta}{mr^2}+\frac{a+3D+3\Delta+d}{mr^3}+\frac{a+4D+6\Delta+4d}{mr^4}$, &c. arising from the division of the terms of a series whose third differences are equal, by those of a geometrical progression, is required?

Now if $\frac{1}{r}=x$; And $\frac{z}{m\times\overline{1-x}|^4}=f$;

Then $\frac{z}{m\times\overline{1-x}|^4}=\frac{a}{m}+\frac{a+D}{mr}+\frac{a+2D+\Delta}{mr^2}$, &c.

Or $\frac{z}{\overline{1-x}|^4}=a+\overline{a+D}\times x+\overline{a+2D+\Delta}\times x^2$, &c.

Th. $z=\overline{1-x}|^4\times\left\{\begin{array}{l}a+\overline{a+D}\times x+\overline{a+2D+\Delta}\times x^2\\+\overline{a+3D+3\Delta+d}\times x^3\text{, \&c.}\end{array}\right.$

But $a-3ax+Dx+3ax^2-2Dx^2+\Delta x^2-ax^3+Dx^3-\Delta x^3+dx^3$, is the product of that multiplication.

Th. $z=a\times\overline{1-x}|^3+Dx\times\overline{1-x}|^2+\Delta x^2\times\overline{1-x}+dx^3$;

And writing $\frac{1}{r}$ for x,

$$f=\frac{a}{m}+\frac{a+D}{mr}+\frac{a+2D+\Delta}{mr^2}+\frac{a+3D+3\Delta+d}{mr^3}\text{, \&c.}$$

$$=\frac{a\times\overline{r-1}|^3+D\times\overline{r-1}|^2+\Delta\times\overline{r-1}+d}{m\times\overline{r-1}|^4}r.$$

Also

Alſo $\mathfrak{f} = \frac{a}{m} - \frac{a+D}{mr} + \frac{a+2D+\Delta}{mr^2} - \frac{a+3D+3\Delta+d}{mr^3}$, &c.

$$= \frac{a \times \overline{r+1}^3 - D \times \overline{r+1}^2 + \Delta \times \overline{r+1} - d}{m \times \overline{r+1}^4} r.$$

Corol. From the ſolutions of the preceding queſtions it will follow, that the ſum of an infinite ſeries of fractions, the numerators of which are a rank of quantities whoſe pth differences are equal, and their denominators the terms of a geometrical progreſſion (putting

$a =$ the numerator of the firſt term;
$m =$ the denominator of the firſt term;
$d' =$ the firſt of the firſt differences;
$d'' =$ the firſt of the ſecond differences, &c.
$d^p =$ the pth difference;

and $r =$ the ratio of the geom. progreſſion) will be

$$\frac{a \times \overline{r-1}^p + d' \times \overline{r-1}^{p-1} + d'' \times \overline{r-1}^{p-2}, \text{ \&c. to } d^p}{m \times \overline{r-1}^{p+1}} r.$$

Quest.

QUEST. CXCIV. The ſum (t) of n terms of the ſeries $\frac{a}{m}+\frac{a+D}{mr}+\frac{a+2D+\Delta}{mr^2}+\frac{a+3D+3\Delta+d}{mr^3}$, &c. is required?

By uſing a proceſs ſimilar to that in queſt. 191. it will appear that

$$t=\frac{1}{mr^n\times\overline{1-x}^4}\times\left\{\begin{array}{l}\overline{r^n-1}\times\overline{a-nD-\frac{1}{2}.n.\overline{n-1}\,\Delta-\frac{1}{6}.n.\overline{n-1}.\overline{n-2}.d}\times\overline{1-x}^3\\ +\overline{r^n-1}\times\overline{D-n\Delta-\frac{1}{2}.n.\overline{n-1}.d}\times x\times\overline{1-x}^2\\ +\overline{r^n-1}\times\overline{\Delta-nd}\times x^2\times\overline{1-x}+\overline{r^n-1}\times dx^3\end{array}\right.$$

Or $$t=\frac{r}{mr^n\times\overline{r-1}^4}\times\left\{\begin{array}{l}\overline{r^n-1}\times\overline{a-nD-\frac{1}{2}.n.\overline{n-1}\,\Delta-\frac{1}{6}.n\,\overline{n-1}.\overline{n-2}.d}\times\overline{r-1}^3\\ +\overline{r^n-1}\times\overline{D-n\Delta-\frac{1}{2}.n.\overline{n-1}.d}\times\overline{r-1}^2\\ +\overline{r^n-1}\times\overline{\Delta-nd}\times\overline{r-1}+\overline{r^n-1}\times d.\end{array}\right.$$

QUEST.

COROL. Hence the sum of n terms of a series of fractions, the numerators of which, are a rank of quantities whose pth differences are equal; and their denominators the terms of a geometrical progression; (the symbols being as in Corol. Quest. 193.) will be the product of $\frac{1}{mr^{n-1} \times \overline{r-1}|^{p+1}}$ into this series;

$$\overline{r-1}|^{p} \times \overline{r^n-1} \times a - na' + \frac{n.\overline{n-1}}{1.2} d'' - \frac{n.\overline{n-1}.\overline{n-2}}{1.2.3} d''', \&c. +$$

$$\overline{r-1}|^{p-1} \times \overline{r^n-1} \times d' - nd'' - \frac{n.\overline{n-1}}{1.2} d''' - \frac{n.\overline{n-1}.\overline{n-2}}{1.2.3} d^{iv}, \&c. +$$

$$\overline{r-1}|^{p-2} \times \overline{r^n-1} \times d'' - nd''' - \frac{n.\overline{n-1}}{1.2} d^{iv} - \frac{n.\overline{n-1}.\overline{n-2}}{1.2.3} d^{v}, \&c. +$$

&c. to

$$1 \times \overline{r^n-1} \times d^p.$$

QUEST. CXCV. It is required to find a series (?) which shall have the same relation to $\frac{1}{1-n}$; that 2L hath to $\overline{\frac{1}{1-n}}\Big|^2$; that is to find a series expressing the logarithm of $\frac{1}{1-n}$?

Since by qu. 185. $\frac{1}{1-n}=1+n+n^2+n^3$, &c. ad infin.

Let - - - $L.\frac{1}{1-n}=xn+yn^2+zn^3$, &c.

Then - $L.\overline{\frac{1}{1-n}}\Big|^2=2xn+2yn^2+2zn^3$, &c. by qu.

But - - $\overline{\frac{1}{1-n}}\Big|^2=\frac{1}{1-2n+nn}=\frac{1}{1-\overline{2n-nn}}$;

And - $L.\frac{1}{1-\overline{2n-nn}}=x\times\overline{2n-nn}+y\times\overline{2n-nn}^2$

$[+z\times\overline{2n-nn}^3$, &c.

By the above assumption; writing therein $2n-nn$, for n:

Now $\overline{2n-nn}\times x=2xn-xnn$;

$\overline{2n-nn}^2\times y= \quad +4ynn-4yn^3+yn^4$;

$\overline{2n-nn}^3\times z= \quad +8zn^3-12zn^4+6zn^5$

$\overline{2n-nn}^4\times u= \quad +16un^4+32un^5$

Th. $L.\overline{\frac{1}{1-n}}\Big|^2=2xn-xn^2-4yn^3+yn^4+6zn^5$, &c.

$+4y+8z-12z-32u$, &c.

$+16n+32e$, &c.

Now

Now by Corol. Quest. 251. Part I.

$$\left.\begin{array}{l} 2x=2x; \\ 2y=4y-x; \\ 2z=8z-4y; \\ 2u=16u-12z+y; \\ 2e=32e-32u+6z; \end{array}\right\} \text{Therefore} \left\{\begin{array}{l} x \text{ may be assumed}; \\ x=2y; \\ 4y=6z; \\ 12z=14u+y; \\ 32u=30e+6z; \end{array}\right.$$

&c. &c. &c. &c.

$$\text{Or} \left\{\begin{array}{l} y=\tfrac{1}{2}x; \\ 2x=6z; \\ 4x=14u+\tfrac{1}{2}x; \\ 8x=30e+2x; \end{array}\right\} \text{Whence} \left\{\begin{array}{l} \tfrac{1}{3}x=z; \\ \tfrac{1}{4}x=u; \\ \tfrac{1}{5}x=e: \end{array}\right.$$

&c. &c. &c.

Th. $L.\frac{1}{1-n}=xn+\frac{x}{2}n^2+\frac{x}{3}n^3+\frac{x}{4}n^4+\frac{x}{5}n^5$, &c.

Corol. By a similar process, the logarithm of

$1+n=xn-\frac{x}{2}n^2+\frac{x}{3}n^3-\frac{x}{4}n^4+\frac{x}{5}n^5$, &c.

QUEST. CXCVI. A series expressing the logarithm of any number (m) is required?

Let - - $m=\frac{1+n}{1-n}$; Then $m-nm=1+n$;

Or - $m-1=mn+n$; Th. $\frac{m-1}{m+1}=n$.

Now $L.\frac{1}{1-n}=xn+\frac{x}{2}n^2+\frac{x}{3}n^3+\frac{x}{4}n^4+\frac{x}{5}n^5$, &c.

And $L.\frac{1+n}{1}=xn-\frac{x}{2}n^2+\frac{x}{3}n^3-\frac{x}{4}n^4+\frac{x}{5}n^5$; &c.

But the sum of two logarithms, is the logarithm of the product of their corresponding numbers:

Th. $L.\frac{1+n}{1-n}=2xn+\frac{2x}{3}n^3+\frac{2x}{5}n^5$, &c.

And - $L.m=2x\times\overline{\frac{m-1}{m+1}+\frac{1}{3}\times\overline{\frac{m-1}{m+1}}\Big|^3+\frac{1}{5}\times\overline{\frac{m-1}{m+1}}\Big|^5}$, &c.

EXAM. What is the logarithm of the number 2?

Assume $x=1$; Then $2x=2$, And $\frac{m-1}{m+1}\left(=\frac{2-1}{2+1}=\right)\frac{1}{3}$ $=n$: Now,

	which powers of n being severally divided by	quote
n =,333333333,	1	,333333333;
n^3 =,037037037,	3	,012345679;
n^5 =,004115226,	5	,000823045;
n^7 =,000457247,	7	,000065321;
n^9 =,000050805,	9	,000005645;
n^{11} =,000005645,	11	,000000513;
n^{13} =,000000627,	13	,000000048;
n^{15} =,000000070,	15	,000000005;
		,346573589:

Th. (,346573589 × 2 =) ,693147178 is the logarithm of 2.

QUEST.

QUEST. CXCVII. The logarithms of two numbers, a and b, whose difference is 2, being given; to find the logarithm of ($\frac{1}{2}\times\overline{a+b}$) the intermediate number?

Now - $\frac{1}{4}\times\overline{a+b}^2=\frac{1}{4}aa+\frac{1}{2}ab+\frac{1}{4}bb$

And $\frac{1}{4}\times\overline{a+b}^2-ab=(\frac{1}{4}aa-\frac{1}{2}ab+\frac{1}{4}bb=)\ \frac{1}{4}\times\overline{a-b}^2$

But - $\frac{1}{2}\times\overline{a-b}=1$, by quest.

Th. $\frac{1}{4}\times\overline{a+b}^2-ab=1$.

By writing $\frac{k}{p}$ for m in quest. 196.

L. $\frac{k}{p}=2x\times\overline{\frac{k-p}{k+p}+\frac{1}{3}\times\left.\frac{k-p}{k+p}\right|^3}$, &c.

Th. L. $\frac{\frac{1}{4}\times\overline{a+b}^2}{ab}$ (putting $\frac{1}{4}\times\overline{a+b}^2+ab=d$ will be)

$=2x\times\overline{\frac{1}{d}+\frac{1}{3d^3}+\frac{1}{5d^5}}$, &c.

Now $\frac{1}{2}$ the logarithm of any square number, is the logarithm of its root;

Th. L. $\frac{\frac{1}{2}\times\overline{a+b}}{\sqrt{ab}}=x\times\overline{\frac{1}{d}+\frac{1}{3d^3}+\frac{1}{3d^5}}$, &c.

But - L. $\frac{a+b}{2}=$L. $\sqrt{ab}+x\times\overline{\frac{1}{d}+\frac{1}{3d^3}}$, &c.

And L. $\sqrt{ab}=\frac{A+B}{2}$; (where $A=L.a$; and $B=L.b$):

Th. - L. $\frac{a+b}{2}=\frac{A+B}{2}+x\times\overline{\frac{1}{d}+\frac{1}{3d^3}}$, &c.

EXAM.

EXAM. I. What is the logarithm of the number 3?

Here - - $a=4$; - - - - $A=1,386294356$;
And - - $b=2$; - - - - $B=0,693147178$;

Th. - $a+b=6$; - - $A+B=2,079441534$;

Th. $\frac{1}{2}\times\overline{a+b}=3$; - $\frac{1}{2}\times\overline{A+B}=1,039720767$;

And $\overline{\frac{1}{2}\times\overline{a+b}}^{2}=9$;

Also $(\overline{\frac{1}{2}\times\overline{a+b}}^{2}+ab=(9+8=)\ 17=d$;
Let $\frac{1}{17}=n$;
Then $n=0,058823529$, $\frac{1}{1}$ of which $0,058823529$;
$n^3=0,000203542$, $\frac{1}{3}$ of which $0,000067847$;
$n^5=0,000000704$, $\frac{1}{5}$ of which $0,000000141$;

Th. the logarithm of 3 is - - 1,098612284.

EXAM. II. What is the logarithm of the number 5?

Here - - - $a=6$; - And $A=1,791759462$;
And - - - $b=4$; - - - $B=1,386294356$;

Th. - - $a+b=10$; - - $A+B=3,178053818$;

And - $\frac{1}{2}\times\overline{a+b}=5$; $\frac{1}{2}\times\overline{A+B}=1,589026909$:

Now $\overline{\frac{1}{2}\times\overline{a+b}}^{2}=25$;

And $(\overline{\frac{1}{2}\times\overline{a+b}}^{2}+ab=25+24=)\ 49=d$:
Let $n=\frac{1}{49}$,
Then $n=0,020408163$, $\frac{1}{1}$ of which $0,020408163$;
$n^3=0,000008500$, $\frac{1}{3}$ of which $0,000002833$;
$n^5=0,000000004$, $\frac{1}{5}$ of which $0,000000001$;

Th. the logarithm of 5 is - - 1,609437906.

COROL. I. Hence L. 10=(L 5+L. 2=) 2,302585084, &c.

SCHOLIUM. Hitherto the quantity x, in the logarithmic series (by many able mathematicians, justly called the *Modulus*) has been assumed $=1$; which assumption produces *Neper*'s logarithms: But experience has shewn that *Brigg*'s logarithms (where 1 is the logarithm of the number 10) are best adapted to practice; it remains therefore to find the *Modulus* which will produce them:

Let x, and y, be the *Modulus*'s of two kinds of logarithms;

And m, any number,

Then L. m, of the $\begin{cases} \text{1st kind } = 2x \times \overline{\dfrac{m-1}{m+1} + \dfrac{1}{3} \times \left.\dfrac{m-1}{m+1}\right|^3}, \text{ \&c.} \\ \text{2d kind } = 2y \times \overline{\dfrac{m-1}{m+1} + \dfrac{1}{3} \times \left.\dfrac{m-1}{m+1}\right|^3}, \text{ \&c.} \end{cases}$

But $2x \times \dfrac{m-1}{m+1}$, &c. : $2y \times \dfrac{m-1}{m+1}$, &c. :: $x : y$;

Th. *Neper*'s L. m : *Brigg*'s L. m :: *Neper*'s *Modulus* : *Briggs*'s *Modulus*:

Th. 2,3025, &c. : 1 :: 1 : 0,43429448, &c. = *Brigg*'s *Modulus*.

Now if the before found Logs. of 2, 3, and 5, be multiplied by 0,43429448, &c. *Briggs*'s Log. of those numbers will be produced.

EXAM. III. What is *Briggs*'s logarithm of the number 7?

Here - - $a = 8$; - - - - $A = 0,90308997$;
And - - $b = 6$; - - - - $B = 0,77815125$;

Th. - $a + b = 14$; - - $A + B = 1,68124122$;

And $\frac{1}{2} \times \overline{a+b} = 7$; - $\frac{1}{2} \times \overline{A+B} = 0,84062061$;

Also $\frac{1}{2} \times \overline{a+b}^2 = 49$;

Th. ($\frac{1}{2} \times \overline{a+b}^2 + ab = 49 + 48 =$) $97 = d$:
Let $\frac{1}{97} = n$.

Now $x = 0,434294482$,

And $n\,x = 0,004477263$, $\frac{1}{1}$ of which $0,00447726$;
$n^3\,x = 0,000000476$, $\frac{1}{3}$ of which $0,00000016$;

Th. the logarithm of 7 is - - - - 0,84509803.

COROL.

COROL. II. From due observation of the 3 last examples; the following rule, for finding *Briggs*'s logarithm to 7 places of figures, of any prime number (p) greater than 7, will be easily deduced:

$$\text{L}.\,p = \frac{\text{L}.\overline{p+1} + \text{L}.\overline{p-1}}{2} + \frac{0.43429448}{pp + \overline{pp-1}};$$

Which may be of use in examining the common tables of logarithms.

EXAM. What is *Briggs*'s logarithm of 11?

$11 \times 11 = 121,$
$11 \times 11 - 1 = 120;$

241

L. 12=1,07918124,
L. 10=1,00000000,

2)2,07918124;

1,03959062;

And 241)0,43429448(0,00180205 = 0,00180205;

L. 11=1,04139267.

```
241)0,43429448(0,00180205
    241
    ----
    1932
    1928
    ----
       494
       482
       ---
        1248
```

QUEST. CXCVIII. The sum (s) of n terms of the series, $\frac{1}{a}+\frac{1}{a-d}+\frac{1}{a-2d}+\frac{1}{a-3d}$, &c. is required?

Now $\frac{1}{a}=\frac{1}{a}$;

$$\frac{1}{a-d}=\frac{1}{a}+\frac{d}{aa}+\frac{d^2}{a^3}+\frac{d^3}{a^4}, \text{ \&c.}$$

$$\frac{1}{a-2d}=\frac{1}{a}+\frac{2d}{aa}+\frac{4d^2}{a^3}+\frac{8d^3}{a^4}, \text{ \&c.}$$

$$\frac{1}{a-3d}=\frac{1}{a}+\frac{3d}{aa}+\frac{9d^2}{a^3}+\frac{27d^3}{a^4}, \text{ \&c.}$$

$$\text{Th. } s=n \text{ terms of} \left\{ \begin{array}{l} \overline{1+1+\ 1+\ 1, \text{ \&c.}} \times \frac{1}{a}+ \\ \overline{0+1+\ 2+\ 3, \text{ \&c.}} \times \frac{1}{a} \times \frac{d}{a}+ \\ \overline{0+1+\ 4+\ 9, \text{ \&c.}} \times \frac{1}{a} \times \frac{d^2}{a^2}+ \\ \overline{0+1+\ 8+27, \text{ \&c.}} \times \frac{1}{a} \times \frac{d^3}{a^3}+ \\ \overline{0+1+16+81, \text{ \&c.}} \times \frac{1}{a} \times \frac{d^4}{a^4}, \text{ \&c.} \end{array} \right.$$

But by quest. 49. n terms of

$1+1+\ 1+\ 1$, &c. $=n$,

$0+1+\ 2+\ 3$, &c. $=\frac{nn}{2}-\frac{n}{2}$,

$0+1+\ 4+\ 9$, &c. $=\frac{n^3}{3}-\frac{nn}{2}+\frac{n}{6}$,

$0+1+\ 8+27$, &c. $=\frac{n^4}{4}-\frac{n^3}{2}+\frac{nn}{4}$,

$0+1+16+81$, &c. $=\frac{n^5}{5}-\frac{n^4}{2}+\frac{n^3}{3}-\frac{n}{30}$;

&c. &c.

Th.

Th. (putting $\frac{d}{a}=q$)

$$s=\frac{1}{a}\times\left\{\begin{array}{l} n+ \\ \frac{n^2q}{2}-\frac{nq}{2}+ \\ \frac{n^3q^2}{3}-\frac{n^2q^2}{2}+\frac{nq^2}{6}+ \\ \frac{n^4q^3}{4}-\frac{n^3q^3}{2}+\frac{n^2q^3}{4}+ \\ \frac{n^5q^4}{5}-\frac{n^4q^4}{2}+\frac{n^3q^4}{3}-\frac{nq^4}{30}, \text{ \&c.} \end{array}\right.$$

$$\text{Or } s=\frac{1}{a}\times\left\{\begin{array}{l} n+\frac{n^2q}{2}+\frac{n^3q^2}{3}+\frac{n^4q^3}{4}+\frac{n^5q^4}{5}, \text{ \&c.} \\ -\frac{nq}{2}-\frac{n^2q^2}{2}-\frac{n^3q^3}{2}-\frac{n^4q^4}{2}, \text{ \&c.} \\ +\frac{nq^2}{6}+\frac{n^2q^3}{4}+\frac{n^3q^4}{3}+\frac{5n^4q^5}{12}, \text{ \&c.} \\ -\frac{nq^4}{30}-\frac{n^2q^5}{12}-\frac{n^3q^6}{6}-\frac{7n^4q^7}{24}, \text{ \&c.} \\ +\frac{nq^6}{42}+\frac{n^2q^7}{12}+\frac{2n^3q^8}{9}+\frac{n^4q^9}{2}, \text{ \&c.} \end{array}\right.$$

$$\text{But }\left\{\begin{array}{ll} nq+\frac{n^2q^2}{2}+\frac{n^3q^3}{3}, \text{ \&c.} \quad - & =\text{L. }\frac{1}{1-nq}; \\ nq+n^2q^2+n^3q^3, \text{ \&c.} \quad - & =\frac{1}{1-nq}-1; \\ 2nq+3n^2q^2+4n^3q^3, \text{ \&c.} & =\frac{1}{\overline{1-nq}^2}-1; \\ 4nq+10n^2q^2+20n^3q^3, \text{ \&c.} & =\frac{1}{\overline{1-nq}^4}-1; \end{array}\right.$$

$$\text{Th.}\left\{\begin{array}{l} n+\frac{n^2q}{2}+\frac{n^3q^2}{3}, \text{ \&c. } = \frac{1}{q}\times \text{L.}\,\frac{1}{1-nq}; \\ \frac{nq}{2}+\frac{n^2q^2}{2}+\frac{n^3q^3}{2}, \text{ \&c. } = \overline{\frac{1}{1-nq}-1}\times\frac{1}{2}; \\ \frac{nq^2}{6}+\frac{n^2q^3}{4}+\frac{n^3q^4}{3}, \text{ \&c. } = \overline{\frac{1}{\overline{1-nq}^2}-1}\times\frac{q}{12}; \\ \frac{nq^4}{30}+\frac{n^2q^5}{12}+\frac{n^3q^6}{6}, \text{ \&c. } = \overline{\frac{1}{\overline{1-nq}^4}-1}\times\frac{q^3}{120}: \end{array}\right.$$

$$\text{Th. } s=\frac{1}{a}\times\left\{\begin{array}{l} \frac{1}{q}\times\text{L.}\,\frac{1}{1-nq}-\overline{\frac{1}{1-nq}-1}\times\frac{1}{2}+\overline{\frac{1}{\overline{1-nq}^2}-1}\times\frac{q}{12} \\ -\overline{\frac{1}{\overline{1-nq}^4}-1}\times\frac{q^3}{120}; \end{array}\right.$$

Or by writing $\frac{d}{a}$ for q

$$s=\frac{1}{a}\times\left\{\begin{array}{l} \frac{a}{d}\times\text{L.}\,\frac{a}{a-nd}-\overline{\frac{a}{a-nd}-1}\times\frac{1}{2}+\overline{\overline{\frac{a}{a-nd}}^2-1}\times \\ \frac{d}{12a}-\overline{\overline{\frac{a}{a-nd}}^4-1}\times\frac{d^3}{120a^3}, \text{ \&c.} \end{array}\right.$$

Lastly, puttig $\alpha=6$; $\beta=30$; $\gamma=42$, &c. that is α, β, γ, &c. = the denominators of those fractions denoted by α, β, γ, &c. in quest. 49; and $Q=\frac{a}{a-nd}$:

Then $s=\frac{L.Q}{d}-\frac{Q-1}{2a}+\frac{\overline{Q^2-1}\times d}{2\alpha aa}-\frac{\overline{Q^4-1}\times d^3}{4\beta a^4}$, &c.

Where the *Neperian* (not the common) logarithm of Q must be used in the first term.

COROL.

COROL. I. By a similar process, the sum ($\mathfrak{S}$) of n terms of the series $\frac{1}{e}+\frac{1}{e+d}+\frac{1}{e+2d}$, &c. may be found, viz. (putting $\frac{e}{e+nd}=\mathfrak{Q}$)

$$\mathfrak{S}=\begin{cases}\frac{1}{d}\times L.\frac{1}{\mathfrak{Q}}+\frac{1-\mathfrak{Q}}{2e}+\frac{\overline{1-\mathfrak{Q}^2}\times d}{2\alpha ee}- \\ \frac{\overline{1-\mathfrak{Q}^4}\times d^3}{4\beta e^4}+\frac{\overline{1-\mathfrak{Q}^6}\times d^5}{6\gamma e^6}-\frac{\overline{1-\mathfrak{Q}^8}\times d^7}{8\delta e^8}, \text{ \&c.}\end{cases}$$

SCHOLIUM. The values of $\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}$, being severally found by division; the quotients added together, make $2,82896825=x$:

Then the sum ($\mathfrak{S}$) of n terms of the series $\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$, &c. will be equal to

$$x+L.\frac{10+n}{10}+\frac{\overline{10+n}-10}{2.\overline{10+n}.10}+\frac{\overline{10+n}^2-\overline{10}^2}{2.\alpha\ \overline{10+n}^2.\overline{10}^2}$$

$$-\frac{\overline{10+n}^4-\overline{10}^4}{4\beta.\overline{10+n}^4.\overline{10}^4}+\frac{\overline{10+n}^6-\overline{10}^6}{6\gamma.\overline{10+n}^6.\overline{10}^6}, \text{ \&c.}$$

Now when n is very great in respect of 10

Then $\left(\frac{\overline{10+n}-10}{10+n}=\right)\frac{n}{10+n}=1$, nearly,

And $\mathfrak{S}=x+L.\frac{10+n}{10}+\frac{1}{20}+\frac{1}{2.\alpha.\overline{10}^2}-\frac{1}{4.\beta.\overline{10}^4}$, &c.

But $\frac{1}{20}+\frac{1}{2\alpha.\overline{10}^2}-\frac{1}{4\beta\,\overline{10}^4}$, &c. $=0,05083250$;

Th. $x+\frac{1}{20}+\frac{1}{2\alpha.\overline{10}^2}-\frac{1}{4\beta.\overline{10}^4}$, &c. $=2,87980075$;

And $\mathfrak{S}=2,87980075+$ Nep. L. $\frac{10+n}{10}$,

Or $\mathfrak{S}=2,87980075+2,3028509\times$ L. $\frac{10+n}{10}$;

Now when $n=\left\{\begin{array}{r}99999990\\999999990\\9999999990\end{array}\right\}$

Then L. $\frac{10+n}{10}=\left\{\begin{array}{l}8-1=7;\\9-1=8;\\10-1=9:\end{array}\right.$

And $\mathfrak{S}=2,87980075+\left\{\begin{array}{c}7\\8\\9\end{array}\right\}\times 2,3025$, &c.

$\left\{\begin{array}{l}=18,99789638;\\=21,30048147;\\=23,60306656.\end{array}\right.$

COROL.

COROL. II. Hence the sum of the whole series $\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$, &c. is infinite; for as often as a place of figures is added to the number of terms n, so often is 2,3025, &c. added to $\mathfrak{S}$; but when n is infinite, the number of its places of figures are infinite; and 2,3025, &c. must be added an infinite number of times.

EXAM. II. The sum of 2000000004 terms of the series $\frac{1}{1}-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\frac{1}{11}$, &c. is required?

That is, putting - - - $\frac{1}{1}-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}=x$,

1000000000 terms of - $\frac{1}{9}+\frac{1}{13}+\frac{1}{17}+\frac{1}{21}=\mathfrak{S}$,

And 1000000000 terms of $\frac{1}{11}+\frac{1}{15}+\frac{1}{19}+\frac{1}{23}=\mathfrak{S}'$;

To find $x+\mathfrak{S}-\mathfrak{S}'$

Now	$\frac{1}{1}=1$	$\frac{1}{3}=0,33333333$
	$\frac{1}{5}=0,2$	$\frac{1}{7}=0,14285714$
Affirm.	$=1,2$	$0,47619048$
Neg.	$=0,47619048$	
	$x=0,72380952$	

To find $\mathfrak{S}$; $e=9$; $n=1000000000$; $d=4$;

$$\mathcal{Q}=\frac{9}{9+4\times 1000000000};\quad \text{L.}\,\frac{1}{\mathcal{Q}}=\text{L.}\,\frac{4000000009}{9};$$

$$\text{And } \mathfrak{S}=\frac{\text{L.}4000000000-\text{L.}9}{4}+\frac{1}{2.9}+\frac{4}{2.\alpha.9^2},\ \&c.$$

To find $\mathfrak{S}'$; $e=11$; $n=1000000000$; $d=4$;

$$\mathcal{Q}=\frac{11}{11+4\times 1000000000};\quad \text{L.}\,\frac{1}{\mathcal{Q}}=\text{L.}\,\frac{4000000011}{11};$$

$$\text{And } \mathfrak{S}'=\frac{\text{L.}4000000000-\text{L.}11}{4}+\frac{1}{2.11}+\frac{4}{2.\alpha.11^2},\ \&c.$$

In both which Q is considered as inconsiderable, in any term after the first,

And L. 4000000000 $= \begin{cases} \text{L. } 4000000009 \\ \text{L. } 4000000011 \end{cases}$

Which is true in logarithms of no more than seven places,

Th. $\mathfrak{S} - \mathfrak{S}'$

$$= \frac{L.11 - L.9}{4} + \frac{1}{2.9} + \frac{4}{2\alpha.9^2} - \frac{4^3}{4\beta.9^4} + \frac{4^5}{6.\gamma.9^6},$$

$$- \frac{1}{2.11} - \frac{4}{2\alpha 11^2} + \frac{4^3}{4\beta.11^4} + \frac{4^5}{6.\gamma.11^6}, \text{ \&c.}$$

L. 11 = 1,0413927
L. 9 = 0,9542425

4)0,0871502

0,0217876 × 2,3025, &c. = 0,0501678:

$\frac{L.11 - L.9}{4} = 0,0501678$; $\frac{1}{2.11} = 0,0454545$;

$\frac{1}{2.9} = 0,0555556$; $\frac{4}{2.6.11^2} = 0,0027548$;

$\frac{4}{2.6.9^2} = 0,0041152$; $\frac{4^3}{4.30.9^4} = 0,0000813$;

$\frac{4^3}{4.30.11^4} = 0,0000364$; $\frac{4^5}{6\,42.11^6} = 0,0000023$;

$\frac{4^5}{6\,42.9^6} = 0,0000079$; $\frac{4^7}{8.30.9^8} = 0,0000015$;

Affirmative = 0,1098829   0,0482944
Negative = 0,0482944

$\mathfrak{S} - \mathfrak{S}'$ = 0,0615885 + (x =) 0,72380952 =
0,7853980 the Answer.

Corol. III. Hence the sum of an infinite series of the above form, may be found: M. *Leibnitz* has proved, that the area of a circle whose diameter is 1, is equal to $\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9}$, &c. ad infinitum; which is a sufficient verification hereof; for the area of such a circle is [illegible]53982, &c.

Quest.

QUEST. CXCIX. The sum (s) of n terms of the series, $\frac{1}{aa}+\frac{1}{\overline{a-d}^2}+\frac{1}{\overline{a-2d}^2}+\frac{1}{\overline{a-3d}^2}$, &c. is required?

Now $\frac{1}{aa}=\frac{1}{aa}$;

$$\frac{1}{\overline{a-d}^2}=\frac{1}{aa}+\frac{2.1d}{a^3}+\frac{3.1d^2}{a^4}+\frac{4.1d^3}{a^5}, \text{ \&c.}$$

$$\frac{1}{\overline{a-2d}^2}=\frac{1}{aa}+\frac{2.2d}{a^3}+\frac{3.4d^2}{a^4}+\frac{4.8d^3}{a^5}, \text{ \&c.}$$

$$\frac{1}{\overline{a-3d}^3}=\frac{1}{aa}+\frac{2.3d}{a^3}+\frac{3.9d^2}{a^4}+\frac{4.27d^3}{a^5}, \text{ \&c.}$$

$$\text{Th. } s = n \text{ terms of} \begin{cases} \overline{1+1+1+\ 1, \text{ \&c.}} \times \frac{1}{aa} + \\ \overline{0+1+2+\ 3, \text{ \&c.}} \times \frac{1}{aa} \times \frac{2d}{a} + \\ \overline{0+1+4+\ 9, \text{ \&c.}} \times \frac{1}{aa} \times \frac{3d^2}{a^2} + \\ \overline{0+1+8+27, \text{ \&c.}} \times \frac{1}{aa} \times \frac{4d^3}{a^3}, \text{ \&c.} \end{cases}$$

Th. (putting $\frac{d}{a}=q$)

$$s = \frac{1}{aa} \times \begin{cases} n + \\ \frac{2n^2q}{2} - \frac{2nq}{2} + \\ \frac{3n^3q^2}{3} - \frac{3n^2q^2}{2} + \frac{3nq^2}{6} + \\ \frac{4n^4q^3}{4} - \frac{4n^3q^3}{2} + \frac{4n^2q^3}{4} + \\ \frac{5n^5q^4}{5} - \frac{5n^4q^4}{2} + \frac{5n^3q^4}{3} - \frac{5nq^4}{30}, \text{ \&c.} \end{cases}$$

$$\text{But}\left\{\begin{array}{l}1+nq+n^2q^2,\ \&c.=\dfrac{1}{1-nq};\\[2mm] 2nq+3n^2q^2+4n^3q^3,\ \&c.=\dfrac{1}{\overline{1-nq}^2}-1;\\[2mm] 3nq+6n^2q^2+10n^3q^3,\ \&c.=\dfrac{1}{\overline{1-nq}^3}-1;\\[2mm] 5nq+15n^2q^2+35n^3q^3,\ \&c.=\dfrac{1}{\overline{1-nq}^5}-1;\end{array}\right.$$

$$\text{Th.}\left\{\begin{array}{l}n+\dfrac{2n^2q}{2}+\dfrac{3n^3q^2}{3},\ \&c.=\dfrac{n}{1-nq};\\[2mm] \dfrac{2nq}{2}+\dfrac{3n^2q^2}{2}+\dfrac{4n^3q^3}{2},\ \&c.=\overline{\dfrac{1}{\overline{1-nq}^2}-1}\times\dfrac{1}{2};\\[2mm] \dfrac{3nq^2}{6}+\dfrac{4n^2q^3}{4}+\dfrac{5n^3q^4}{3},\ \&c.=\overline{\dfrac{1}{\overline{1-nq}^3}-1}\times\dfrac{q}{6};\\[2mm] \dfrac{5nq^4}{30}+\dfrac{6n^2q^5}{12}+\dfrac{7n^3q^6}{6},\ \&c.=\overline{\dfrac{1}{\overline{1-nq}^5}-1}\times\dfrac{q^3}{30};\end{array}\right.$$

$$\text{and } t=\frac{1}{a^2}\times\left\{\begin{array}{l}\dfrac{n}{1-nq}-\overline{\dfrac{1}{\overline{1-nq}^2}-1}\times\tfrac{1}{2}+\overline{\dfrac{1}{\overline{1-nq}^3}-1}\times\dfrac{q}{6}\\[2mm] -\overline{\dfrac{1}{\overline{1-nq}^5}-1}\times\dfrac{q^3}{30}+\overline{\dfrac{1}{\overline{1-nq}^7}-1}\times\dfrac{q^5}{42};\end{array}\right.$$

Or by writing $\frac{d}{a}$ for q; and α, β, γ, &c. Q as in the laſt;

$$t=\frac{Q-1}{ad}-\frac{Q^2-1}{2aa}+\frac{\overline{Q^3-1}\times d}{\alpha a^3}-\frac{\overline{Q^5-1}\times d^3}{\beta a^5},\ \&c.$$

QUEST.

QUEST. CC. The sum (v) of n terms of the series, $\frac{1}{a^3}+\frac{1}{\overline{a-d}^3}+\frac{1}{\overline{a-2d}^3}+\frac{1}{\overline{a-3d}^3}+\frac{1}{\overline{a-4d}^3}$, &c. is required?

Now $\frac{1}{a^3}=\frac{1}{a^3}$;

$$\frac{1}{\overline{a-d}^3}=\frac{1}{a^3}+\frac{3.1d}{a^4}+\frac{6.1d^2}{a^5}+\frac{10.1d^3}{a^6}, \text{ \&c.}$$

$$\frac{1}{\overline{a-2d}^3}=\frac{1}{a^3}+\frac{3.2d}{a^4}+\frac{6.4d^2}{a^5}+\frac{10.8d^3}{a^6}, \text{ \&c.}$$

$$\frac{1}{\overline{a-3d}^3}=\frac{1}{a^3}+\frac{3.3d}{a^4}+\frac{6.9d^2}{a^5}+\frac{10.27d^3}{a^6}, \text{ \&c.}$$

Th, (putting $\frac{d}{a}=q$).

$$v=\frac{1}{a^3}\times\left\{\begin{array}{l} n+ \\ \frac{3n^2q}{2}-\frac{3nq}{2}+ \\ \frac{6n^3q^2}{3}-\frac{6n^2q^2}{2}+\frac{6nq^2}{6}+ \\ \frac{10n^4q^3}{4}-\frac{10n^3q^3}{2}+\frac{10n^2q^3}{4}+ \\ \frac{15n^5q^4}{5}-\frac{15n^4q^4}{2}+\frac{15n^3q^4}{3}-\frac{15nq^4}{30}, \text{ \&c.} \end{array}\right.$$

$$\text{But }\left\{\begin{array}{l} 1+\frac{3}{2}nq+2n^2q^2, \text{ \&c. } =\frac{2-nq}{2\times\overline{1-nq}^2}; \\ 3nq+6n^2q^2+10n^3q^3, \text{ \&c. } =\frac{1}{\overline{1-nq}^3}-1; \\ 4nq+10n^2q^2+20n^3q^3, \text{ \&c. } =\frac{1}{\overline{1-nq}^4}-1; \\ 6nq+21n^2q^2+56n^3q^3, \text{ \&c. } =\frac{1}{\overline{1-nq}^6}-1; \end{array}\right.$$

$$\text{Th.}\left\{\begin{aligned}
&n + \tfrac{3}{2}n^2q + \tfrac{6}{3}n^3q^2 = \frac{n\times\overline{2-nq}}{2\times\overline{1-nq}^2}\\
&\frac{3nq}{2} + \frac{6n^2q^2}{2} + \frac{10n^3q^3}{2} = \overline{\frac{1}{\overline{1-nq}^3}-1}\times\tfrac{1}{2}\\
&\frac{6nq^2}{6} + \frac{10n^2q^3}{4} + \frac{15n^3q^4}{3} = \overline{\frac{1}{\overline{1-nq}^4}-1}\times\frac{3q}{2\alpha}\\
&\frac{15nq^4}{30} + \frac{21n^2q^5}{12} + \frac{28n^3q^6}{6} = \overline{\frac{1}{\overline{1-nq}^6}-1}\times\frac{5q^3}{2.\beta}
\end{aligned}\right.$$

$$\text{Th. } v = \frac{1}{a^3}\times\left\{\begin{aligned}
&\frac{n\times\overline{2-nq}}{2\times\overline{1-nq}^2} - \overline{\frac{1}{\overline{1-nq}^3}-1}\times\tfrac{1}{2} + \overline{\frac{1}{\overline{1-nq}^4}-1}\times\frac{3q}{2\alpha}\\
&-\overline{\frac{1}{\overline{1-nq}^6}-1}\times\frac{5q^3}{2\beta} + \overline{\frac{1}{\overline{1-nq}^8}-1}\times\frac{7q^5}{2\gamma},\ \&c.
\end{aligned}\right.$$

Or uſing the ſame ſymbols as in the preceding queſtions.

$$v = \frac{\mathcal{Q}^2-1}{2a^2d} - \frac{\mathcal{Q}^3-1}{2a^3} + \frac{\overline{\mathcal{Q}^4-1}\times 3d}{2\alpha a^4} - \frac{\overline{\mathcal{Q}^6-1}\times 5d^3}{2\beta a^6},\ \&c.$$

Corol.

COROL. I. If $s =$ sum of the reciprocals, of the first power; t, of the second powers; v, of the third powers, &c. and z of mth powers, of an arithmetical progression: Then,

$$s = \frac{L.Q.}{d} - \frac{Q-1}{2a} + \frac{\overline{Q^2-1} \times d}{2\alpha a^2} - \frac{\overline{Q^4-1} \times d^3}{4\beta a^4} + \frac{\overline{Q^6-1} \times d^5}{6\gamma a^4}, \text{ \&c.}$$

$$t = \frac{Q-1}{ad} - \frac{Q^2-1}{2aa} + \frac{\overline{Q^3-1} \times d}{\alpha a^3} - \frac{\overline{Q^5-1} \times d^3}{\beta a^5} + \frac{\overline{Q^7-1} \times d^5}{\gamma a^7}, \text{ \&c.}$$

$$v = \frac{Q^2-1}{2a^2 d} - \frac{Q^3-1}{2a^3} + \frac{\overline{Q^4-1} \times 3d}{2\alpha a^4} - \frac{\overline{Q^6-1} \times 5d^3}{2\beta a^6} + \frac{\overline{Q^8-1} \times 7d^5}{2\gamma a^8}, \text{ \&c.}$$

$$w = \frac{Q^3-1}{3a^3 d} - \frac{Q^4-1}{2a^4} + \frac{\overline{Q^5-1} \times 2d}{\alpha a^5} - \frac{\overline{Q^7-1} \times 5d^3}{\beta a^7} + \frac{\overline{Q^9-1} \times 28d^5}{3\gamma a^9}, \text{ \&c.}$$

&c. &c.

$$z = \left\{ \begin{array}{l} \dfrac{Q^{m-1}-1}{\overline{m-1} \times a^{m-1} d} - \dfrac{Q^m-1}{2a^m} + \dfrac{\overline{Q^{m+1}-1} \times md}{2\alpha a^{m+1}} \\ - \dfrac{\overline{Q^{m+3}-1} \times m.\overline{m+1}.\overline{m+2}.d^3}{2.3.4.\beta a^{m+3}} \\ + \dfrac{\overline{Q^{m+5}-1} \times m.\overline{m+1}.\overline{m+2}.\overline{m+3}.\overline{m+4}.d^5}{2.3.4.5.6.\gamma a^{m+5}}, \text{ \&c.} \end{array} \right.$$

COROL.

Corol. II. If $\mathcal{Z}$ represent the sum of n terms of the series, $\frac{1}{e^m}+\frac{1}{\overline{e+d}^m}+\frac{1}{\overline{e+2a}^m}+\frac{1}{\overline{e+3d}^m}$, &c. (where $m \sqsubset 1$) put $\frac{e}{e+nd}=\mathcal{Q}$; Then,

$$\mathcal{Z}=\begin{cases} \dfrac{\overline{1-\mathcal{Q}^{m-1}}}{\overline{m-1}\times e^{m-1}d}+\dfrac{1-\mathcal{Q}^m}{2e^m}+\dfrac{\overline{1-\mathcal{Q}^{m+1}}\times md}{2ae^{m+1}} \\ -\dfrac{\overline{1-\mathcal{Q}^{m+3}}\times m.\overline{m+1}.\overline{m+2}\,d^3}{2.3.4.\beta.e^{m+3}}+ \\ \dfrac{\overline{1-\mathcal{Q}^{m+5}}\times m.\overline{m+1}.\overline{m+2}.\overline{m+3}.\overline{m+4}\,d^5}{2.3.4.5.6.\gamma.e^{m+5}}-\text{\&c.} \end{cases}$$

See Corol. I. Quest. 198.

Corol. III. But when n is a very great number, $\frac{e}{e+nd}=\mathcal{Q}$ will be very small, and may be rejected; and Then,

$$\mathcal{Z}=\frac{1}{\overline{m-1}\times e^{m-1}d}+\frac{1}{2e^m}+\frac{md}{2ae^{m+1}}-\frac{m.\overline{m+1}.\overline{m+2}.d^3}{2.3.4.\beta e^{m+3}},$$
&c.

Exam.

EXAM. I. What is the ſum of the infinite ſeries $\frac{1}{1}+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}$, &c.?

Let the ſum of the firſt 9 terms be found by ſo many diviſions - - - - $\Big\}=1,53976773$

And the ſum of the remaining terms beginning with the tenth, by Cor. III. $\Big\}=0,10516634$

Then $\frac{1}{1}+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}$, &c. ad infinit. $=1,64463407$

SCHOL. I. M. *John Bernoulli* has proved that the above ſeries is $\frac{1}{6}$ of the ſquare of the circumference of a circle whoſe diameter is 1: *See his* WORKS, Vol. IV. Page 20, 21, 22, &c.

Now the circumference of a circle, whoſe diameter is $1=3,14159265$, &c.

And $\frac{\overline{3,14159265,\text{ \&c.}}^2}{6}=1,64493406$, &c.

SCHOL. II. By proceeding in like manner the ſums of

$\frac{1}{1}+\frac{1}{8}+\frac{1}{27}+\frac{1}{64}$, &c. ad infinit. $=1,20205691$

$\frac{1}{1}+\frac{1}{27}+\frac{1}{125}+\frac{1}{343}$, &c. ad infinit. $=1,05179980$

$\frac{1}{8}+\frac{1}{64}+\frac{1}{216}+\frac{1}{512}$, &c. ad infinit. $=0,15025711$

And by queſt. 184.

$7:1::\frac{1}{1}+\frac{1}{27}+\frac{1}{125}$, &c. $:\frac{1}{8}+\frac{1}{64}+\frac{1}{216}$, &c.

But $1,05179980=0,15025711\times 7$

Which is a verification of the proceſs uſed in both claſſes of queſtions.

The two following queſtions were deſigned for the firſt part, but were omitted for want of room.

QUEST.

QUEST. CCI. It is required to find the values of x, y, and z, in the following equations, *viz.*

$$\left.\begin{aligned} xx+xy&=10 \\ yy+yz&=21 \\ zz+zx&=24 \end{aligned}\right\}?$$

Substitute $y=nx$; And $z=mx$:

$$\text{Then}\left\{\begin{aligned} xx+\ nxx&=10, \\ n^2x^2+nmx^2&=21, \\ m^2x^2+\ mx^2&=24, \end{aligned}\right\}\text{Or}\left\{\begin{aligned} xx&=\frac{10}{1+n}, \\ xx&=\frac{21}{n^2+nm}, \\ xx&=\frac{24}{m^2+m}; \end{aligned}\right.$$

$$\text{Th. } \frac{24}{m^2+m}=\frac{10}{1+n}; \quad \text{Th. } n=\frac{5m.\overline{m+1}-12}{12}:$$

$$\text{Also } \frac{24}{m^2+m}=\frac{21}{n^2+nm}; \quad \text{Th. } 8n^2+8nm=7m.\overline{m+1}:$$

$$\text{Now } n^2=\frac{25m^2.\overline{m+2}^2-120m.\overline{m+1}+144}{144},$$

$$\text{And } nm=\frac{5m^2\ \overline{m+1}-12m}{12};$$

$$\text{Th. } 7m.\overline{m+1}=\frac{25m^2.\overline{m+1}^2-120m.\overline{m+1}+144}{18}$$

$$+\frac{10m^2.\overline{m+1}-24m}{3},$$

Or $126m.\overline{m+1}=25m^2.\overline{m+1}^2-120m.\overline{m+1}+144$
$+60m^2.\overline{m+1}-144m$,

Or $246m.\overline{m+1}=25m^2.\overline{m+1}^2+144+60m^2.\overline{m+1}$
$-144m$,

Or $246m^2+246m=25m^4+50m^3+25m^2+144+60m^3$
$+60m^2-144m$,

Th. $0=25m^4+110m^3-161m^2-390m+144$:

$$\text{But } \frac{25m^4+110m^3-161m^2-390m+144}{m-2}=25m^3$$
$+160m^2+159m-72$,

Th. $m=2$: $n=\left(\frac{10.3-12}{12}=\right)\frac{3}{2}$; And $x=2$.

QUEST.

QUEST. CCI. How many ways may 200 be divided into five whole numbers, x, y, u, e, and z, so that $12x+3y+u+\frac{e}{2}+\frac{z}{3}=200$?

$$\left.\begin{array}{ll}\text{Now first} & y+u+e+z=200-x \\ \text{And} & 3y+u+\frac{e}{2}+\frac{z}{3}=200-12x\end{array}\right\}\text{ by transposition.}$$

$$\text{Also}\left\{\begin{array}{l}18y+6u+3e+2z=1200-72x \\ 2y+2u+2e+2z=400-2x \\ 18y+18u+18e+18z=3600-18x\end{array}\right\}\text{ by multiplication.}$$

$$\text{But}\left\{\begin{array}{l}16y+4u+e=800-70x \\ 12u+15e+16z=54x+2400\end{array}\right\}\text{ by subtraction.}$$

$$\text{Therefore - }\left\{\begin{array}{l}x\sqsupset\left(\frac{800}{70}=\right)11\tfrac{3}{7}, \\ x\sqsubset 0:\end{array}\right.$$

$$\text{Secondly}\left\{\begin{array}{l}6u+3e+2z=1200-72x-18y \\ 2u+2e+2z=400-2x-2y\end{array}\right\}\text{ by transp.}$$

$$\text{And - }6u+6e+6z=1200-6x-6y\text{ by multip.}$$

$$\text{But - }\left\{\begin{array}{l}4u+e=800-70x-16y \\ 3e+4z=12y+66x\end{array}\right\}\text{ by subtr.}$$

$$\text{Th. - - - }\left\{\begin{array}{l}y\sqsupset\left(\frac{800-70x}{16}=\right)50-3\tfrac{5}{8}x, \\ y\sqsubset 0;\end{array}\right.$$

From

From which limits it will appear; that when

$$\left.\begin{array}{l} x = 11;\ y \sqsupset 1\tfrac{7}{8} \\ x = 10;\ y \sqsupset 6\tfrac{2}{8} \\ x = 9;\ y \sqsupset 10\tfrac{5}{8} \\ x = 8;\ y \sqsupset 15 \\ x = 7;\ y \sqsupset 19\tfrac{3}{8} \\ x = 6;\ y \sqsupset 23\tfrac{6}{8} \\ x = 5;\ y \sqsupset 28\tfrac{1}{8} \\ x = 4;\ y \sqsupset 32\tfrac{4}{8} \\ x = 3;\ y \sqsupset 36\tfrac{7}{8} \\ x = 2;\ y \sqsupset 41\tfrac{2}{8} \\ x = 1;\ y \sqsupset 45\tfrac{5}{8} \end{array}\right\} \text{Th. } y \text{ will have } \left\{\begin{array}{c} 1 \\ 6 \\ 10 \\ 14 \\ 19 \\ 23 \\ 28 \\ 32 \\ 36 \\ 41 \\ 45 \end{array}\right\} \text{values.}$$

$$\text{Thirdly} \left\{\begin{array}{l} 3t+2z=1200-72x-18y-6u \\ 2t+2z=\ \ 400-\ \ 2x-\ \ 2y-2u \end{array}\right\} \text{by transp.}$$

And by $\frac{3}{2}$: $\quad 3t+3z= 600-3x-3y-3u$, by multipl.

$$\text{But by substr.} \left\{\begin{array}{l} t= 800-70x-16y-4u, \\ z= 3u+15y+69x-600; \end{array}\right.$$

$$\text{Th.} \left\{\begin{array}{l} u \sqsupset \left(\dfrac{800-70x-16y}{4}=\right) 200-\dfrac{35x}{2}-4y, \\[2ex] u \sqsubset \left(\dfrac{600-69x-15y}{3}=\right) 200-23x-5y; \end{array}\right.$$

To apply which, let, First $x=11$; Then $y=1$; $u \sqsupset 3\frac{1}{2}$; $u \sqsubset 0$; and u has 3 values.

Secondly, let $x=10$; Then

$$\left.\begin{array}{ll} y=6; & u \sqsupset 1; \\ y=5; & u \sqsupset 5; \\ y=4; & u \sqsupset 9; \\ \&c. & \&c. \end{array}\right\} u \sqsubset 0; \text{ and } u \text{ has } \left\{\begin{array}{c} 0 \\ 4 \\ 8 \\ \&c. \end{array}\right\} \text{ values;}$$

Put V= the number of values of u,

Then V=(6 terms of 0+4+8, &c. =) 60. See qu. 3d.

Thirdly, let $x=9$; Then

$$\left.\begin{array}{ll} y=10; & u \sqsupset 2\frac{1}{2} \\ y=9; & u \sqsupset 6\frac{1}{2} \\ y=8; & u \sqsupset 10\frac{1}{2} \\ \&c. & \&c. \end{array}\right\} u \sqsubset 0; \text{ and } u \text{ has } \left\{\begin{array}{c} 2 \\ 6 \\ 10 \\ \&c. \end{array}\right\} \text{ values;}$$

Th. V= (10 terms of 2+6+10, &c. =) 200.

Fourthly, let $x=8$; Then

$y=14$; $u \sqsupset 4$; $u \sqsubset -54$; and u has 3 values,
$y=13$; $u \sqsupset 8$; $u \sqsubset -49$; and u has 7 values,
&c. &c. &c. &c.
$y=4$; $u \sqsupset 44$; $u \sqsubset -4$; and u has 43 values,

---

$y=3$; $u \sqsupset 48$; $u \sqsubset 1$; and u has 46 values,
$y=2$; $u \sqsupset 52$; $u \sqsubset 6$; and u has 45 values,
$y=1$; $u \sqsupset 56$; $u \sqsubset 11$; and u has 44 values,

$$\text{Th. } V=\left\{\begin{array}{l} 11 \text{ terms of } 3+7+11, \&c. \\ +\ 3 \text{ terms of } 46+45+44 \end{array}\right\}=388.$$

Fifthly

Fifthly, let $x=7$; Then

$y=19$; $u \sqsupset 1\frac{1}{2}$; $u \sqsubset -56$; and u has 1 value,
&c. &c. &c. &c.
$y=8$; $u \sqsupset 45\frac{1}{2}$; $u \sqsubset -1$; and u has 45 values,

---

$y=7$; $u \sqsupset 49\frac{1}{2}$; $u \sqsubset 4$; and u has 45 values,
$y=6$; $u \sqsupset 53\frac{1}{2}$; $u \sqsubset 9$; and u has 44 values,
&c. &c. &c. &c.

$$\text{Th. } V=\left\{\begin{array}{l} 12 \text{ terms of } 1+5+9, \text{ \&c.} \\ +\ 7 \text{ terms of } 45+44+43, \text{ \&c.} \end{array}\right\}=750.$$

Sixthly, let $x=6$; Then

$y=23$; $u \sqsupset 3$; $u \sqsubset -53$; and u has 2 values,
&c. &c. &c. &c.
$y=13$; $u \sqsupset 43$; $u \sqsubset -3$; and u has 42 values,

---

$y=12$; $u \sqsupset 47$; $u \sqsubset 2$; and u has 44 values,
&c. &c. &c. &c.

$$\text{Th. } V=\left\{\begin{array}{l} 11 \text{ terms of } 2+6+10, \text{ \&c.} \\ +12 \text{ terms of } 44+43+42, \text{ \&c.} \end{array}\right\}=704.$$

Seventhly, let $x=5$; Then

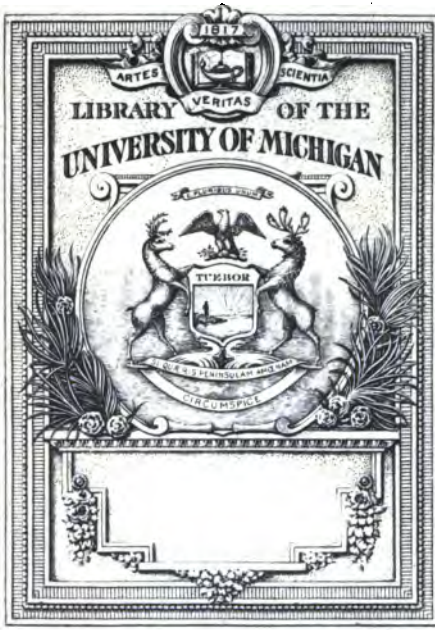
$y=28$; $u \sqsupset \frac{1}{2}$; $u \sqsubset -55$; and u has 0 value,
&c. &c. &c. &c.
$y=17$; $u \sqsupset 44\frac{1}{2}$; $u \sqsubset 0$; and u has 44 values,

---

$y=16$; $u \sqsupset 48\frac{1}{2}$; $u \sqsubset 5$; and u has 43 values,
&c. &c. &c. &c.

$$\text{Th. } V=\left\{\begin{array}{l} 12 \text{ terms of } 0+4+8, \text{ \&c.} \\ +16 \text{ terms of } 43+42+41, \text{ \&c.} \end{array}\right\}=832.$$

Eghthly,


1817
ARTES
SCIENTIA
VERITAS
LIBRARY OF THE
UNIVERSITY OF MICHIGAN
TUEBOR
CIRCUMSPICE